Student Name: ______________________________

Date: ______________________________

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Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 6

Arithmetic Properties and Polynomials

TAKS Objective 2 – Demonstrate an understanding of the properties and attributes of functions

Lesson Objectives:

- Explore how the commutative, associative, and distributive properties apply to algebra
- Simplify polynomials
- Solve linear equations algebraically

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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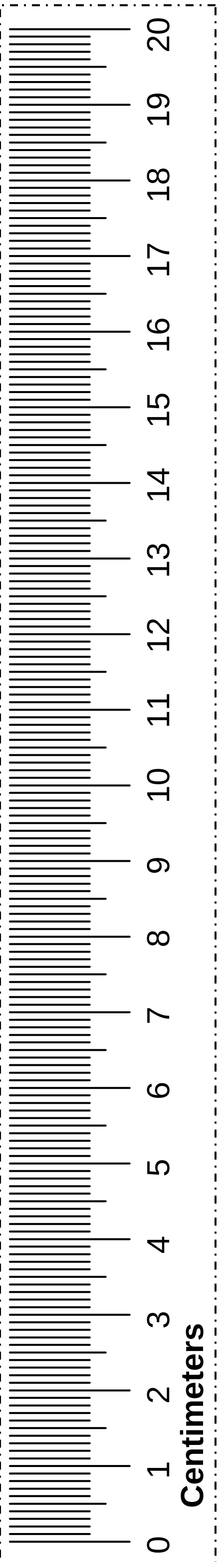


TAKS Mathematics Chart

Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

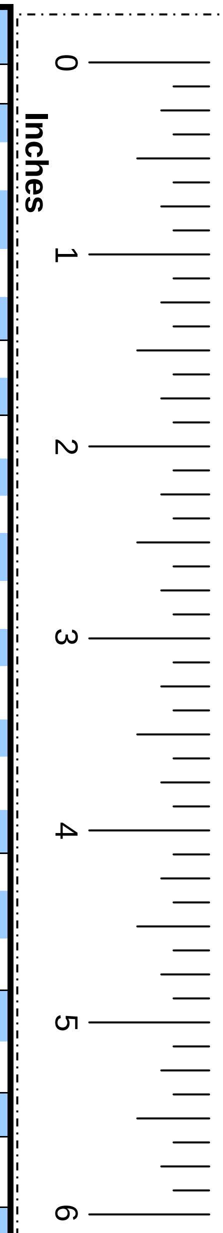
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
Triangles	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



When observing the equation form of a function, we usually set the dependent variable, *y*, equal to an **algebraic expression** involving the independent variable, *x*.

> **Algebraic expressions** are made up of constants and variables. They are connected by addition, subtraction, multiplication, and division.

Algebraic Expression	Constant(s)	Variable(s)
$3x$	3	*x*
$5y - 6$	5 and 6	*y*
$\frac{(3s + 1)}{4}$	3, 1, and 4	*s*
$4b + 2c$	4 and 2	*b and c*

Expressions are made of terms. Each of the terms is separated by addition or subtraction. A single term is called a **monomial**, and several terms combined by addition and/or subtraction are called a **polynomial**.

Monomials:

$$x, \quad 8, \quad 5x, \quad \frac{2}{3}xy, \quad 3a^2bc^3$$

Monomials have only one term.

Polynomials:

$$x + 2, \quad 8 - y, \quad 5x^2 - x + 3, \quad 3x^2 + \frac{2}{3}xy - 5y^2$$

2 terms, 2 terms, 3 terms, 3 terms

Polynomials can have any number of terms.

The algebraic expressions listed above are all in simplest form. However, sometimes we need to combine terms in order to simplify an algebraic expression.

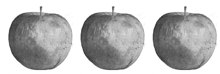
Example

Simplify the expression $3a + 2a$.

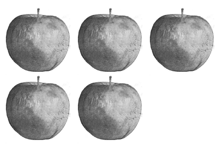
Solution

Let's use a picture to help us. Suppose that the variable *a* is an object.

Let  $= a$

This means that we can show $3a$ as 

Now we'll add $3a + 2a$

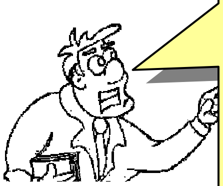


$$\begin{array}{r} 3a \\ \underline{+2a} \\ 5a \end{array}$$

If we had 3 apples and added 2 apples, we would not say "We have three apples and two apples." We would simplify things and say "We have five apples." We simplify algebraic expressions the same way – by combining the **like-terms**.

