Student Name: ______________________	**Contact Person Name:** ______________________
Date: ______________________	**Phone Number:** ______________________

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 9

Understanding $y = mx + b$

TAKS Objective 3 – Demonstrate an understanding of linear functions.

Lesson Objectives:

- Understand the slope-intercept form of a line $y = mx + b$
- Identify the effect of a change in the values of *m* or *b*
- Write the equation of a line given either
  - Its slope and *y*-intercept
  - Two points it passes through

Authors:

Tim Wilson, B.A.
Jason March, B.A., M.S.Ed

Editor:

Linda Shanks

Graphics:

Tim Wilson
Jason March

The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

National PASS Center
Geneseo Migrant Center
3 Mt. Morris – Leicester Road
Leicester, NY 14481
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass

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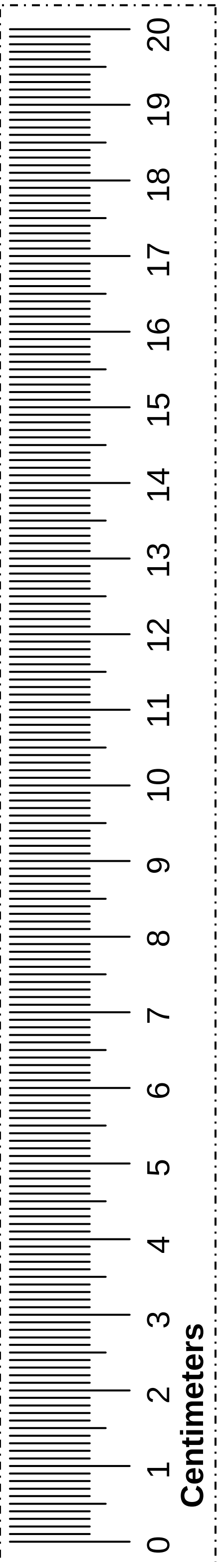
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

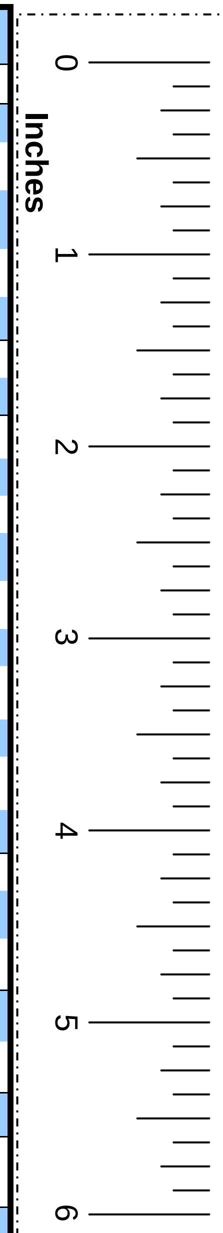
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
Triangles	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Think Back

Linear functions are defined by two things:

- ***Slope = rate of change = m*** $= \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
- ***Where they cross the y-axis, the y-intercept***

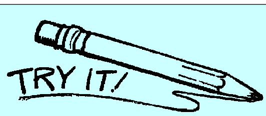
<u>Slope-intercept form of a line:</u>

$$y = mx + b$$

m is the slope.

b is the *y*-intercept.

The line $y = 4x + 5$ has a slope of 4 and a *y*-intercept of 5. The line $y = -x + \frac{4}{7}$ has a slope of -1 and a *y*-intercept of $\frac{4}{7}$. The line $y = \frac{2}{3}x - 9$ has a slope of $\frac{2}{3}$ and a *y*-intercept of -9. What about the line $y = x$? Since we can rewrite $y = x$ to be $y = 1x + 0$, its slope is 1, and its *y*-intercept is 0. ($y = x$ is the <u>linear parent function</u>; verify its slope and *y*-intercept on your own using a sketch of its graph.)



Determine the slope and *y*-intercept of each of the following lines.

