Student Name:	Contact Person Name:
Date: ____________	Phone Number: ____________

Texas Assessment
of Knowledge and Skills

Exit Level Math Review

Lesson 4

Parent Functions, Domain, & Range

TAKS Objective 2 – Demonstrate an understanding of the properties and attributes of functions

Lesson Objectives:

- Identify a linear and quadratic parent function based on a graph
- Find the domain and range of a function

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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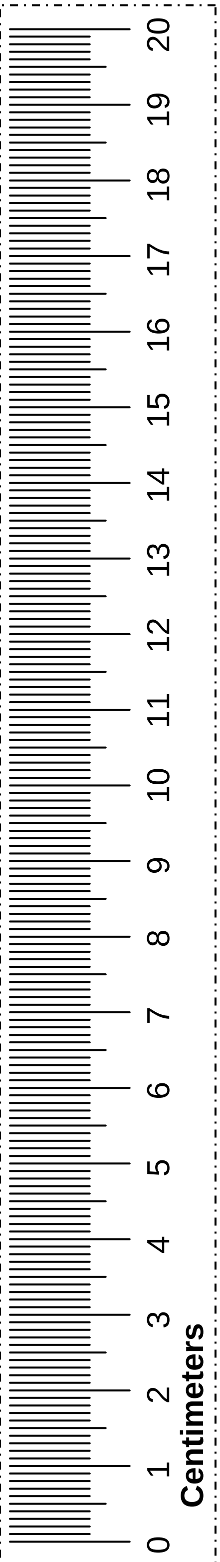
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

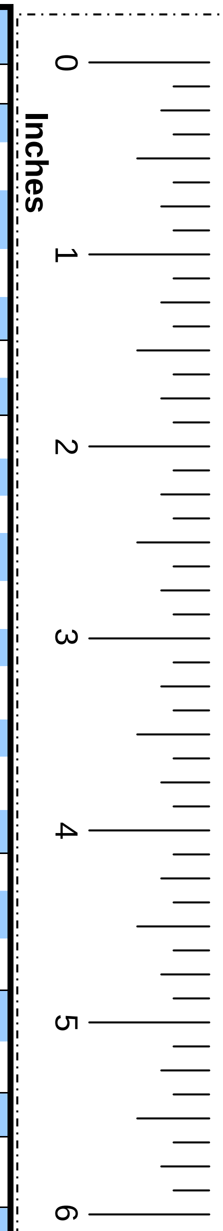
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
Triangles	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

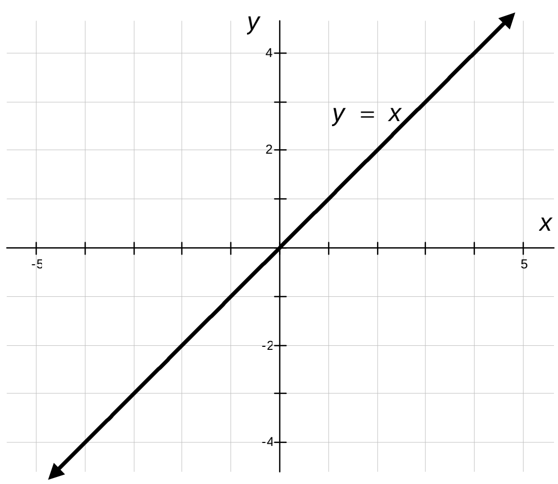


You will be successful on the exam if you can: (1) graph a function, and (2) create a function based on a graph. To continue understanding the relationship between functions and graphs, you must understand **parent functions**.

A **parent function** is the most basic way to represent a certain type of function.

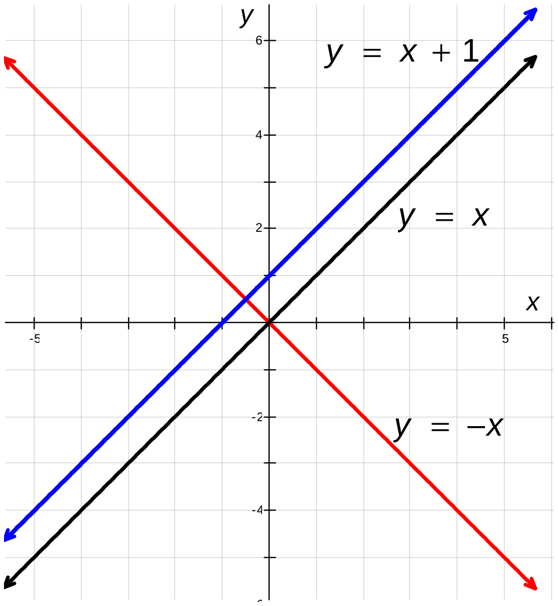
The **linear parent function** is $y = x$.

