Student Name: _____	Contact Person Name: _____
Date: _____	Phone Number: _____

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 11
Equations and Inequalities

TAKS Objective 4 – Formulate and use linear equations and inequalities

Lesson Objectives:

- Graph an inequality
- Write or identify a linear equation or inequality to represent a given real-world situation
- Solve real-world problems involving linear equations and inequalities

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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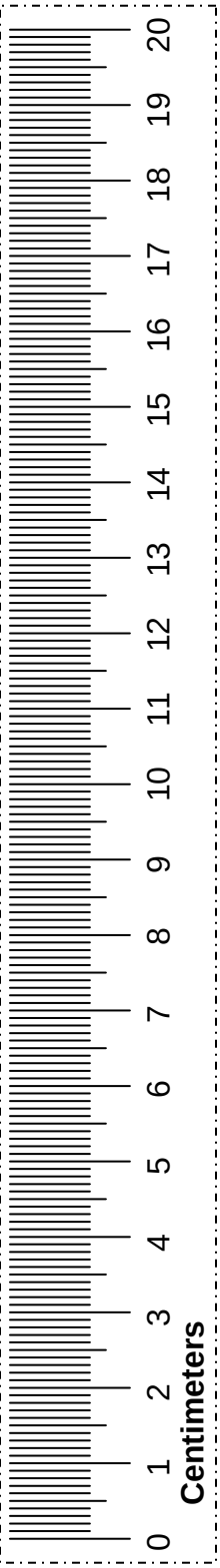
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

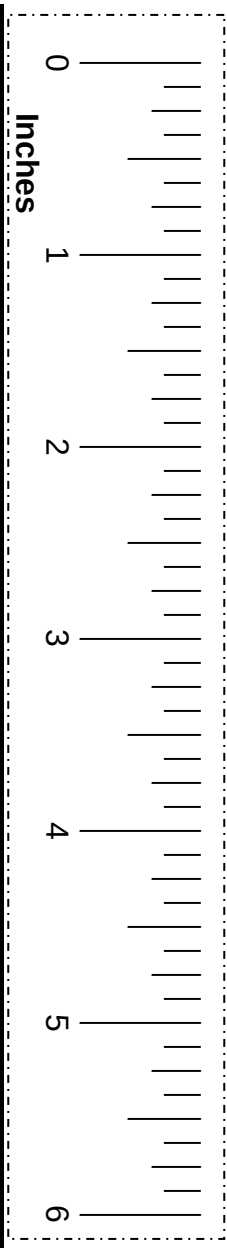
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



We will begin by working with **inequalities**.

FACT

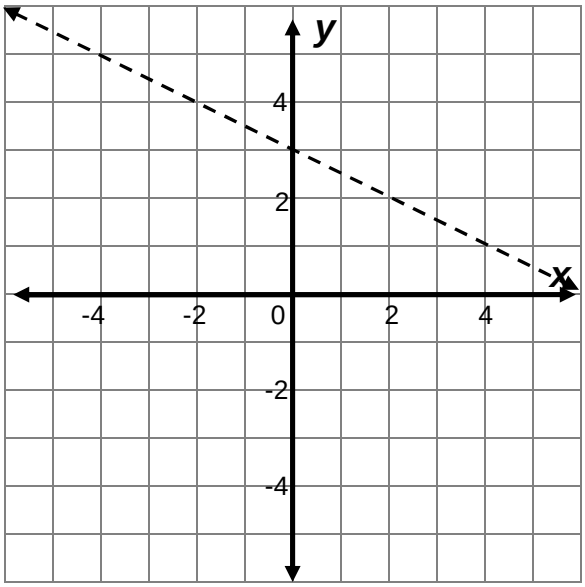
*If an algebraic sentence uses one of the following symbols, it is called an **inequality**.*

$<$	$\leq$	$>$	$\geq$
less than	less than or equal to	greater than	greater than or equal to

Graphing an inequality is very similar to graphing a line.