Like terms are terms with the exact same combination of variables and exponents. Like terms do not need to have the same **coefficient**.

Like Terms	NOT Like Terms
$4x$, $6x$, x, $-9x$	$4x$, $4y$
$9a^2b$, $4a^2b$, $2a^2b$	$3a^2b$, $3ab$, $3ab^2$
1, 7, 9, 12	4, $3n$



FACT

*The **coefficient** is the number in front of the variable. It multiplies the variable(s). 4 is the coefficient in the term $4x^2$. 1 is the coefficient of x.*

Example

Simplify the expression $2x^2 + 3 - 3x^2$

Solution

To simplify this expression, you must combine the like terms. The best way to do that is by drawing different shapes around the like terms. Make sure you include the sign in front of each term.

$$2x^2 + 3 - 3x^2$$

Now we can combine the like terms. $2x^2 - 3x^2 = -1x^2 = -x^2$

$$2x^2 + 3 - 3x^2$$

$$-x^2 + 3$$

Since no other terms can be combined, the answer is $-x^2 + 3$.

Notice that the exponent did not change in the term with the x^2. Adding and subtracting like terms is similar to adding and subtracting fractions. The variables and exponents need to be the same, just as the fraction denominators need to be the same.

The commutative property allows us to rearrange the order of the terms, as long as we keep the signs in front of each term the same. For instance, $-x^2 + 3 = 3 - x^2$.

FACT

The commutative property allows us to add terms in any order. The associative property allows us to group terms in a sum.

Example

Simplify the expression $2x(x - 2) - (x + 2) + 9$.

Solution

When an expression is written next to parentheses, it means we need to multiply. To multiply terms into parentheses, use the distributive property. A negative sign in front of parentheses means to multiply each term inside the parentheses by –1.

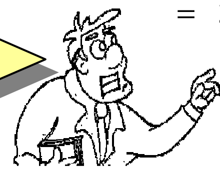
$$2x(x - 2) - (2 + x) + 9 = 2x(x - 2) + -1(2 + x) + 9$$

$$= 2x^2 - 4x - 2 - x + 9$$

FACT

$x \cdot x = x^2$ *just as*

$2 \cdot 2 = 2^2$

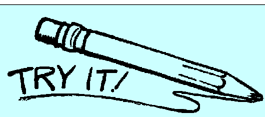


Now that we have removed the parentheses, we can combine the like-terms.

$$2x^2 - 4x - 2 - x + 9$$

$$= 2x^2 - 5x + 7$$

None of the other terms can be combined.



1) A triangle has a base that is 6 inches more than its height. The area of the triangle is represented by $A = \frac{1}{2}(h + 6)h$. Which formula is equivalent to the area formula above?

A $A = \frac{1}{2}h + 6h$

B $A = \frac{1}{2}h^2 + 6h$

C $A = \frac{1}{2}h + 3$

D $A = \frac{1}{2}h^2 + 3h$

Example

Simplify the expression $(x - 2)(x + 4)$.

Solution

To simplify this we need to distribute the first parenthesis into the second one. We will do this using two methods.

Method 1: FOIL

The foil method only works when multiplying two **binomials**.

FACT

There are three special types of polynomials: monomials, binomials, and trinomials. If a polynomial has more than three terms, we simply call it a polynomial. Notice that each word has a different prefix. Mono- means one, bi- means two, and tri- means three. The prefix poly- means many.

The letters F-O-I-L stand for the terms within two binomials.

"F" stands for first. Multiply the first terms in each of the binomials.

$$(x - 2)(x + 4)$$
$$x \cdot x$$
$$x^2$$

"O" stands for outer. Multiply the outer two terms in the set of binomials, and add it to the product of the first terms.

$$(x - 2)(x + 4)$$
$$x \cdot 4$$
$$x^2 + 4x$$

"I" stands for inner. Multiply the inner two terms in the set of binomials, and add it to the other terms.

$$(x - 2)(x + 4)$$
$$-2 \cdot x$$
$$x^2 + 4x - 2x$$

> **Problem Solving Tip**
>
> Remember to carry the sign in front of the number.