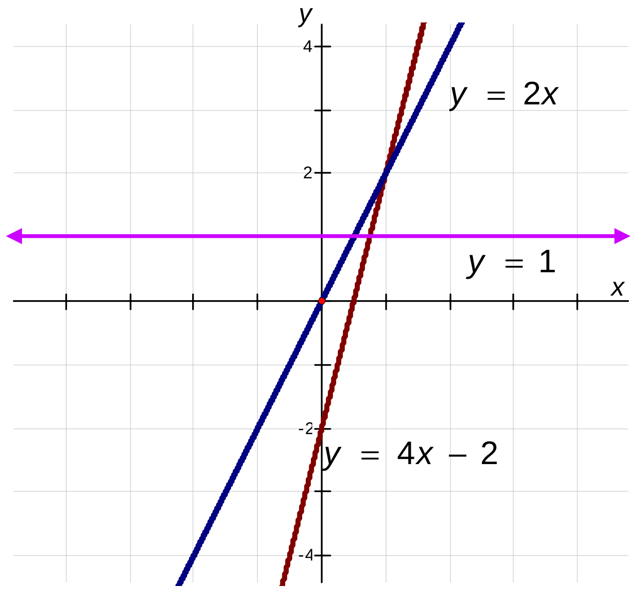
As its name suggests, the linear parent function $y = x$ is the most basic form of a line.



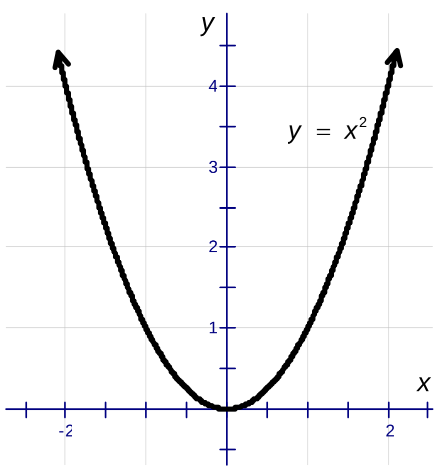
All graphs of lines are some variation of the linear parent function, $y = x$. In fact, the equation of every line can be written in the form $y = mx + b$. We will learn more behind the meaning of m and b later on.

Observe the graphs of other lines and note how their equations all contain some form of the linear parent function, $y = x$.





Linear Functions

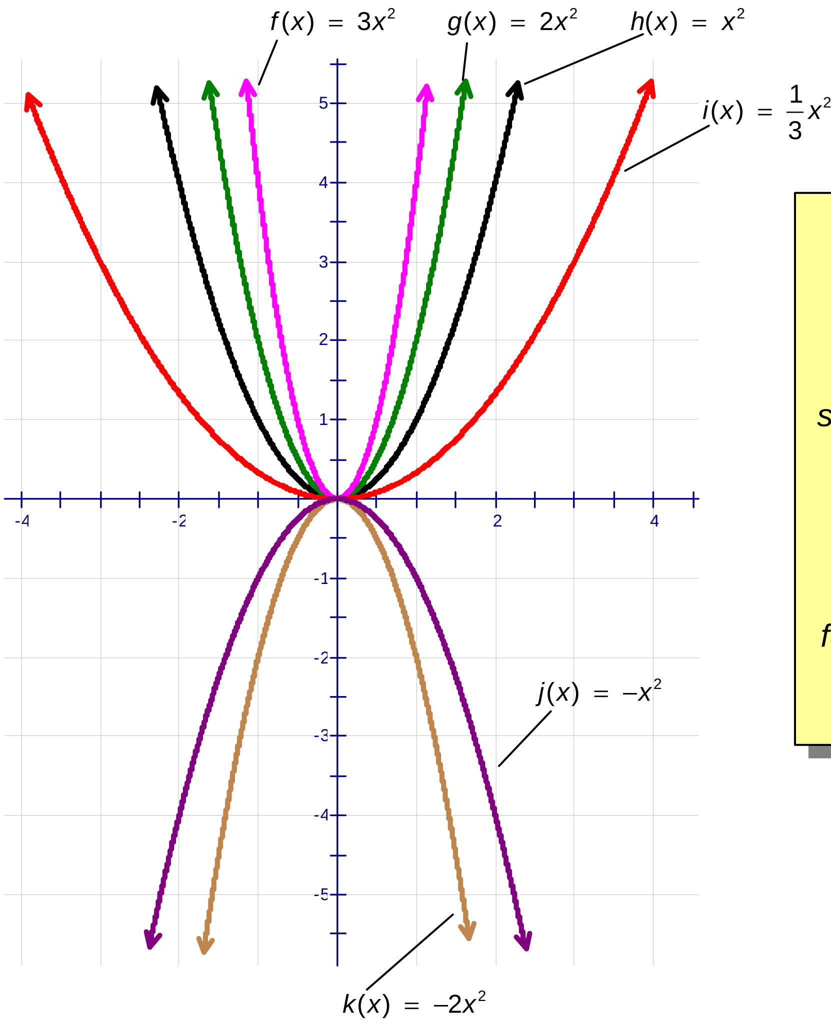


The **quadratic parent function** is $y = x^2$.

The graph of a quadratic function is called a **parabola**. The equations of all graphs of parabolas are some variation of the quadratic (or parabolic) parent function, $y = x^2$.