Example

Graph the inequality $y > -\frac{1}{2}x + 3$

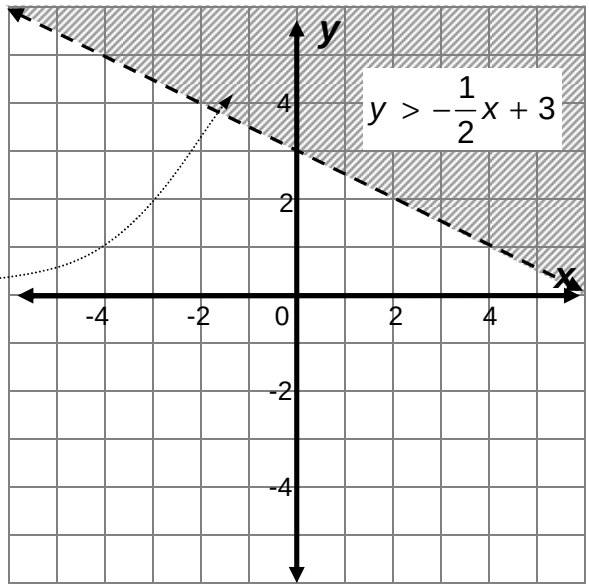


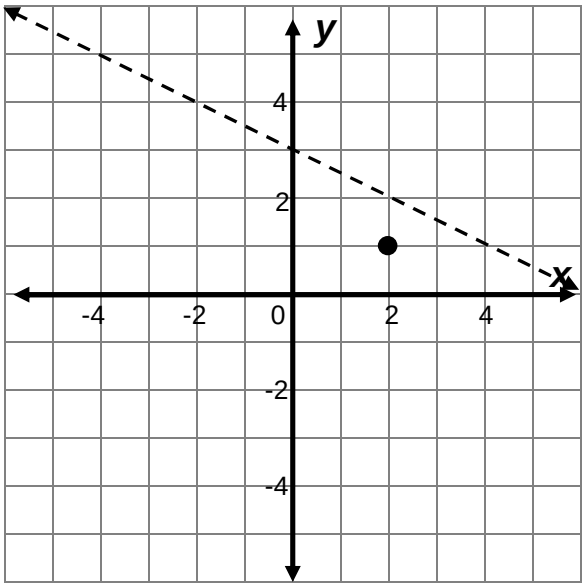
***Step 1*:** Graph as if graphing the line $y = -\frac{1}{2}x + 3$, except use a dotted line. This is because this example uses >, greater than, so values directly on the line are not part of the solution. A dotted line shows this.

***Step 2*:** Shade the graph using one of two methods.

1. Remember that $y > mx + b$ and $y \geq mx + b$ are always shaded above the line. $y < mx + b$ and $y \leq mx + b$ are always shaded below the line.

2. Test any point above or below the graph with the original inequality.





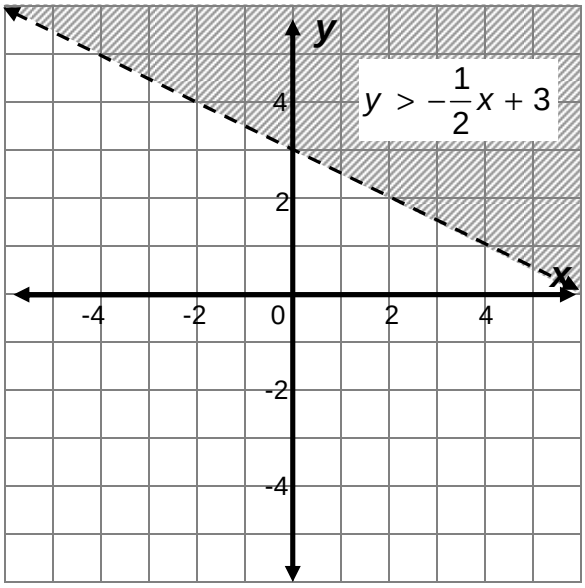
Test (2,1):