"L" stands for last. Multiply the last terms in each of the binomials, and add it to the other terms.

$$(x - 2)(x + 4)$$
$$-2 \cdot 4$$
$$x^2 + 4x - 2x - 8$$

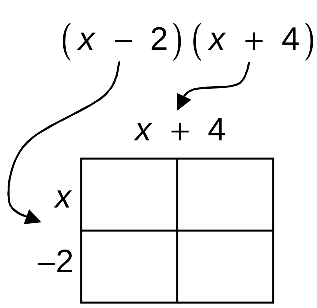
Now we combine the like terms.

$$x^2 + 4x - 2x - 8$$
$$x^2 + 2x - 8$$

Since $4x - 2x = 2x$, which is positive, we put a plus sign in front of it when we combine all the terms to make the polynomial.

Method 2: Punnett Square

The Punnett-square method can be used to multiply two polynomials with any number of terms. Set up the polynomials as follows:



Multiply the terms within each box.

	x	$+4$
x	$x \cdot x$	$x \cdot 4$
-2	$-2 \cdot x$	$-2 \cdot 4$

→

	x	$+4$
x	x^2	$4x$
-2	$-2x$	-8

Combine the like terms.

x^2	$4x$
$-2x$	-8

$$x^2 + 2x - 8$$

FACT

It is standard to write the term with the biggest exponent first.

Example

Simplify the expression $5x(x + 2)(x + 9)$

Solution

The best way to simplify this is by multiplying the binomials together first.

$$5x(x + 2)(x + 9)$$

	x	$+9$
x	x^2	$9x$
$+2$	$2x$	18

$$5x(x^2 + 11x + 18)$$

Next, we have to distribute the 5*x* into the **trinomial**.

$$5x\left(x^2 + 11x + 18\right) = 5x^3 + 55x^2 + 90x$$

No other terms can be combined, so the answer is $5x^3 + 55x^2 + 90x$.

Problem Solving Tip

When multiplying variables that are the same letter, we add the exponents. If no exponent is present, it means the exponent is 1. For example, $x^2y \cdot x^3y^2 = x^5y^3$.

TRY IT!

2) Which of the following expressions is equivalent to $x\left(x - 1\right)\left(x + 1\right)$?

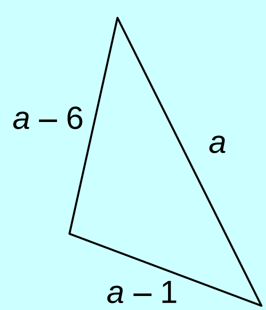
A $x^3 - x$

B $x^3 - 1$

C $x^3 - 2x^2 + 1$

D $x^3 - 2x^2 + x$

3) Which polynomial represents the perimeter of the given triangle?



A $3a - 4$

B $3a - 7$

C $a^3 - 7a^2 + 6a$

D $a^2 - 7a + 6$

Simplifying polynomials can useful for solving equations.

Example

Solve the following equation for x. $4(x+3) - 8x = 24$

Solution

To solve this equation, we need to simplify the left side of the equal sign.

$$4(x+3) - 8x = 24$$

$$4x + 12 - 8x = 24$$

$$-4x + 12 = 24$$

Both sides of the equation are in simplest form, so now we can solve for x. We need to get the x by itself on the left side of the equal sign. Use opposite operations to remove the constants that are added and multiplied.

Undo addition by subtracting

$$-4x + 12 = 24$$
$$-12 \quad -12$$
$$\frac{-4x}{-4} = \frac{12}{-4}$$
$$x = -3$$

Undo multiplication by dividing

The answer is $x = -3$. To check this answer, we substitute $x = -3$ in the **original equation**. Use the correct order of operations to check your answer.

FACT

PEMDAS is the order of operations:

Parentheses,

Exponents,

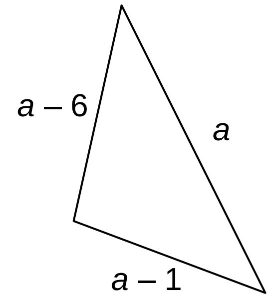
Multiplication and Division from left to right,

Addition and Subtraction from left to right.

$$4(x+3) - 8x = 24$$
$$4((-3)+3) - 8(-3) = 24$$
$$4(0) - 8(-3) = 24$$
$$0 + 24 = 24$$
$$24 = 24$$

Example

If the perimeter of the given triangle is 35, what is the value of *a*.