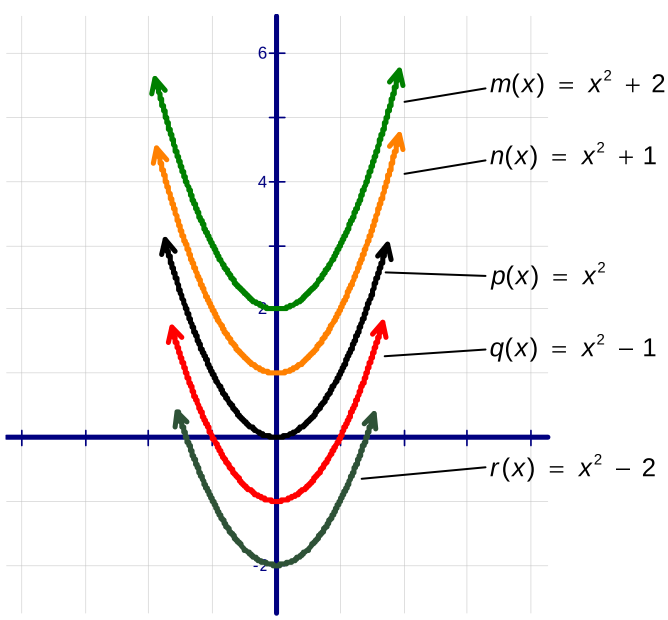
In fact, the equations of all parabolas can be written in the form $y = ax^2 + bx + c$. As with lines, we will learn more about parabolas later on.

Observe how each graph is a parabola and each equation contains the quadratic parent function, $y = x^2$



FACT

When different functions are graphed on the same set of axes, it is common to write their equations with function notation to distinguish them. Each function is given a different letter (f, h, g).

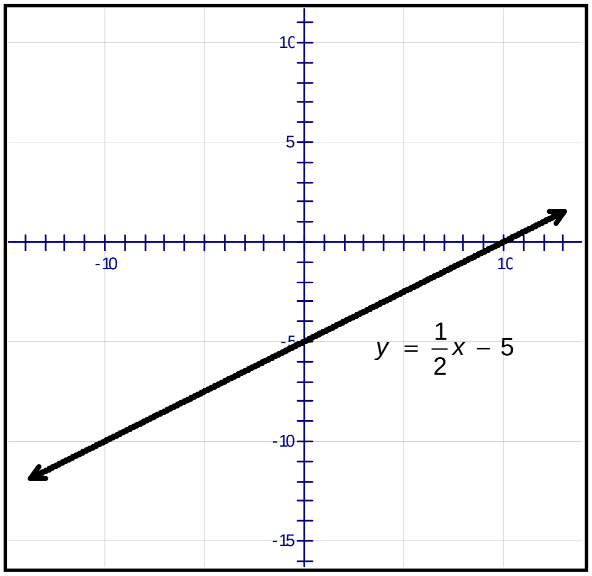


Problem Solving Tip

Make a guess about how the graph of a function changes when different values add to or multiply the parent function.

Example

Which equation is the parent function of the graph shown below?



A $y = x$

B $y = \frac{1}{2}x - 5$

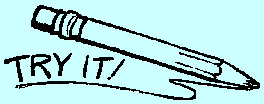
C $y = x^2$

D $y = \sqrt{x}$

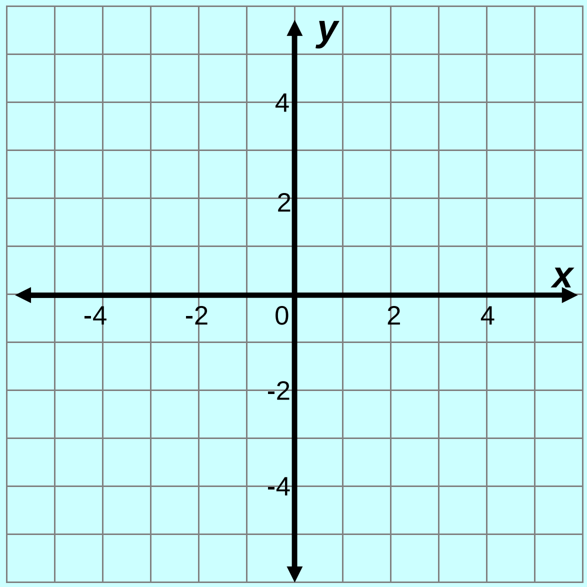
Solution

Since this is a linear function, the answer is the linear parent function, $y = x$, or choice **A**.

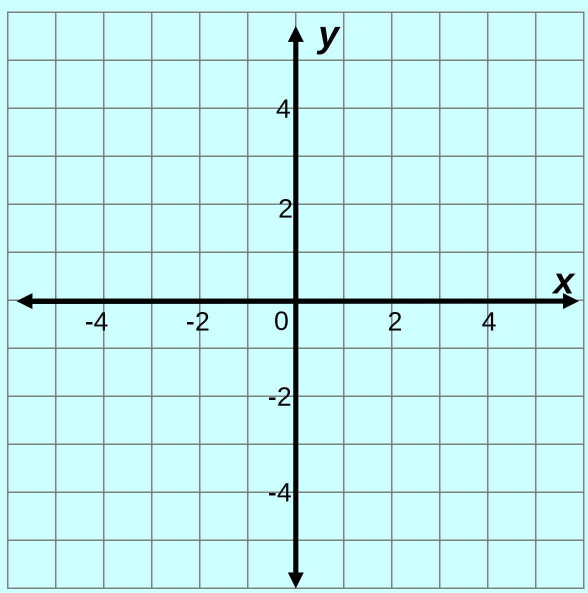
Although this answer comes naturally from reading the lesson, on the test you will face a large variety of questions. The key to answering this question is to know the definition of "parent function." Notice the answer is not choice **B**. $y = \frac{1}{2}x - 5$ is the exact equation of the line in the graph. The term "parent function" is why the answer is choice **A**, $y = x$, the most basic equation of a line.