$$y > -\frac{1}{2}x + 3$$

$$1 \stackrel{?}{>} -\frac{1}{2}(2) + 3$$

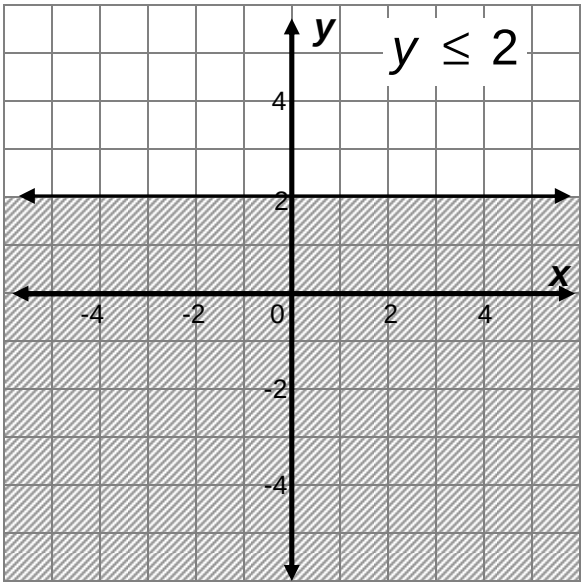
$$1 \stackrel{?}{>} -1 + 3$$

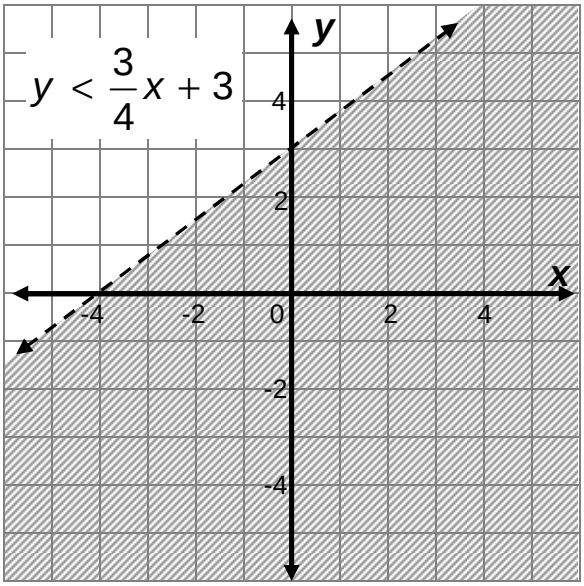
$$1 \not> 2$$

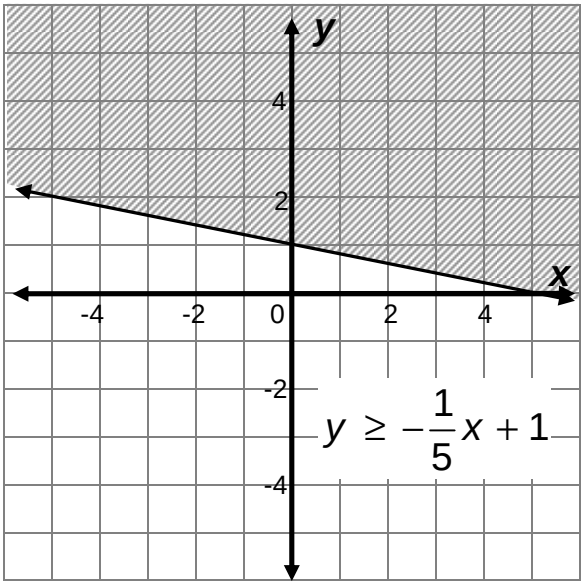


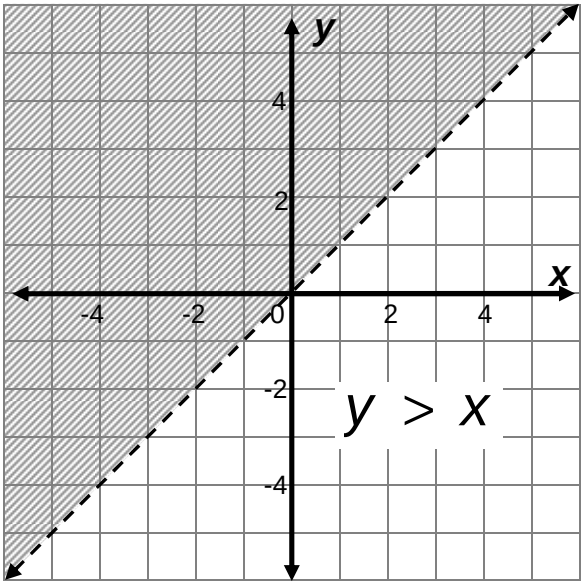
This shows that the point (2,1) is not part of the solution, so we shade the opposite region of the graph, above the dotted line.

