**Student Name:** ______________________________

**Date:** ______________________________

**Contact Person Name:** ______________________________

**Phone Number:** ____________________

**Texas Assessment of Knowledge and Skills**

# Exit Level Math Review

# Lesson 12

## Systems of Equations

**TAKS Objective 4 –** Formulate and use linear equations and inequalities

**Lesson Objectives:**

- Solve a system of equations graphically and algebraically
- Model a situation using a system of linear equations
- Determine the number of solutions to a system of linear equations

***Authors:***

Tim Wilson, B.A.
Jason March, B.A., M.S.Ed

***Editor:***

Linda Shanks

***Graphics:***

Tim Wilson
Jason March


The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**


National PASS Center
Geneseo Migrant Center
3 Mt. Morris – Leicester Road
Leicester, NY 14481
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass



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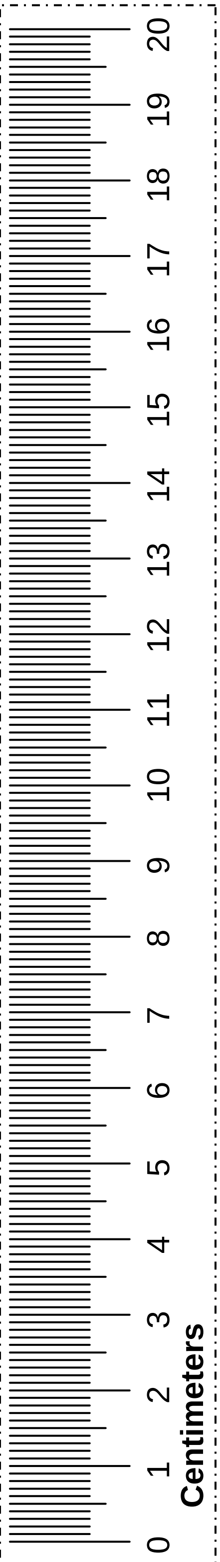
# *TAKS* Mathematics Chart




## Length

| Metric | Customary |
|---|---|
| 1 kilometer = 1000 meters | 1 mile = 1760 yards |
| 1 meter = 100 centimeters | 1 mile = 5280 feet |
| 1 centimeter = 10 millimeters | 1 yard = 3 feet |
| | 1 foot = 12 inches |

## Capacity and Volume

| Metric | Customary |
|---|---|
| 1 liter = 1000 milliliters | 1 gallon = 4 quarts |
| | 1 gallon = 128 fluid ounces |
| | 1 quart = 2 pints |
| | 1 pint = 2 cups |
| | 1 cup = 8 fluid ounces |

## Mass and Weight

| Metric | Customary |
|---|---|
| 1 kilogram = 1000 grams | 1 ton = 2000 pounds |
| 1 gram = 1000 milligrams | 1 pound = 16 ounces |

## Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

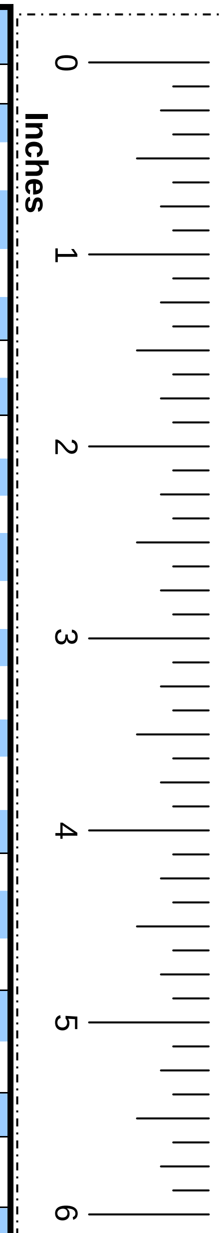
1 day = 24 hours

1 hour = 60 minutes

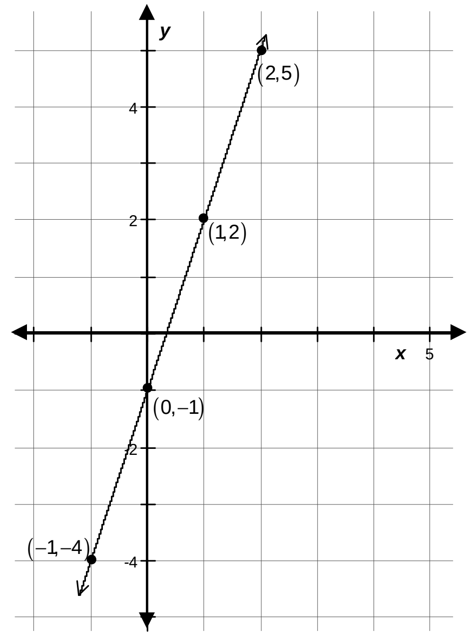
1 minute = 60 seconds

# *TAKS* Mathematics Chart

| | | |
|---|---|---|
| **Perimeter** | Rectangle | $P = 2l + 2w$ or $P = 2(l + w)$ |
| **Circumference** | Circle | $C = 2\pi r$ or $C = \pi d$ |
| **Area** | Rectangle | $A = lw$ or $A = bh$ |
| | Triangle | $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$ |
| | Trapezoid | $A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$ |
| | Regular polygon | $A = \frac{1}{2}aP$ |
| | Circle | $A = \pi r^2$ |
| ***P*** represents the perimeter of the base of a three-dimensional figure. | | |
| ***B*** represents the area of the base of a three-dimensional figure. | | |
| **Surface Area** | Cube (total) | $S = 6s^2$ |
| | Prism (lateral) | $S = Ph$ |
| | Prism (total) | $S = Ph + 2B$ |
| | Pyramid (lateral) | $S = \frac{1}{2}Pl$ |
| | Pyramid (total) | $S = \frac{1}{2}Pl + B$ |
| | Cylinder (lateral) | $S = 2\pi rh$ |
| | Cylinder (total) | $S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$ |
| | Cone (lateral) | $S = \pi rl$ |
| | Cone (total) | $S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$ |
| | Sphere | $S = 4\pi r^2$ |
| **Volume** | Prism or Cylinder | $V = Bh$ |
| | Pyramid or Cone | $V = \frac{1}{3}Bh$ |
| | Sphere | $V = \frac{4}{3}\pi r^3$ |
| **Special Right** | 30°, 60°, 90° | $x,\ x\sqrt{3},\ 2x$ |
| **Triangles** | 45°, 45°, 90° | $x,\ x,\ x\sqrt{2}$ |
| **Pythagorean Theorem** | | $a^2 + b^2 = c^2$ |
| **Distance Formula** | | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ |
| **Slope of a Line** | | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| **Midpoint Formula** | | $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ |
| **Quadratic Formula** | | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| **Slope-Intercept Form of an Equation** | | $y = mx + b$ |
| **Point-Slope Form of an Equation** | | $y - y_1 = m(x - x_1)$ |
| **Standard Form of an Equation** | | $Ax + By = C$ |
| **Simple Interest Formula** | | $I = prt$ |



In a linear equation, an ordered pair $(x, y)$ is a solution if the $x$-coordinate corresponds to the given $y$-coordinate. Graphically, each solution is a point on the line.




The graph at right shows the function $y = 3x - 1$ and four ordered pairs that are solutions to the equation.

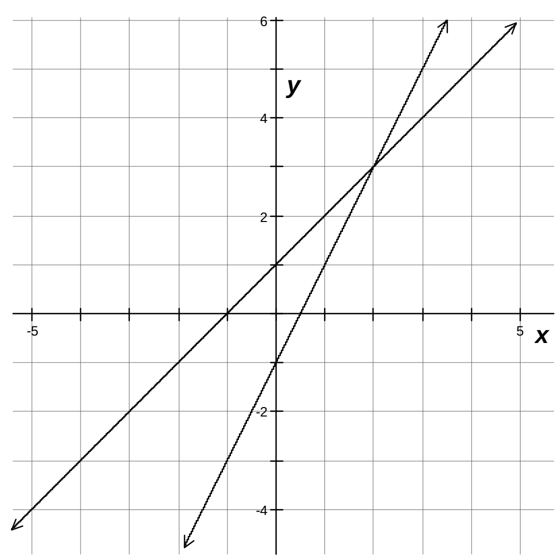
Sometimes, we are given a graph of two lines. This is called a **system of linear equations**.

