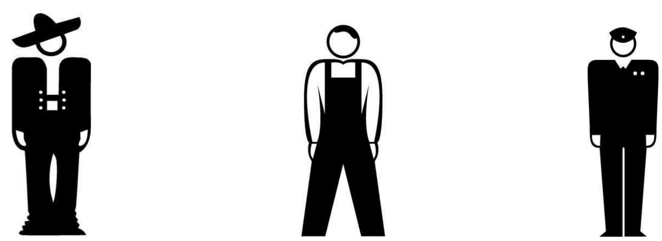
Axioms

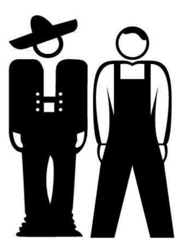
Axioms are statements about quantities in general that are accepted as true without proof. (Things we may take for granted.)

1. ***Quantities that are equal to the same quantity or to equal quantities are equal to each other.***

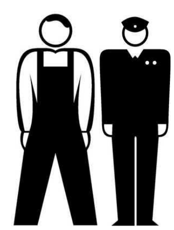
   For example, if $a = b$, and $b = c$, then we know that $a = c$ as well. Consider three people,



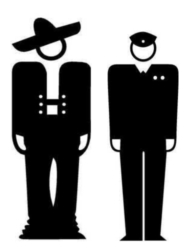
Hector José Ramón



If Hector and José are the same height,



and José and Ramón are the same height,



then we know Hector and Ramón must be the same height, without measuring them.

2. ***If equals are added to equals, the sums are equal.***

If we start with two equal things: $a = b$

and we add the same amount (5) to each one: $a + 5$, $b + 5$

then the results will still be equal: $a + 5 = b + 5$.

3. ***If equals are subtracted from equals, the remainders are equal.***

This axiom is almost the same as the previous one. It means that if we have two equal things $(a = b)$ and we <u>subtract</u> the same amount from each, then the resulting things will still be equal $(a - 4 = b - 4)$.

4. ***If equals are multiplied by equals, the products are equal.***

If two things are equal: $a = b$,

and we <u>multiply</u> them by the same number (7): $7a$, $7b$

their products will equal each other: $7a = 7b$.

5. ***If equals are divided by equals (not zero), the quotients are equal.***

If we have equal quantities: $a = b$

and we <u>divide</u> them by the same number (2): $a \div 2$, $b \div 2$

the resulting quotients will also be equal: $a \div 2 = b \div 2$.

Note: We have to say "(not zero)" because dividing by zero is impossible to do.

6. ***The whole of a quantity is equal to the sum of all of its parts, and if all the values are positive, the whole is greater than any of its parts.***

Consider this example:

$= \$\ 0.41$

Here, the whole is the total value of these coins ($0.41).

The whole value is equal to the sum of all the parts, or the value of each individual coin added together $(0.41 = 0.25 + 0.05 + 0.10 + 0.01)$.

The whole value ($0.41) is always more than the value of any group of one, two or three coins.

7. ***A quantity may be substituted for its equal in any expression.***

This axiom says that if two things are equal, say $x = y$, then in an expression, say, $3x + 4$, we can switch the *x* for *y*. That is to say,

Given: $x = y$, and an expression, $3x + 4$, we know,

$$3x + 4 = 3y + 4$$

8. ***If equals are added or subtracted from unequals or if unequals are multiplied or divided by the same positive number, the results are unequal in that same order.***

This axiom is the same as Axioms 2, 3, and 4, except that now we start with two things that are not equal. Say, $12 < 36$. This axiom states that if we add, subtract, multiply, or divide each by the same positive number, the inequality sign will stay in the same direction.

Addition: $12 + 3 < 36 + 3$

$15 < 39$ True.

Subtraction: $12 - 7 < 36 - 7$

$5 < 29$ True.

Multiplication: $12 \times 2 < 36 \times 2$

$24 < 72$ True.

Division: $12 \div 6 < 36 \div 6$

$2 < 6$ True.

9. ***If unequals are subtracted from equals, the results are unequal in the opposite order.***

Take two equal things: $10 = 10$

and two unequal things: $6 > 3$

and subtract them in the order: $10 - 6$, *then* $10 - 3$

the differences will have a switched inequality sign. $4 < 7$

10. ***If unequals are added to unequals in the same order, the results are unequal in the same order.***

Take two things that aren't equal. $4 < 7$

Take two more unequal things, using the same sign. $5 < 6$

Then add each left number, and each right number. $4 + 5$ $7 + 6$

The sums will always have the same sign. $9 < 13$

TIP: *To help yourself understand the axioms, particularly axioms eight, nine, and ten, invent your own examples, and test them out yourself.*

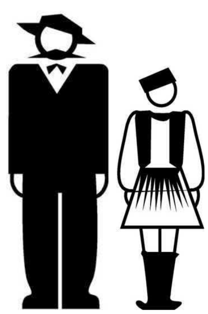
11. ***If the first of three quantities is greater than the second and the second is greater than the third, then the first is greater than the third.***

This axiom is saying that if we have three numbers, *a*, *b*, and *c*, and all we know is that $a > b$, and $b > c$, then we also know that $a > c$. This is similar to Axiom 1, but with inequalities. Observe:

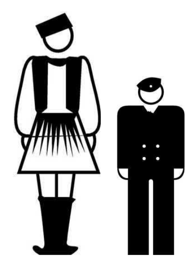
Consider three people:

Jim Marta Steve

And we only know that,

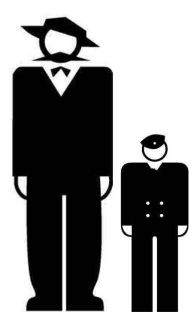


Jim is taller than Marta,



and Marta is taller than Steve.

Without taking another measurement, we also know that,



Jim is taller than Steve.

12. ***Like powers or like positive roots of equals are equal.***

This just means that if we have two equal things, say $a = b$, then we also know,

$$a^2 = b^2 \qquad \sqrt{a} = \sqrt{b}$$

$$a^3 = b^3 \qquad \sqrt[3]{a} = \sqrt[3]{b}$$

$$a^4 = b^4 \qquad \sqrt[4]{a} = \sqrt[4]{b}$$

$$a^5 = b^5 \qquad \sqrt[5]{a} = \sqrt[5]{b}$$

$$\vdots \qquad \vdots$$

$$a^n = b^n \qquad \sqrt[n]{a} = \sqrt[n]{b}$$

Powers of each are equal, and roots of each are equal.

End of Axioms