These example graphs should help you understand how to graph an inequality.









FACT

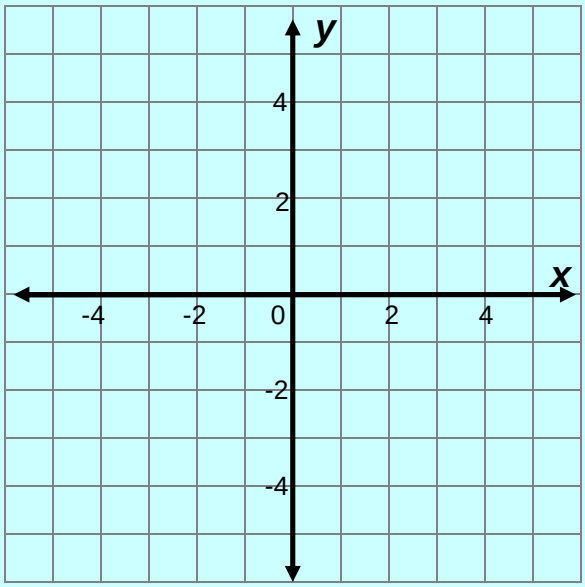
$y < mx + b$ Dotted line, shaded below

$y \leq mx + b$ Solid line, shaded below

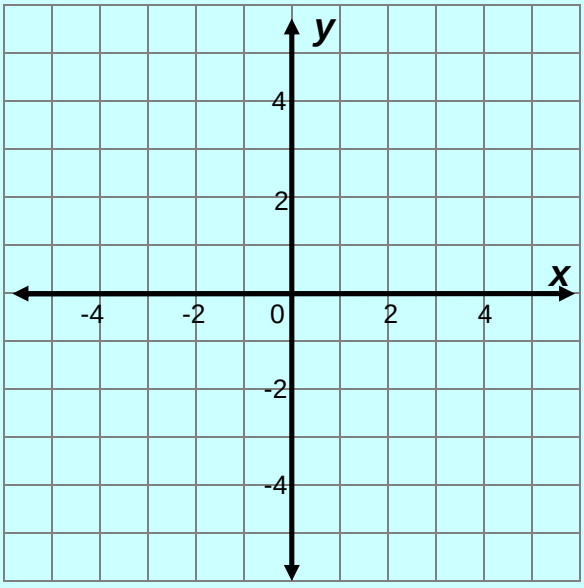
$y > mx + b$ Dotted line, shaded above

$y \geq mx + b$ Solid line, shaded above

1) Graph the inequality $y < -\frac{1}{3}x + 4$ on the axes below.



2) Which of the following ordered pairs is not in the solution set of $y > \frac{1}{2}x - 1$?



A (0,2)

B (1,4)

C (1,–2)

D (–3,1)

Example

Graph the inequality $-2y - x \geq 6$

Solution

First, we must use algebra to solve the inequality for *y*.

$-2y - x \geq 6$

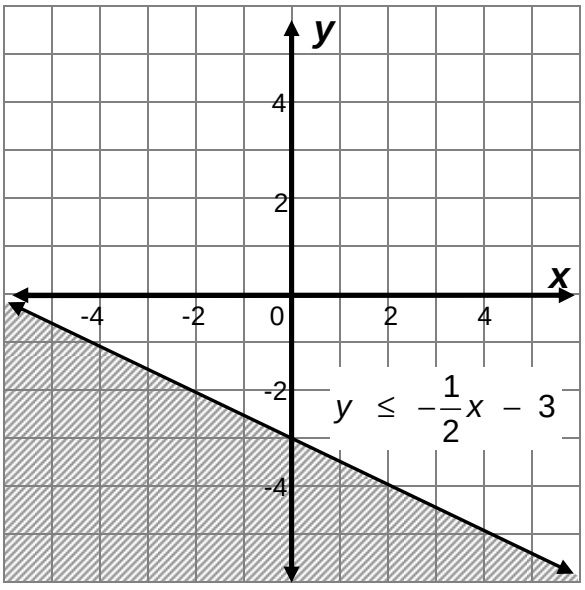
$+x \quad +x$ Begin isolating *y* by adding *x* to both sides.