> A **system of linear equations** is two or more linear equations set equal to each other. They can have one solution, no solution, or infinitely many solutions.

When a system of linear equations is graphed, the solution is the point where the lines intersect.

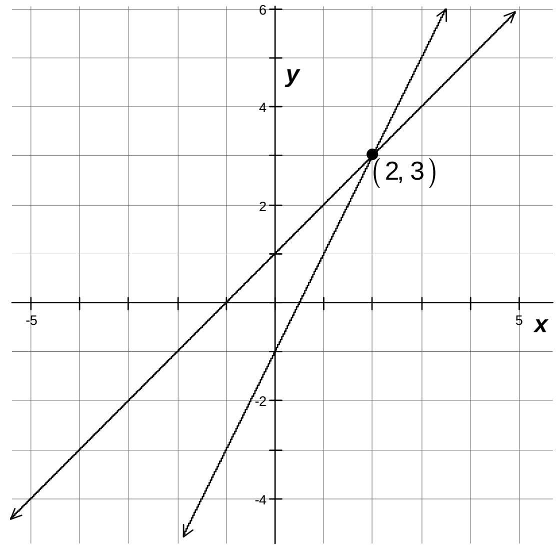
***Example***

The graph shows the equations $y = x + 1$ and $y = 2x - 1$. What is the solution to this system of equations?



***Solution***

The solution to this system of equations is the point where the two lines intersect, $(2, 3)$.




If we substitute these values for *x* and *y* in each of the equations, they work for both. This is how to check your answers.

$$(2, 3)$$

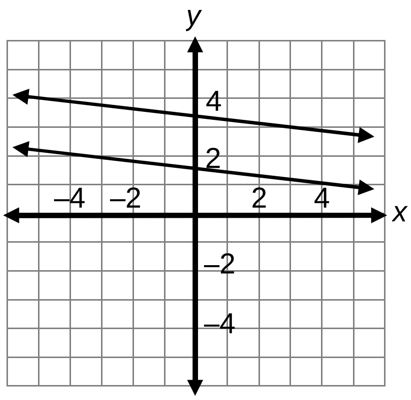
$$\begin{aligned} y &= x + 1 \\ 3 &= 2 + 1 \end{aligned} \qquad \begin{aligned} y &= 2x - 1 \\ 3 &= 2(2) - 1 \\ 3 &= 4 - 1 \end{aligned}$$

No other point works in both of the equations. For example, $(1, 2)$ is a solution for $y = x + 1$, but not for the equation $y = 2x - 1$.

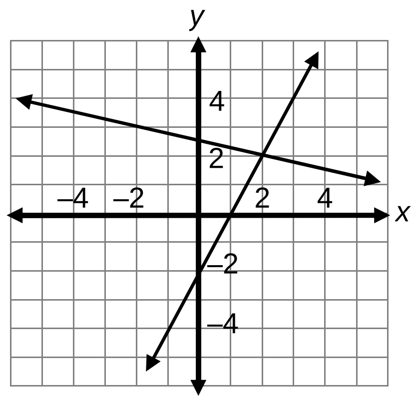
$$(1, 2)$$

$$\begin{aligned} y &= x + 1 \\ 2 &= 1 + 1 \end{aligned} \qquad \begin{aligned} y &= 2x - 1 \\ 2 &= 2(1) - 1 \\ 2 &\neq 2 - 1 \end{aligned}$$

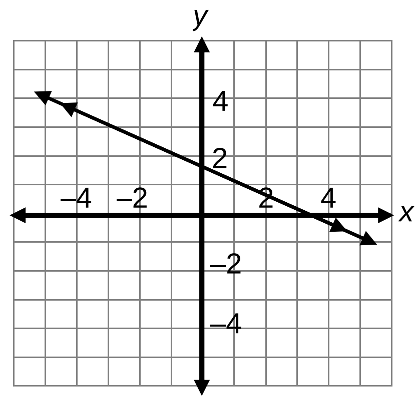
This system of linear equations had one solution. This is not always the case. A system of linear equations can either have one solution, no solution, or infinitely many solutions.