1) $y = x + 1$

2) $y = -x$

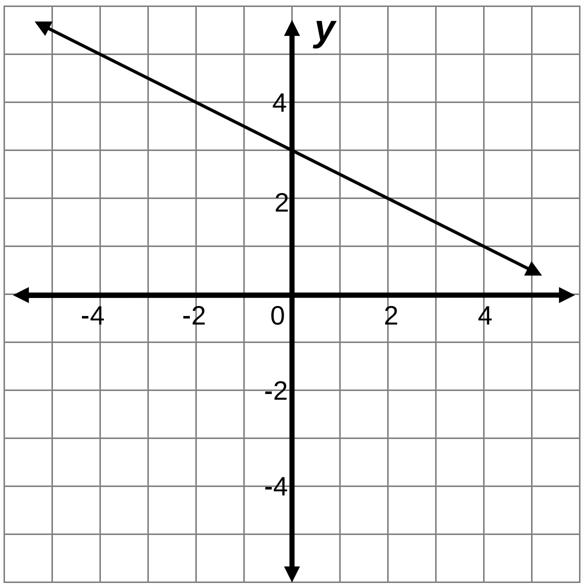
3) $y = \frac{9}{4}x - 8$

4) $y = 3x + \frac{5}{2}$

Using the graph of a line, we can write its equation by finding its slope and *y*-intercept.

Example

Write the equation of the graph below.



Solution

First, find the slope using any two points on the graph. How about $(2,2)$ and $(4,1)$?

You may wish to choose two different points, and verify that the slope is the same. Using the formula, we see $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{4-2} = -\frac{1}{2}$. Next, we must identify *b*, the *y*-intercept. $b = 3$.

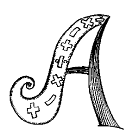
Finally, we will substitute these values of *m* and *b* into the equation $y = mx + b$ to get the equation of the line in the problem.

$$y = mx + b$$

$$y = -\frac{1}{2}x + 3$$

Problem Solving Tip

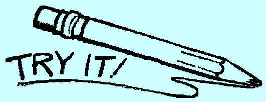
Always write the formula before you substitute.



Algorithm

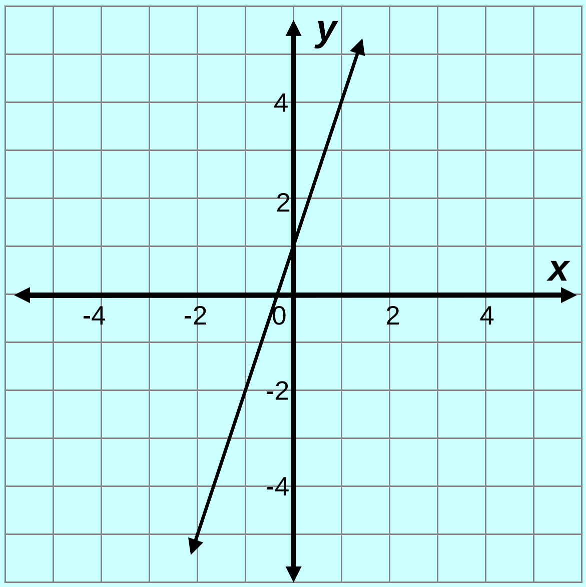
To write the equation of a line graph:

1) Use two points to find the slope, $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2) Identify *b*, the *y*-intercept.
3) Substitute the values of *m* and *b* into the general equation of a line, $y = mx + b$. (*x* and *y* will not change.)

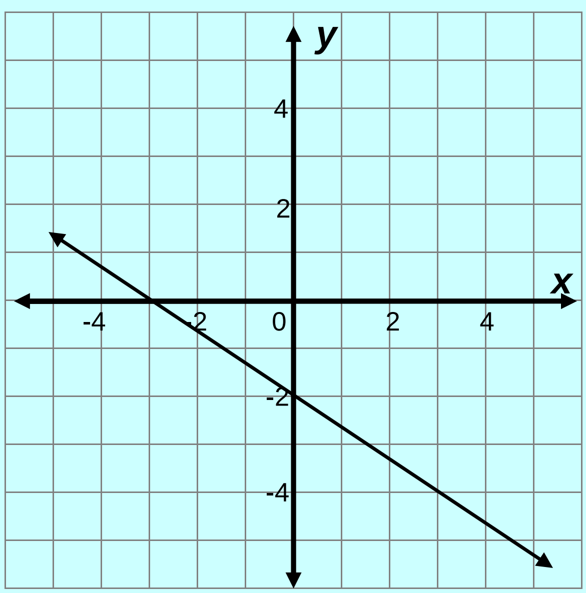


Write the equation of each line graph below.