$-2y \geq x + 6$ Turn the coefficient of *y* to 1 by dividing each side by -2.

$\frac{-2y}{-2} \leq \frac{x + 6}{-2}$ **When you multiply or divide an inequality by a negative number, flip the inequality sign.**

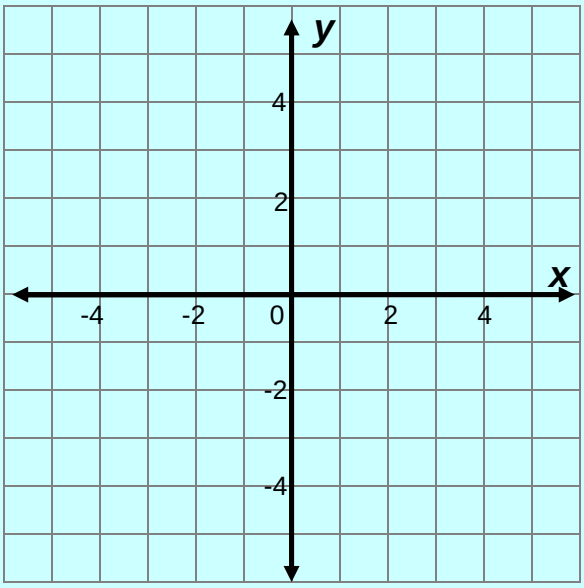
$\frac{-2y}{-2} \leq \frac{x}{-2} + \frac{6}{-2}$ Remember to divide each term by -2.

$y \leq -\frac{1}{2}x - 3$ Simplify your answer to standard form. Notice that *x* divided by -2 equals negative one-half *x*.



Finally, graph and label your result.

3) Graph the inequality $-4y - 12x < 4$ on the axes below.



Example

Bridget is buying used DVD movies at a garage sale. Action movies are priced at $2, and drama movies are priced at $1.50. Bridget can spend no more than $25. Write an inequality that represents the number of action movies, *x*, and drama movies, *y*, that Bridget can buy.

Solution

The total price of action movies is the number of action movies, *x*, times the price per action movie, 2.

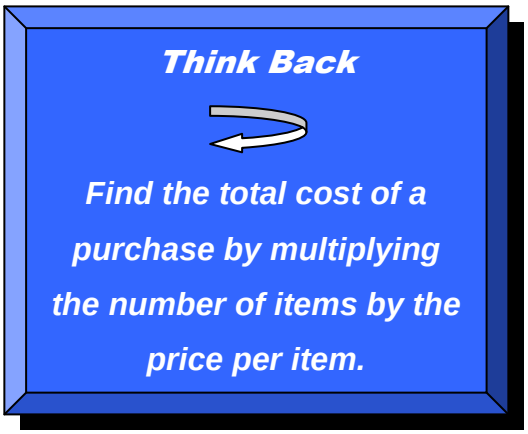
$$x \cdot 2 = 2x$$

The price of drama movies is the number of drama movies, *y*, times the price per drama movie, 1.50.

$$y \cdot 1.50 = 1.5y$$

The price of action movies and drama movies can be no more than 25.

$$2x + 1.5y \leq 25$$



Example

Garfield is on a diet. He can eat no more than 1,500 calories a day. Garfield's lasagna has 400 calories per serving, and his other food has 250 calories per serving. Which inequality could be used to find *s*, the number of servings of lasagna, and *f*, the number of servings of other food, that Garfield is allowed to eat each day?

A $(400 + 250)(s + f) > 1{,}500$

B $400s + 250f \leq 1{,}500$

C $(s + 400)(f + 250) \leq 1{,}500$

D $250s + 400f > 1{,}500$

Solution

One method to solve this is directly, as we solved the previous example. Another method is this: think of an example that Garfield's diet allows. For instance, he may eat 1 serving of lasagna and 4 servings of other food. This is 400 lasagna calories and 1,000 calories of other food for a total of 1,400 calories. Next, substitute 1 for *s* and 4 for *f* in each of the answer choices to see if it gives you 1,400 calories on the left side of the inequality.