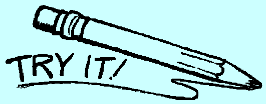


1) Sketch the linear parent function on the axes below and label its equation.



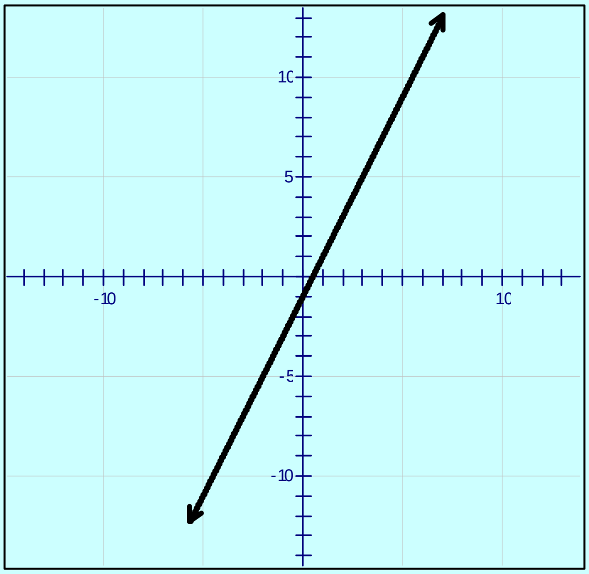
2) Sketch the quadratic parent function below and label its equation.



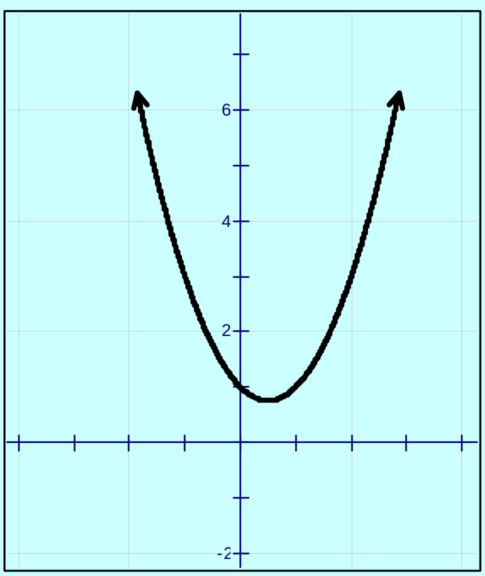


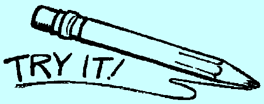
Write the equation of the parent function next to each graph.

3)

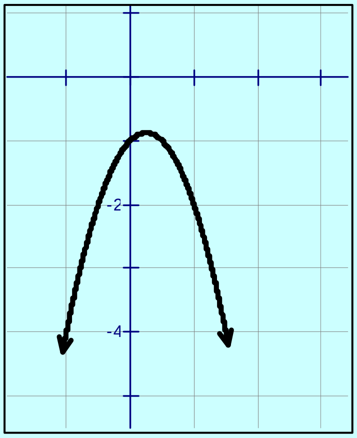


4)





5) Which equation is the parent function of the graph shown below?

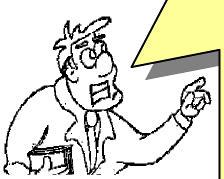


A $x - y$

B $y = x^2$

C $y = x^3$

D $y = |x|$



FACT

*Linear functions are **degree 1**. This means that the variable is raised to the first power (exponent of 1).*

$$y = x^1$$

*Quadratic functions are **degree 2**. The highest degree variable is raised to the second power (squared).*

$$y = x^2$$

Cubic functions are degree 3. Can you guess what the cubic parent function looks like?

The **domain** is the set of all *x*-values for which a function is defined.

The **range** is the set of all *y*-values for which a function is defined.

Think Back

x-values are values of the independent variable.

y-values are values of the dependent variable.

Example

Identify the domain and range of the function below.

$$\{(0,5),(1,7),(2,9),(3,11),(4,13)\}$$

Solution

Domain is the set of all *x*-coordinates. {0, 1, 2, 3, 4}

Range is the set of all *y*-coordinates. {5, 7, 9, 11, 13}

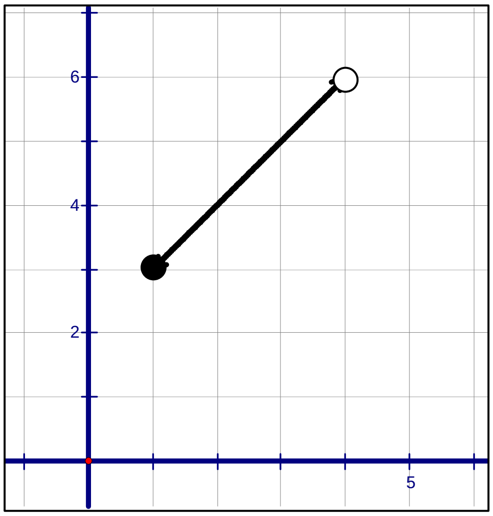
FACT

When listing elements of a ***set****, use curly brackets.*

"{ }"

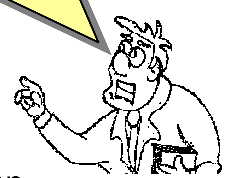
Example

Identify the domain and range of the function below using inequalities.