5)

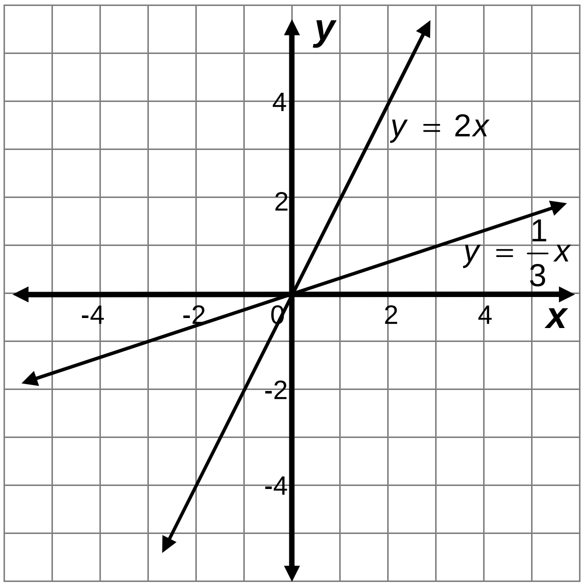


6)



For the general equation of a line, $y = mx + b$, changing the values of m and b creates a different looking graph. Observe how changes in m (slope) and b (y-int.) affect the graph of a line in ways you can predict.

Change in *m* (slope)



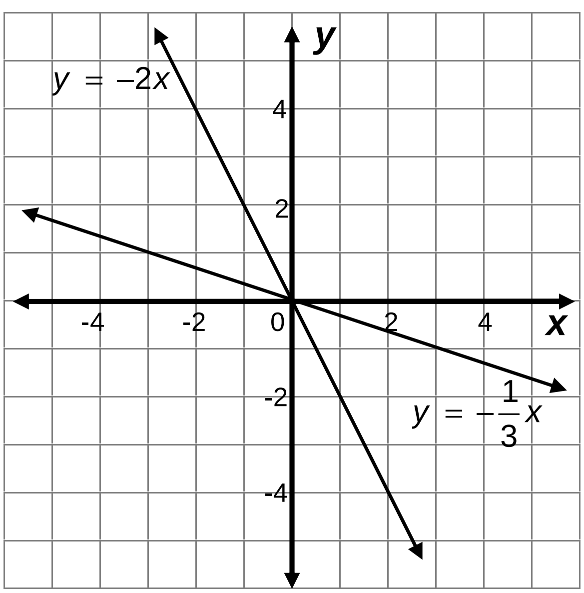
Large values of m produce steep graphs.

Here, since $|2| > \left|\frac{1}{3}\right|$, the graph of $y = 2x$ is steeper than that of $y = \frac{1}{3}x$.

FACT

The absolute value is a number's distance from zero. The absolute value of x is $|x|$. Absolute values are always positive.

$|7| = 7$, $|-2| = 2$



We use absolute values to show that large negative values also produce steep slopes. At left, since $|-2| > \left|-\frac{1}{3}\right|$, the graph of $y = -2x$ is steeper than that of $y = -\frac{1}{3}x$.

Example

If the function $y = x - 2$ is changed to $y = -4x - 2$, how would the graph of the new function compare with the original?

Solution

First, understand that both equations are written in slope-intercept form:

$y = mx + b$, where *m* is slope.

From this, we can see that the slope of $y = x - 2$ is 1, since there is an invisible 1 in front of *x*.

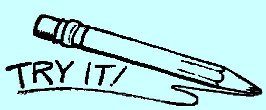
The slope of $y = -4x - 2$ is -4.

Next, use the absolute values.

Notice $|-4| > |1|$.

The graph of the new function will be steeper, because $|-4| > |1|$.

> ***Think Back***
>
> ***x = 1x.***
>
> ***If no coefficient is shown in front of a variable, it is assumed to be 1.***



For 7 & 8, circle the equation of the steeper graph.