Choice A:

$(400 + 250)(s + f) > 1{,}500$

$(400 + 250)(1 + 4)$ Only check the side with variables.

$= (650)(5)$ Since choice A does not give 1,400 when $s = 1$ and $f = 4$, it is incorrect.

$= 3{,}250$

Choice B:

$400s + 250f \leq 1{,}500$ Choice B gives us the correct number of calories, and at this point you may choose this answer. However, it is recommended you test each answer choice so you are absolutely certain of which answer is correct.

$400(1) + 250(4)$

$= 400 + 1{,}000$

$= 1{,}400$

Choice C:

$(s + 400)(f + 250) \leq 1{,}500$

$(1 + 400)(4 + 250)$

$= (401)(254)$

$= 101{,}854$ As we suspect, choice C is incorrect.

Choice D:

$250s + 400f > 1{,}500$

$250(1) + 400(4)$

$= 250 + 1{,}600$

$= 1{,}850$ Choice D is also incorrect.

Therefore, the answer is choice **B**.

Sometimes drawing a picture helps reveal how to set up a problem.

Example

Aldo is building a rectangular chain-link fence around his swimming pool. He has at most 300 feet of chain-link fencing to use. Aldo wants the length of the fence to be three times its width, x. Which inequality best identifies the amount of fencing Aldo can use for his fence?

A $3x^2 \leq 300$

B $4x \leq 300$

C $8x \leq 300$

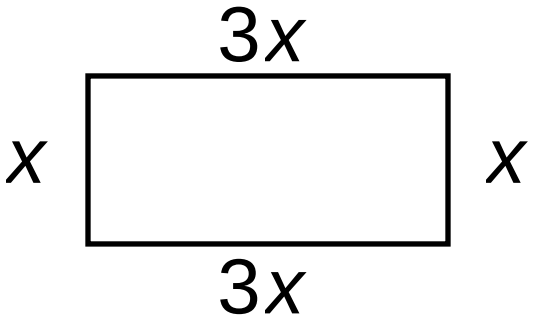
D None of these

Solution

First, draw a picture of the situation described: a rectangular fence.

Next, we are told the width is described by x. Write this on your picture for each width.

Next, write the length of the fence. We are told it is three times the width.



Next, be careful! What does the question ask about? It asks to represent the length of the fence, not the area inside the fence. The length around the fence is the sum of each side.

$$3x + x + 3x + x = 8x.$$

Notice one of the answer choices uses $8x$. Be careful, it may not be correct, since "none of these" is also an answer choice. From the question, we know that the length around the fence can be no more than 300. It can be 300, or it can be less than 300. As an inequality, this is

$$8x \leq 300.$$

Therefore, the answer is choice **C**.

4) Marco works at a trophy store. He is filling a box with bowling trophies and tennis trophies. The box can hold no more than 30 pounds. Bowling trophies weight 2.5 pounds each and tennis trophies weigh 3.1 pounds each. Write an inequality to show the number of bowling trophies, b, and the number of tennis trophies, t, that can fit in the box.

5) The Irondequoit High School volleyball team needs 48 team shirts that have both the high school mascot and the letters IHS on them. Each shirt with a printed mascot costs \$6.75. The letters IHS cost \$0.95 per letter. A supplier will sell every eighth shirt with the mascot and letters for half price. What is a reasonable conclusion about p, the price of all 48 shirts before taxes?

A $400 \leq p < 412$

B $412 \leq p < 424$

C $424 \leq p < 436$

D $436 \leq p < 448$

Example

The energy of a certain particle is 7.92 joules. If this particle loses 0.14 joules of energy every 30 seconds, what will its energy be after 5 minutes?

A 6.52 joules

B 7.22 joules

C 3.45 joules

D 2.26 joules

Solution

We can solve this numerically, using a calculator.

There are ten 30-second intervals in 5 minutes. Thus, the particle will lose 0.14 joules ten times over a 5 minute period. That is, the particle will lose $0.14 \cdot 10 = 1.4$ joules. Subtract this from the initial energy value.

$7.92 - 1.4 = 6.52$ joules. Choice **A** is the answer.