Solution

The perimeter of a triangle is the sum of all the sides. If we add up all the sides of the triangle, it should equal 35.

$$(a - 6) + (a - 1) + a = 35$$

Because terms in parentheses are being added, do not use FOIL. In fact, since all of the parentheses have plus signs in front of them, we can remove them.

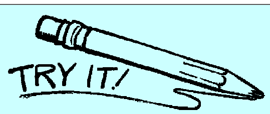
Combine like terms.

$$a + a - 6 + a - 1 = 35$$

$$3a - 7 = 35$$

Solve for *a*.

$$3a - 7 = 35$$
$$\quad +7 \quad +7$$
$$\frac{3a}{3} = \frac{42}{3}$$
$$a = 14$$



4) Which of the following is the solution for the equation $4(3x + 1) - 18 = 22$

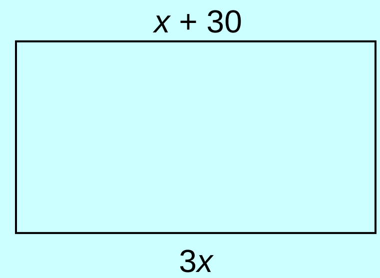
A $x = 3$

B $x = 0$

C $x = -3$

D $x = 4$

5) In a rectangle, the opposite sides have the same length. Find the length of one of these sides.



A 15

B 45

C 30

D 25

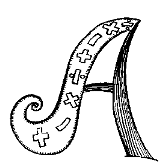
Review

Know these concepts:

1. Like terms have the exact same combination of variables and exponents.

2. To simplify an expression, distribute the through parentheses and combine like terms.

3. Use the following algorithm for solving equations where the variable has no exponent.



Algorithm

To solve an equation:

1) Combine like terms.

2) Undo addition or subtraction.

3) Undo multiplication or division.

Solve for y:

$$2y + 3y - 5 = 15$$
$$5y - 5 = 15$$
$$+5 \quad +5$$
$$5y = 20$$
$$\frac{5y}{5} = \frac{20}{5}$$
$$y = 4$$

4. When asked to solve for a variable, if the question is multiple choice, you can substitute the given values in for the variable and see which one works.

Practice Problems
Lesson 6

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 6.

1) Which of the following expressions is equivalent to $4n(3 - n) - 2(n^2 - 3n)$?

A $12n - 4n^2$ **B** $18n - 6n^2$

C $6n - 6n^2$ **D** $-2n^2 - 8n$

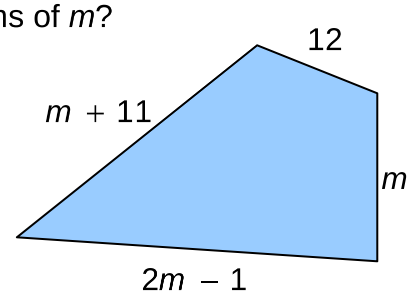
2) What is the perimeter of the given quadrilateral in terms of *m*?

A $4m + 22$

B $3m + 12$

C $4m - 10$

D $10 - m$



3) Which of the following expressions is equivalent to $(x - y)^2$

A $x^2 - y^2$ **B** $2x - 2y$

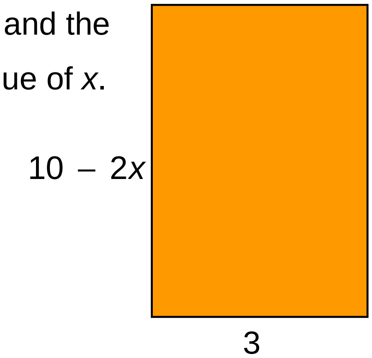
C $x^2 - 2xy + y^2$ **D** $x^2 + y^2$

4) Which of the following is the solution for the equation $4(x - 2) - (x + 3) = 17$?

A $x = 1$ **B** $x = 2$

C $x = 3$ **D** $x = 4$

5) In a rectangle, the area is found by multiplying the length and the width. If the area of the given rectangle is 24, find the value of x.



A $x = 24$

B $x = 1$

C $x = 4$

D $x = 8$

6) Which of the following equations is equivalent to $2y - 3x = 5x + 10$

A $y = 5x + 10$

B $y = 8x + 10$

C $y = x + 5$

D $y = 4x + 5$

ANSWERS TO TRY IT

1) D

2) A

3) B

4) A

5) B

NOTES

End of Lesson 6