7) $y = \frac{1}{5}x - 9$

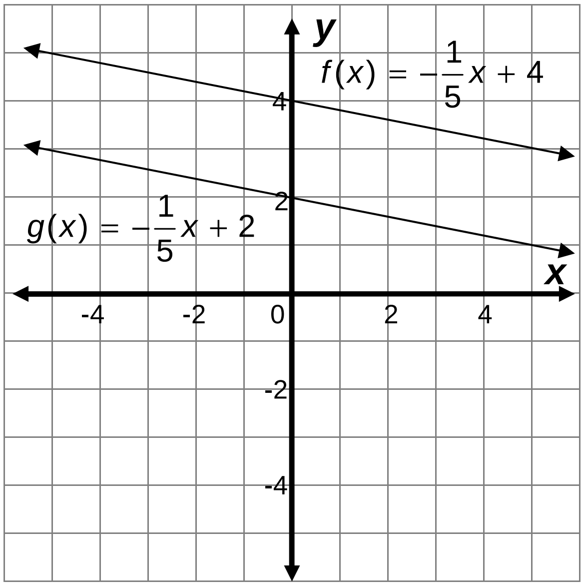
$y = -2x - 12$

8) $y = x + 2$

$y = \frac{2}{3}x - 5$

9) When the function $y = \frac{3}{4}x - 7$ is changed to $y = \frac{1}{2}x - 7$, describe how the new graph compares with the original.

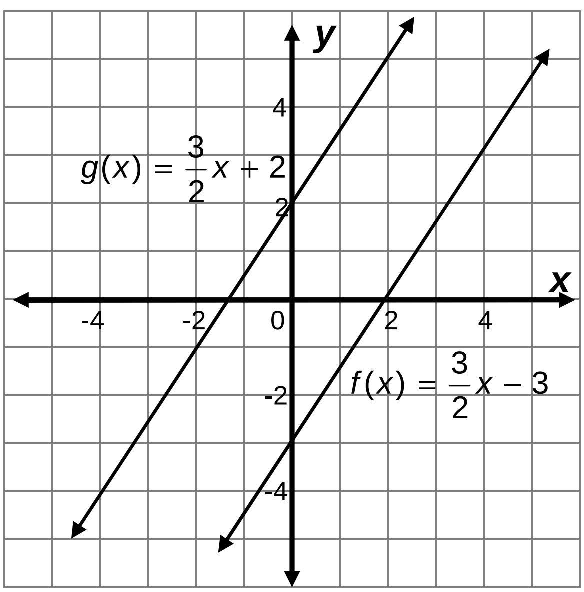
Change in *b* (*y*-intercept)



For the function $f(x) = -\frac{1}{5}x + 4$, $b = 4$.

For the function $g(x) = -\frac{1}{5}x + 2$, $b = 2$.

Notice that $f(x)$ crosses the *y*-axis at a point 2 units higher than $g(x)$ because the difference of their *y*-intercepts is $4 - 2 = 2$.



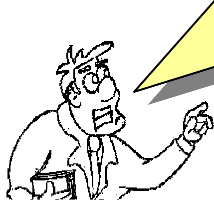
The *y*-intercept of $f(x)$ is $b = -3$.

The *y*-intercept of $g(x)$ is $b = 2$.

We see $f(x)$ crosses the *y*-axis at a point 5 units lower than $g(x)$. This is because the difference of their *y*-intercepts is $-3 - 2 = -5$.

FACT

Lines with equal slope and different y-intercepts are ***parallel***.



Example

The graph of the function $y = 4x - 1$ is changed to $y = 4x + 4$. How does the graph of the new function compare with the graph of the original?

Solution

(1) Decide what changes between the two graphs.

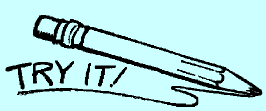
The y-intercept changes.

(2) By how much does this value change?

Find the difference of the y-intercept of the new graph and the y-intercept of the original graph: $4 - -1 = 5$.

(3) Write a sentence.

The new graph crosses the y-axis at a point 5 units higher than the original graph does.



For 10 & 11, circle the equation of the graph with the lowest *y*-intercept.

10) $y = \frac{1}{5}x - 15$

$y = \frac{1}{5}x - 12$

11) $y = x + 2$

$y = x$

12) When the function $y = \frac{6}{5}x + 1$ is changed to $y = \frac{6}{5}x - 7$, describe how the new graph compares with the original.

We can find the equation of a line given only two points it passes through.

Example

Write the equation of the line that passes through the points $(2,5)$ and $(6,8)$.

Solution

We need to use these points to find two things: the slope and the *y*-intercept. Finding the slope from two points should be very familiar to you by now.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{6-2} = \frac{3}{4}$$

To find the *y*-intercept, we must be creative. Start by substituting the slope into $y = mx + b$.

$$y = \frac{3}{4}x + b$$

Next, since we are looking to find *b*, the *y*-intercept, we need to fill in values for *x* and *y* in the equation. How? Use a point given to be on the graph. We will use $(2,5)$. If you wish, use $(6,8)$ and see if you get the same *y*-intercept.