Example

Melissa works as a waitress. Melissa earns $7.00 per hour plus tips. She serves an average of 29 customers per hour. If Melissa earns $215 during a 5-hour shift, which amount best represents the average tip per customer?

Solution

We could use the same method as above, subtracting the hourly amount earned over 5 hours, then dividing the difference by the total number of customers. We can also solve this algebraically, writing an equation to model the money Melissa makes and solving for the average amount per customer.

Total money in a shift = (number of hours worked)(amount earned in 1 hour)

Let x be the average tip per customer.

$$215 = (5)(7 + 29x)$$

Now we can directly solve for x.

$$215 = 5(7 + 29x)$$

$$215 = 35 + 145x$$

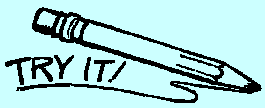
$$-35 \quad -35$$

$$180 = 145x$$

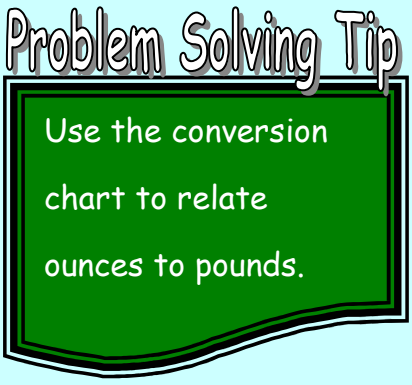
$$\frac{180}{145} = \frac{145x}{145}$$

$$1.24 \approx x$$

Melissa earns an average of \$1.24 per customer.



6) Apu wants to buy a box of tofu hot dogs for \$13.99. Each hot dog weighs about 1.7 ounces and will be packed into a carton that holds between 13 and 15 pounds. What is the greatest number of hot dogs that the carton can hold without its weight limit being exceeded?



A 141

B 8

C 140

D 9

Review

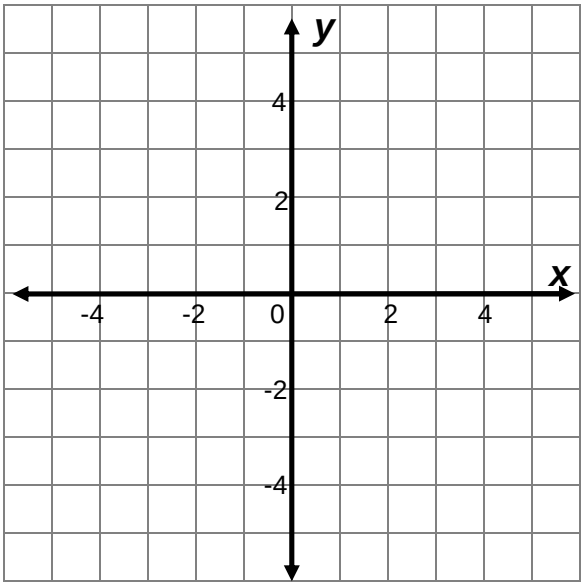
Know these concepts:

1. Graphing an inequality
    - **a.** First, solve the inequality for *y* algebraically
        - **i.** Flip the inequality sign if multiplying or dividing by a negative number
2. Identify an inequality or equation from a given situation.
3. A variety of methods can be used to discover an answer.
    - **a.** Solve directly
    - **b.** Use a calculator
    - **c.** Draw a picture
    - **d.** Write an equation
    - **e.** Eliminate incorrect answer choices

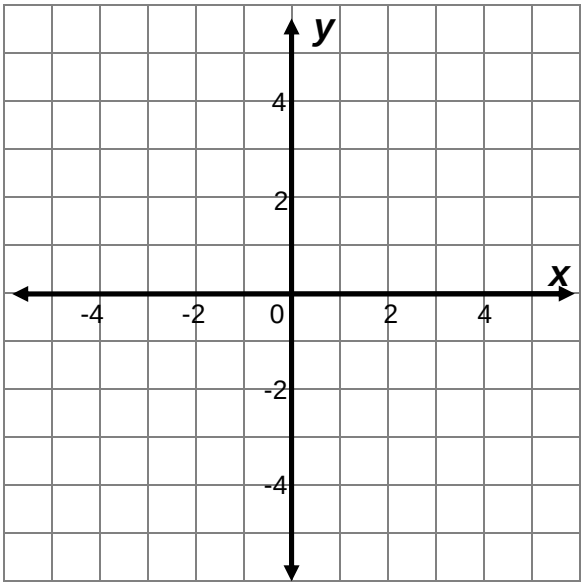
Practice Problems
Lesson 11

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 11.