Lines that do not intersect are parallel. They have zero solutions.




Intersecting lines have only one solution. Here, the solution is $(2,2)$.




Lines that intersect at every point (coincident lines) have infinitely many solutions.

We can determine the number of solutions a pair of linear equations has by putting them in $y = mx + b$ form and observing their slopes and *y*-intercepts.

***Example***

How many solutions exist for the following system of equations?

$$y = 3x - 7$$
$$y = 3x + 1$$

***Solution***

We are only given equations to work with in this problem. We can graph the two equations or we can use our knowledge of linear equations to solve this. Both equations are written in $y = mx + b$ form. The two equations have the same slope and different *y*-intercepts.

$$y = 3x - 7$$
$$y = 3x + 1$$

The lines are parallel. Therefore, there is no solution to this system of equations.

***Think Back***

***Parallel lines have equal solve and never intersect.***

***Example***

How many solutions exist for the following system of equations?

$$3y = 3x - 12$$
$$2y - 2x = -8$$

**A** one solution

**B** two solutions

**C** no solution

**D** infinitely many solutions

***Solution***

A system of linear equations cannot have two solutions, so we can eliminate choice **B**. We have to put both of the equations into $y = mx + b$ form by solving each for $y$.

$$\frac{3y}{3} = \frac{3x - 12}{3}$$
$$y = x - 4$$

$$2y - 2x = -8$$
$$+2x \quad +2x$$
$$\frac{2y}{2} = \frac{2x - 8}{2}$$
$$y = x - 4$$

Notice that both equations are the same. This means the two lines are coincident. Thus, there are infinitely many solutions. Choice **D** is the answer.

Any point that is on the line $y = x - 4$ is a solution to both equations. For example,

$$(1, -3)$$

$$3y = 3x - 12$$
$$3(-3) = 3(1) - 12$$
$$-9 = 3 - 12$$
$$-9 = -9$$

$$2y - 2x = -8$$
$$2(-3) - 2(1) = -8$$
$$-6 - 2 = -8$$
$$-8 = -8$$

**Algorithm**

**To find the number of solutions to a system of linear equations problem:**

1) Put both equations in $y = mx + b$ form.
2) Look at the slope ($m$) of each equation.
   a. If they are different, there is one solution. You are done.
   b. If they are the same, move on to step 3.
3) Look at the $y$-intercepts ($b$).
   a. If they are the same, there are infinitely many solutions.
   b. If they are different, the lines are parallel. There is no solution.

***Example***

How many solutions exist for the following system of equations?

$$3y + 12 = 2x$$
$$2x - y = 4$$

**A** one solution

**B** two solutions

**C** no solution

**D** infinitely many solutions

***Solution***

Choice **B** is impossible for two linear equations. Cross it out

*Step 1*: Put both equations in $y = mx + b$ form.

$$\begin{aligned} 3y + 12 &= 2x \\ \underline{-12} \quad & \underline{-12} \\ \frac{3y}{3} &= \frac{2x - 12}{3} \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

$$\begin{aligned} 2x - y &= 4 \\ \underline{-2x} \quad & \underline{-2x} \\ \frac{-y}{-1} &= \frac{-2x + 4}{-1} \\ y &= 2x - 4 \end{aligned}$$

*Step 2*: Compare the slopes.

$$y = \boxed{\tfrac{2}{3}}x - 4 \qquad y = \boxed{2}x - 4$$

The slopes are different, so there is only one solution. Choice **A** is the answer.

Sometimes you may be asked to find a solution for a system of linear equations.