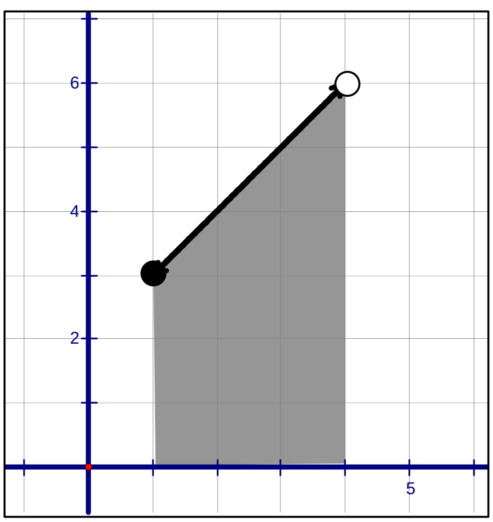
FACT

An open circle on a graph means that point is not part of the solution, domain, or range.



Solution

First we will identify the domain, or the *x*-values of the function.



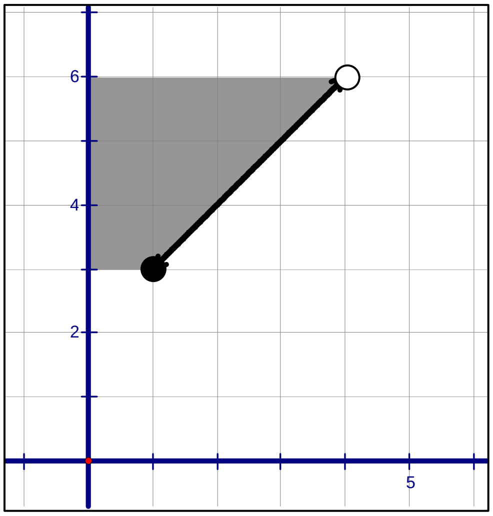
The *x*-values are from 1 to 4, not including 4. With inequalities, we show the domain as $1 \leq x < 4$.

FACT

Inequality symbols are:

- $\leq$ *less than or equal to*
- $<$ *less than*
- $\geq$ *greater than or equal to*
- $>$ *greater than*

Next, we find the range, or the *y*-values of the function.



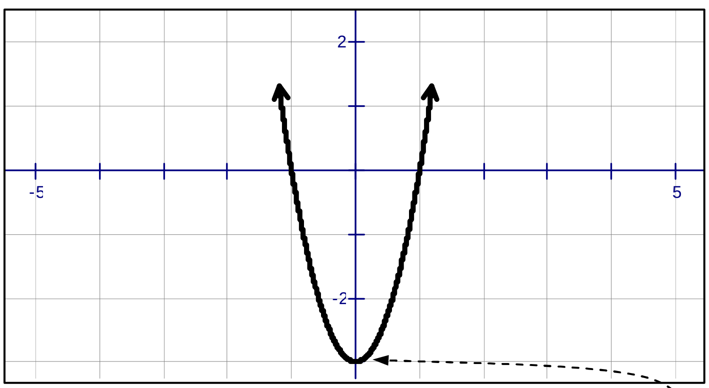
The *y*-values are from 3 to 6, not including 6. Using inequalities, the range is $3 \le y < 6$.

including

not including

Example

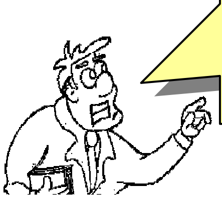
Identify the domain and range of the function below using inequalities.



Solution

There is no restriction on the domain of the function. The arrows show that the graph can be extended forever, covering every *x*-value. Therefore, the domain is "**all real numbers**."

The range, however, has a lowest value, -3. The range is $y \ge -3$.



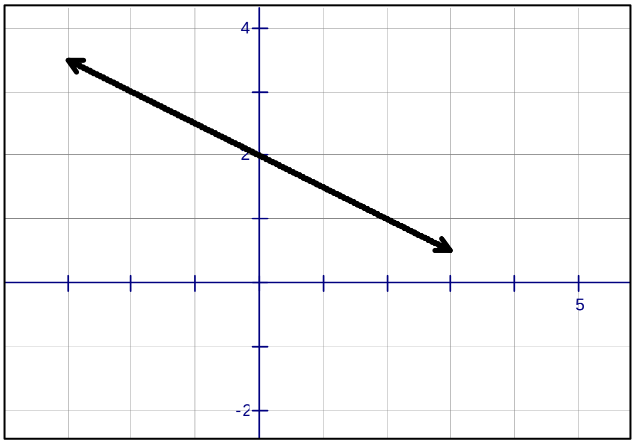
FACT

*If a function can extend forever left and right, its <u>domain</u> is **all reals**.*

*If a function can extend forever up and down, its <u>range</u> is **all reals**.*

Example

Identify the domain and range of the function below.



Solution

The domain is all real numbers.

The range is all reals.

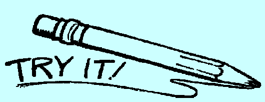
When the domain or range is the set of all real numbers, you will not see it written in inequality form. Instead, you may see **set notation**:

$D = \{x : x \in \mathbb{R}\}$

"Domain is the set of every *x*-value such that *x* is a real number."