1) Graph the inequality $y > x + 2$



2) Graph the inequality $2x + 2y \leq -2$



3) The owners of a pet store want to spend a maximum of $700 for at least 6 minutes of advertising. The cost to advertise on a local TV station is $120 per minute. The cost to advertise on a local radio station is $75 per minute. Which inequality best represents the cost of buying *r* radio minutes and *t* TV minutes?

A $r + t \leq 6$

B $75t + 120r \leq 6$

C $75r + 120t \leq 700$

D $r + t \leq 700$

4) Barry works a job delivering mattresses. He is able to deliver 2 mattresses per hour on average. The mattress company pays him $6 per hour and customers tip him for each mattress he delivers. In a typical 40-hour work week, Barry makes $612. What is Barry's average tip per mattress delivered?

A $372

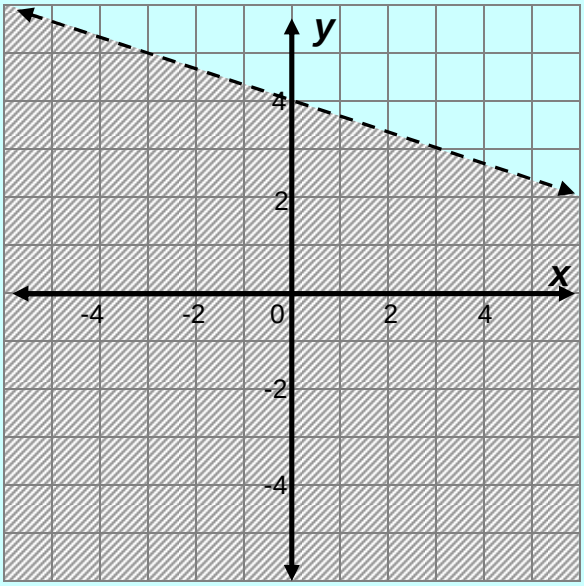
B $4.65

C $240

D $8.00

ANSWERS TO TRY IT

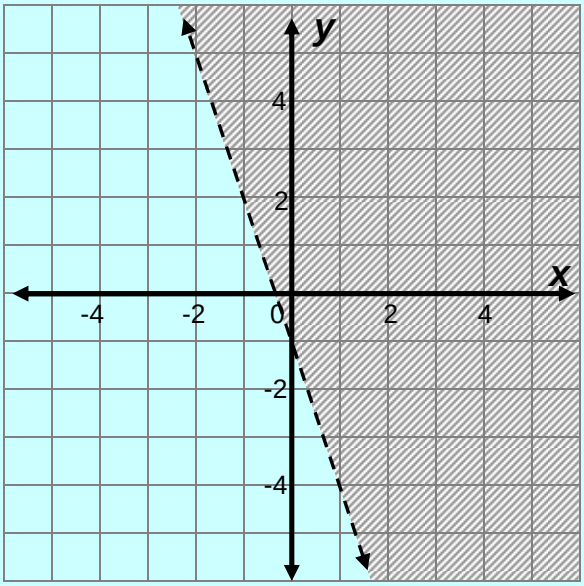
1)



2) **C**

Note: You may solve this question by substituting points into the given inequality or by graphing the inequality and plotting the points.

3)



$-4y - 12x < 4$

$+12x \quad +12x$

$-4y < 12x + 4$

$\frac{-4y}{-4} > \frac{12x + 4}{-4}$

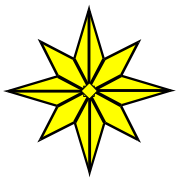
$\frac{-4y}{-4} > \frac{12x}{-4} + \frac{4}{-4}$

$y > -3x - 1$

4) $2.5b + 3.1t \leq 30$ (or something equivalent)

5) **C** (The cost of shirts before tax is $432)

6) **A**



End of Lesson 11