***Example***

Which is the correct solution to the following system of linear equations?

$$3y + 9 = 3x$$
$$2x - y = 4$$

**A** $(1, -2)$

**B** $(0, -3)$

**C** There is no solution

**D** There are infinitely many solutions

***Solution***

As with previous problems, we will put both these equations in $y = mx + b$ form.

$$\begin{aligned} 3y + 9 &= 3x \\ -9 \quad & \quad -9 \\ \frac{3y}{3} &= \frac{3x - 9}{3} \\ y &= x - 3 \end{aligned}$$

$$\begin{aligned} 2x - y &= 4 \\ -2x \quad & \quad -2x \\ \frac{-y}{-1} &= \frac{-2x + 4}{-1} \\ y &= 2x - 4 \end{aligned}$$

The slopes are different, so we know there is a unique solution. Since both of these equations equal *y*, we can set them equal to each other.

$$y = x - 3 \qquad y = 2x - 4$$

$$x - 3 = 2x - 4$$

Now we can solve for *x*.

$$\begin{aligned} x - 3 &= 2x - 4 \\ -x \ +4 & \quad -x \ +4 \\ 1 &= x \end{aligned}$$

We know the answer must be **A**, but what if we needed to find the *y*-value? Input the *x* value into either equation, and see what you get as the *y*-value.

$$\begin{aligned} y &= x - 3 \\ y &= 1 - 3 \\ y &= -2 \end{aligned}$$

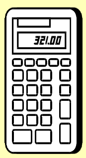
$$\begin{aligned} y &= 2x - 4 \\ y &= 2(1) - 4 \\ y &= -2 \end{aligned}$$

The answer is choice **A** $(1, -2)$.

We substituted 1 for *x* in both equations just to prove that we would get the same *y* value.

You cannot use the guess-and-check method to solve this type of problem. If neither choice **A** nor **B** works, it does not necessarily mean that there are no solutions.

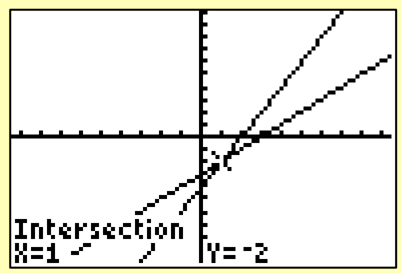
***Calculator Tip***

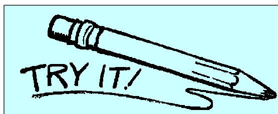



*We can use the calculator to find the point where two lines intersect. After you put both equations into y = mx + b form, enter them as $y_1$ and $y_2$ on the calculator. Press* Y= *and enter them as follows.*


*Graph the function and press* 2nd TRACE *.*

*Then, scroll down and highlight intersection and press* ENTER*. Move the cursor near where the two lines intersect and press* ENTER *three times. The calculator will display the point where the lines intersect.*




**1)** How many solutions exist for the following system of equations?

$$x = 8 + 2y$$
$$2x - 4y = 8$$

**A** one solution

**B** two solutions

**C** no solution

**D** infinitely many solutions

**2)** Which is the correct solution to the following system of linear equations?

$$2y - 3x = 12$$
$$6x = 4y - 24$$

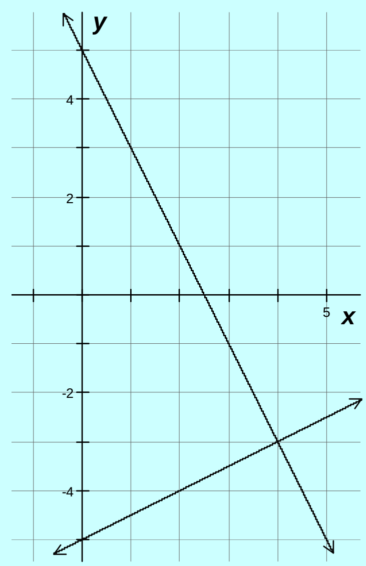
**A** $(0, -4)$

**B** $(1, 1)$

**C** There is no solution

**D** There are infinitely many solutions

**3)** The accompanying graph shows the equations $y = -2x + 5$ and $y = \frac{1}{2}x - 5$. What is the solution to this system of equations?



Sometimes you may be asked to model a situation using a system of equations.