$R = \{y : y \in \mathbb{R}\}$

"Range is the set of every *y*-value such that *y* is a real number."

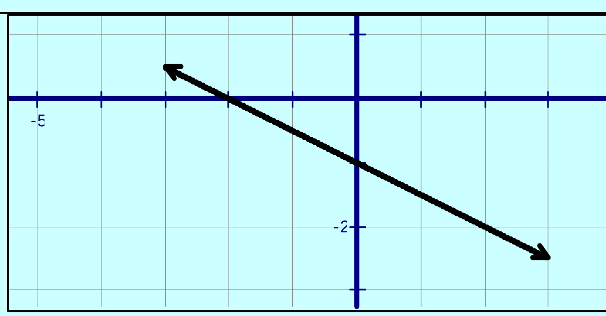


Write the domain and range of each function. Use inequalities or set notation.

6) $\{(-1,3),(1,1),(3,-1),(4,-2)\}$

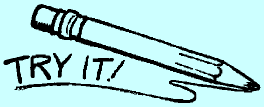
Domain: Range:

7)



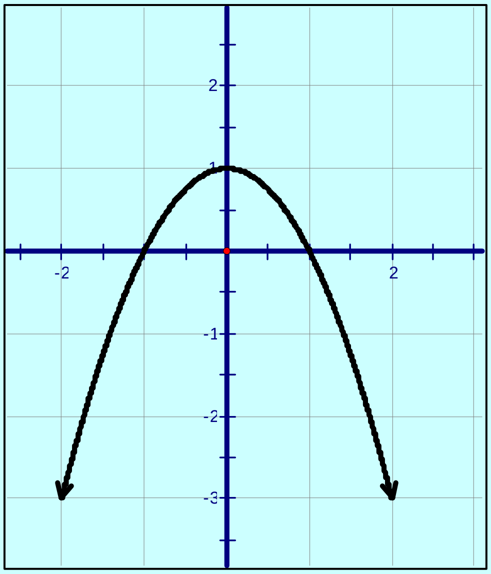
Domain:

Range:



Identify the domain and range of the function below using inequalities.

8)



Domain:

Range:

Example

Rachel is buying movie tickets that cost $12 each. There is a limit of 4 tickets per person. Let *x* represent the number of tickets Rachel buys. Let *y* be the number of dollars she pays for the tickets. This function can be represented by $y = 12x$. What is a reasonable domain for this function?

A $\{x : x \in \mathbb{R}\}$

B $\{y : 0 \le y \le 4\}$

C $\{0, 1, 2, 3, 4\}$

D $\{12, 24, 36, 48\}$

Solution

(1) Read the question. What is it asking you to find?

We need to find the <u>domain</u> of the function.

(2) Recall facts about the domain.

Domain is the set of x-values in a function.

(3) Decide which information is important to answer the question. Re-read the question if you need to.

"Let x represent the number of tickets Rachel buys.

There is a limit of 4 tickets per person."

(4) Read the answer choices. Eliminate answers that cannot be true.

A $\{x : x \in \mathbb{R}\}$

~~**B**~~ $\{y : 0 \le y \le 4\}$

C $\{0, 1, 2, 3, 4\}$

D $\{12, 24, 36, 48\}$

*Choice **B** is false, since domain is the set of <u>x-values</u>.*

(5) Use information from steps 2 and 3 to select the correct answer.

*Domain is all x-values. In this problem, x is the number of movie tickets. We are also told Rachel can buy a maximum of 4 movie tickets. (Now we can eliminate choice **A**.) This means she can buy 0, 1, 2, 3, or 4 movie tickets. (This eliminates choice **D** as well.) Aha! That means the answer is choice **C**.*

Problem Solving Tip

In a multiple-choice question, cross out answers that you know are false.

Example

A rectangular garden is being installed in an office park. The garden is to be 5 feet longer than it is wide. The garden must be at least 15 feet wide, but no more than 28 feet wide. The function $f(x) = x^2 + 5x$ describes the area of the courtyard in terms of its width in feet, x. What is a reasonable range for this function?

A $15 \le x \le 28$

B $15 \le f(x) \le 28$

C $f(x) \le 924$

D $300 \le f(x) \le 924$

Solution

<u>The Direct Method</u>:

The range of the function is every value in between and including the minimum and maximum area of the garden.

The minimum area occurs when $x = 15$. We find this by finding $f(15)$

$$\begin{aligned} f(x) &= x^2 + 5x \\ f(15) &= (15)^2 + 5(15) \\ &= 225 + 5(15) \\ &= 225 + 75 \\ &= 300 \end{aligned}$$

The maximum area occurs when $x = 28$. Therefore, find $f(28)$.

$$\begin{aligned} f(x) &= x^2 + 5x \\ f(28) &= (28)^2 + 5(28) \\ &= 784 + 5(28) \\ &= 784 + 140 \\ &= 924 \end{aligned}$$

The range is everything between 300 and 924, including these values. We write this using inequalities. The range is $300 \le f(x) \le 924$

The answer is choice **D**.

The Indirect Method:

Eliminate false answer choices.