Using the point $(2,5)$, substitute 2 for *x* and 5 for *y*.

$$y = \frac{3}{4}x + b$$

$$5 = \frac{3}{4}(2) + b$$

$$5 = \frac{6}{4} + b$$

$$5 = \frac{3}{2} + b$$

$$-\frac{3}{2} \qquad -\frac{3}{2}$$

$$\frac{7}{2} = b$$

FACT

$$5 - \frac{3}{2}$$

$$= \frac{5}{1}\left(\frac{2}{2}\right) - \frac{3}{2}$$

$$= \frac{10}{2} - \frac{3}{2}$$

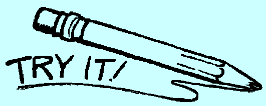
$$= \frac{7}{2}$$

Algorithm

Given two points, find the equation of a line:	$(2,5)$ and $(3,7)$
1) Find the slope.	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{3-2} = \frac{2}{1} = 2$
2) Substitute the slope into the equation of a line, $y = mx + b$.	$y = mx + b$ $y = 2x + b$
3) Substitute one of the given points for x and y, and simplify.	Using $(3,7)$, $y = 2x + b$ $7 = 2(3) + b$ $7 = 6 + b$
4) Solve for b.	$-6 \quad -6$ $1 = b$
5) Substitute the values of m and b into the equation $y = mx + b$.	$y = mx + b$ $y = 2x + 1$

Problem Solving Tip

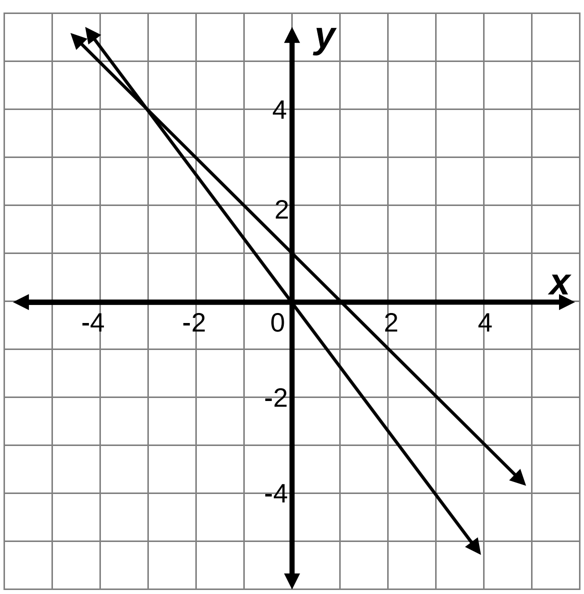
Check your work and draw the graph. Verify that it intersects the two given points.



13) Write the equation of the graph that passes through the points $(4,7)$ and $(5,9)$.

Example

Two lines are shown on the grid. The two lines pass through (-3,4). One line passes through the origin, and the other passes through the point (4,-2). Which choice below identifies these lines?



A $y = -x + 1$ and $y = \frac{4}{3}x + \frac{3}{2}$

B $y = \frac{4}{3}x - 2$ and $y = x - 1$

C $y = -\frac{3}{4}x + 1$ and $y = -x$

D $y = -\frac{4}{3}x$ and $y = -x + 1$

Solution

From this lesson, you have learned how to write the equation of a line from its graph. However, we can analyze the answer choices to find the answer in a faster way than writing both equations. Start with what is simplest to find on both lines: their *y*-intercepts. They are 1 and 0. From the answer choices, we see that only one choice has equations with these intercepts.

A $y = -x + 1$ and $y = \frac{4}{3}x + \frac{3}{2}$

B $y = \frac{4}{3}x - 2$ and $y = x - 1$

C $y = -\frac{3}{4}x + 1$ and $y = -x - 4$

(D) $y = -\frac{4}{3}x$ and $y = -x + 1$

$\left(= -\frac{4}{3}x + 0\right)$

Sometimes you will need to solve problems the direct, long way. This problem, however, allows you to use reasoning to eliminate every answer choice but the correct one.

FACT

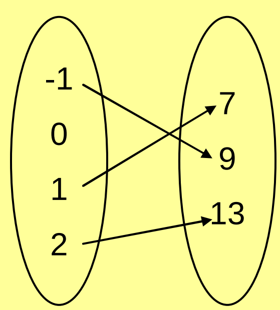
You will not always see the graph of a function. You may be expected to write the equation of a linear function given a set of ordered pairs

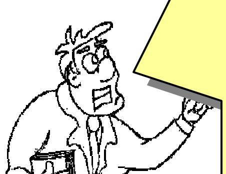
$\{(1,3),(3,2),(5,1),(7,0)\}$

a table

x	y
0	1
1	3
3	0
5	3

or a mapping





The process to write the equation is the same: use 2 points.