A $15 \le x \le 28$

*The question asks us to find range, and x-values correspond to the domain. Choice **A** must be false.*

B $15 \le f(x) \le 28$

*This may be correct, since it uses f(x) as the range. However, in the question, we are given the domain, x, to be the width of the garden, and these values, 15 and 28, to be the width in feet. These are not the correct numbers, so choice **B** is false.*

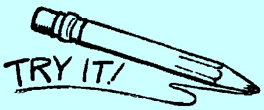
C $f(x) \le 924$

This choice is telling us the range, or the area of the garden can be any value less than or equal to 924 square feet. What about -7? While it is true that $-7 \le 924$*, we know that area can never be negative. Choice **C** must be false.*

This leaves only choice **D** as the correct answer.

Problem Solving Tip

Think of a <u>counterexample</u> to show that something is false. Here, we used -7 to show something wrong about the answer choice.



9) Yolanda is buying steel rod. Each foot of rod is 15 pounds. She has enough money to buy up to 6 feet of rod. Let *x* be the number of feet of steel rod Yolanda buys. The total weight of rod Yolanda buys is represented by $f(x) = 15x$. What is a reasonable domain for this function?

A $0 \le f(x) \le 15$

B $0 \le f(x) \le 90$

C $0 \le x \le 6$

D $\{0, 1, 2, 3, 4, 5, 6\}$

10) A rectangular parking lot is being constructed outside a museum. The parking lot must be two times wider than it is long. The parking lot must be at least 25 m wide, but no more than 50 m wide. The function $f(x) = 2x^2$ describes the area of the parking lot in terms of its length in meters, *x*. What is a reasonable range for this function?

A $1{,}250 \le f(x) \le 5{,}000$

B $1{,}250 \le x \le 5{,}000$

C $25 \le f(x) \le 50$

D $25 \le x \le 50$

Review

Know these concepts:

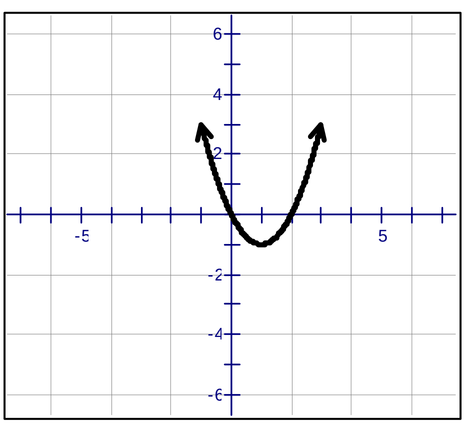
1. A **parent function** is the most basic equation of a type of a graph.
   - **a.** The linear parent function is $y = x$. All graphs with this parent function are straight lines.
   - **b.** The quadratic parent function is $y = x^2$. All graphs with this parent function are parabolas.
2. **Domain** and **Range**
   - **a.** Domain = *x*-values = independent variable
   - **b.** Range = *y*-values = dependent variable

Practice Problems

Lesson 4

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 4.

1) Which equation is the parent function of the graph below?



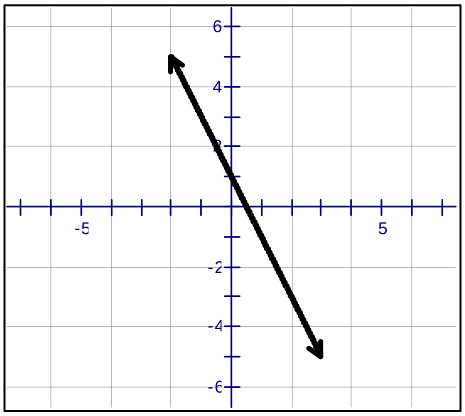
A $y = 2x$

B $y = x^2$

C $y = x$

D $y = \pm\sqrt{x}$

2) Which equation is the parent function of the graph shown below?



A $y = x^2$

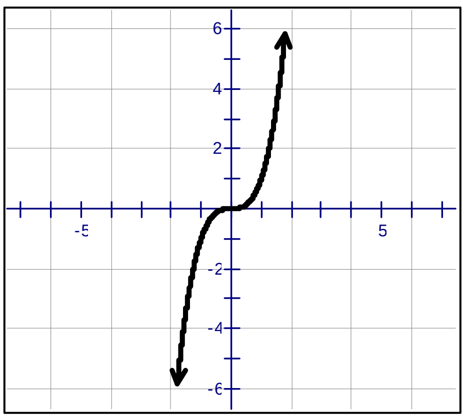
B $y = \frac{1}{2}x$

C $y = x$

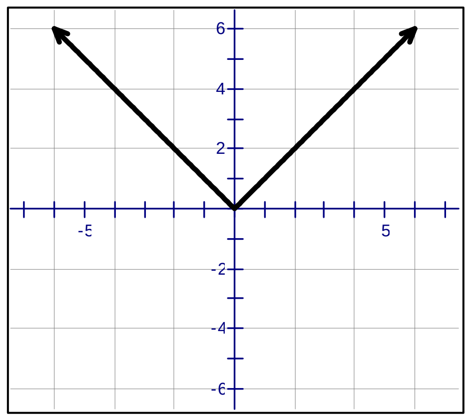
D $y = |x|$

3) Which graph has the parent function $y = x^2$?