Review

Know these concepts:

1. The general equation of a line, sometimes called the slope-intercept form, is $y = mx + b$.

a. *m* is the slope

b. *b* is the *y*-intercept.

2. Write the equation of a line, given its graph.

3. Changing *m* results in a change in steepness.

a. Large values of *m* result in steep graphs.

4. Changing *b* results in a parallel line above or below the original.

5. Find the equation of a line given two points it passes through.

a. Find the slope using the given points.

b. Use the slope and one of the points to solve for *b*.

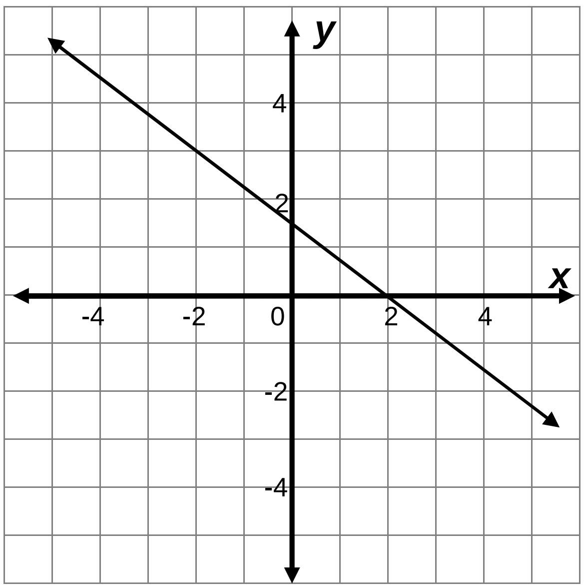
c. Substitute the values of *m* and *b* into $y = mx + b$.

Practice Problems
Lesson 9

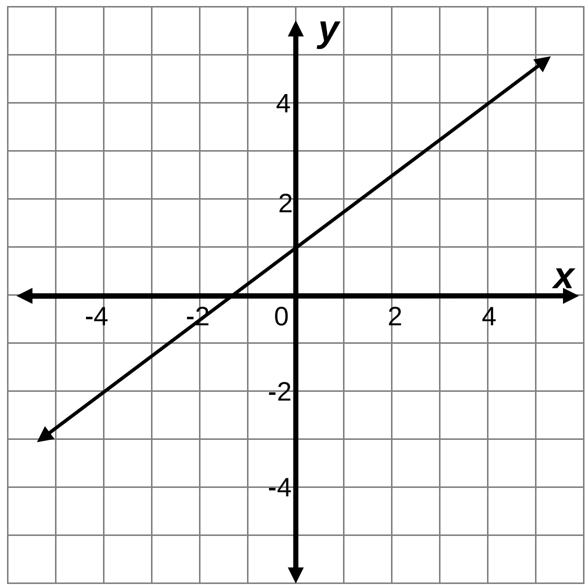
Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 9.

1) Which graph best represents the line that has a slope of $-\frac{3}{4}$ and contains the point (3,-4)?

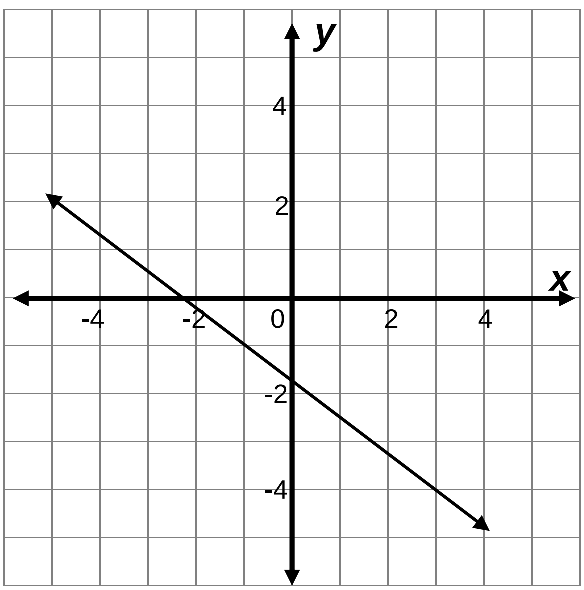
A



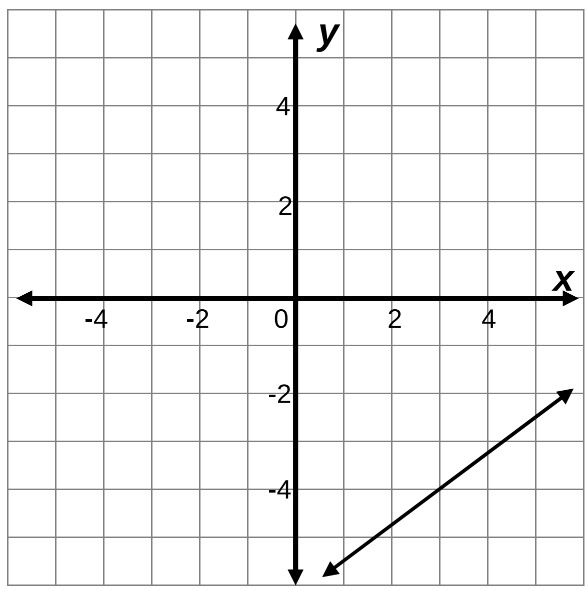
B



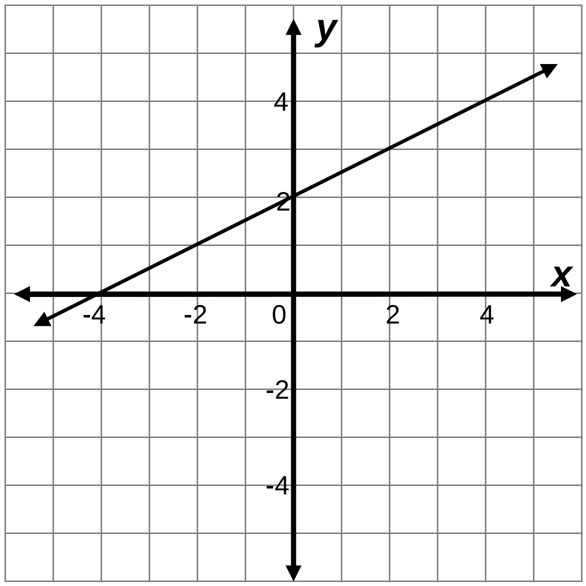
C



D



2)



If the line is moved 2 units up, which equation will best describe the new line?

A $y = 2x + 4$

B $y = \frac{1}{2}x + 4$

C $y = 2x$

D $y = \frac{1}{2}x$

3) Which best describes the effect of changing the graph of $y = \frac{1}{2}x - 5$ to $y = -3x + 2$?

A The new graph will be steeper and will have a *y*-intercept 7 units higher than the original.

B The new graph will be steeper and will have a *y*-intercept 7 units lower than the original.

C The new graph will be less steep and will have a *y*-intercept 7 units higher than the original.

D The new graph will be less steep and will have a *y*-intercept 7 units lower than the original.

4) What is the *y*-intercept of the function that passes through the points (4,-6) and (5, -8)?

A 2

B -4

C $\frac{1}{2}$

D $\frac{-1}{4}$

5) Which function has a slope of $\frac{5}{9}$ and a *y*-intercept of -4

A $y = \frac{5}{9}x + 4$

B $y = \frac{5}{9}x - 4$

C $y = -4x + \frac{5}{9}$

D $y = 4x - \frac{5}{9}$

ANSWERS TO TRY IT

1) $m = 1$

y**-intercept** $= 1$

2) $m = -1$

y**-intercept** $= 0$

3) $m = \dfrac{9}{4}$

y**-intercept** $= -8$

4) $m = 3$

y**-intercept** $= \dfrac{5}{2}$

5) $y = 3x + 1$

6) $y = -\dfrac{2}{3}x - 2$

7) $y = -2x - 12$

8) $y = x + 2$

9) The new function is less steep than the original because $\left|\dfrac{1}{2}\right| < \left|\dfrac{3}{4}\right|$.

10) $y = \dfrac{1}{5}x - 15$

11) $y = x$

12) $y = \dfrac{6}{5}x - 7$ has a y-intercept 8 units lower than that of $y = \dfrac{6}{5}x + 1$. This is because $-7 - 1 = -8$.

13) $y = 2x - 1$

End of Lesson 9