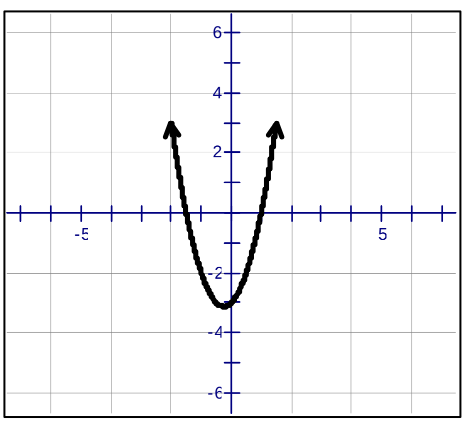
A



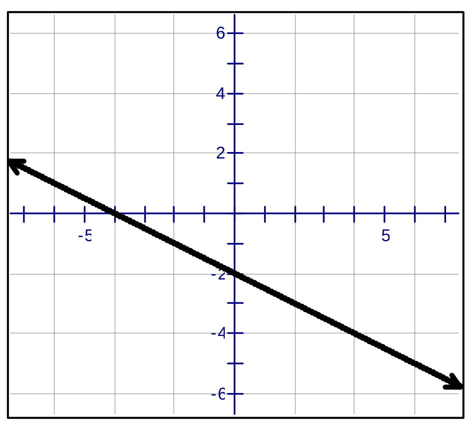
B



C



D



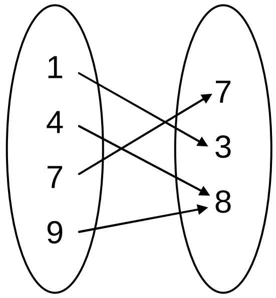
Identify the domain and range of each function below. Use either inequality or set notation as appropriate.

4) $\{(-6,0), (-4,1), (0,1), (3,-8)\}$

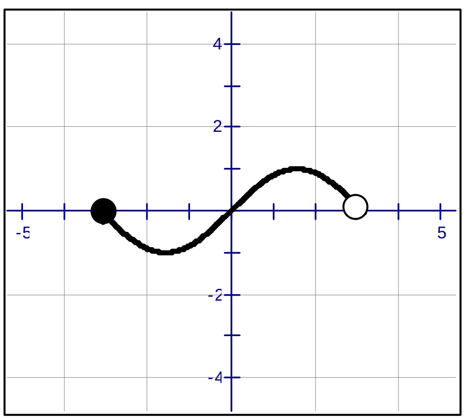
5)

x	y
0	1
1	3
3	0
5	3

6)



7) Mr. Wilson asked his students to identify the domain represented by the function graphed below. Which of the following student responses is correct?



A $-3 \leq x < 3$

B $-1 \leq x \leq 1$

C $-3 \leq x \leq 3$

D $-1 \leq y \leq 1$

8) Dorothy is planting a garden at the corner of her lawn in the shape of an isosceles right triangle. She wants the shorter, equal sides of the garden to measure at least 10 feet each, but no more than 15 feet each. The function that shows the area of her garden is $g(s) = \frac{1}{2}s^2$, where s represents the length of one of the equal sides in feet. What is a reasonable range for this function?

A $10 \leq g(s) \leq 15$

B $15 \leq g(s) \leq 10$

C $50 \leq g(s) \leq 112.5$

D $112.5 \leq g(s) \leq 50$

9) Bob is buying bales of hay to feed his horses for \$4 each. There is a limit of 10 bales of hay per customer. Let x represent the number of bales of hay Bob buys. The function to represent how much Bob spends on hay is represented by $h(x) = 4x$. Assuming each customer must buy whole bales of hay, what is a reasonable domain for this function?

A $\{0,1,2,3,4,5,6,7,8,9,10\}$

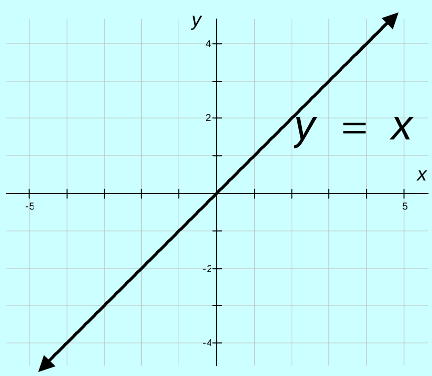
B $\{4,8,12,16,20,24,28,32,36,40\}$

C $\{x : x \in \mathbb{R}\}$

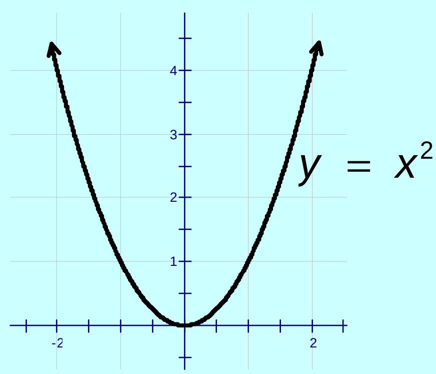
D $0 \leq x \leq 10$

ANSWERS TO TRY IT

1)



2)



3) $y = x$ **4)** $y = x^2$ **5)** **B**

6) $D = \{-1,1,3,4\}$ $R = \{3,1,-1,-2\}$ (numbers may be in any order)

7) $D = \{x : x \in \mathbb{R}\}$ $R = \{y : y \in \mathbb{R}\}$

8) $D = \{x : x \in \mathbb{R}\}$ Range: $y \leq 1$

9) **C** **10)** **A**

End of Lesson 4