***Example***

Sondra is making cookies that contain both peanut butter chips and chocolate chips. She used a total of 20 ounces of chips. There are 10 chocolate chips in each ounce of chocolate chips, and 15 peanut butter chips in each ounce of peanut butter chips. If Sondra used a total of 400 chips in her cookies, which system of linear equations could be used to find the number of ounces chocolate chips, *c*, and the number of ounces of peanut butter chips, *p*, Sondra used?

**A** $c + p = 20$
$10c + 15p = 400$

**B** $c + p = 400$
$10c + 15p = 20$

**C** $20c + 10p = 400$
$c + p = 20$

**D** $20c + 15p = 10$
$c + p = 400$

***Solution***

It is helpful to first write let statements for the variables given to us.

Let *c* = ounces of chocolate chips

Let *p* = ounces of peanut butter chips

Next, we need to analyze the paragraph to find information about these variables.

"She used a total of 20 ounces of chips."

This statement means if we add the ounces of chocolate chips, *c*, and the peanut butter chips, *p*, it will equal 20.

$$c + p = 20$$

Immediately, we can eliminate choice **B** and **D**.

~~**B** $c + p = 400$~~
$10c + 15p = 20$

**D** $20c + 15p = 10$
~~$c + p = 400$~~

Cross them out.

Now we need to determine the other equation.

"There are 10 chocolate chips in each ounce of chocolate chips, and 15 peanut butter chips in each ounce of peanut butter chips."

If Sondra uses $c$ ounces of chocolate chips, $10c$ is the number of chocolate chips. Similarly, if she uses $p$ ounces of peanut butter chips, $15p$ is the number of peanut butter chips.

Let $10c$ = the <u>number</u> of chocolate chips

Let $15p$ = the <u>number</u> of peanut butter chips

Next, we are given information relating the number of chips used.

"Sondra used a total of 400 chips in her cookies"

If we add the number of chocolate chips and the number of peanut butter chips, it equals 400.

$$10c + 15p = 400$$

This means choice **A** is the answer.

***Example***

Diego has $1.50 in nickels and dimes. If he has a total of 20 coins, which system of linear equations could be used to find the number of nickels, $n$, and the number of dimes, $d$, Diego has?

**A** $n + d = 20$
$.10n + .05d = 1.50$

**B** $n + d = 1.50$
$.10n + .05d = 20$

**C** $.05n + .10d = 20$
$n + d = 1.50$

**D** $.05n + .10d = 1.50$
$n + d = 20$

***Solution***

Write let statements for the variables given to us.

Let $n$ = number of nickels

Let $d$ = number of dimes

Read the question for information about the number of coins.

"He has a total of 20 coins."

Therefore, the number of nickels plus dimes is 20.

$$n + d = 20$$

We can eliminate choices **B** and **C**.

**B** ~~$n + d = 1.50$~~
$.10n + .05d = 20$

Cross them out.

**C** $.05n + .10d = 20$
~~$n + d = 1.50$~~

Next, we will use the value of the coins to come up with the second equation. A nickel is worth \$.05 and a dime worth \$.10.

Let $.05n$ = the dollar amount of nickels

Let $.10d$ = the dollar amount of dimes

Use the information given about the value of the coins.

"Diego has \$1.50 in nickels and dimes."

Thus, the dollar amount of the nickels and dimes should sum to \$1.50.

$$.05n + .10d = 1.50$$

This means choice **D** is the answer.

Suppose we were asked to determine how many of each coin Diego has.

***Example***

Diego has \$1.50 in nickels and dimes. If he has a total of 20 coins, how many of each coin does he have?

***Solution***

We know the system of equations used to solve this problem is as follows.

$$.05n + .10d = 1.50$$
$$n + d = 20$$

To solve this system of equations, we will solve the second equation for the variable *d* in terms of *n*.

$$\begin{aligned} n + d &= 20 \\ \underline{-n} \quad & \quad \underline{-n} \\ d &= 20 - n \end{aligned}$$

Now that we know what *d* is, we substitute it into the other equation.

$$d = 20 - n \qquad .05n + .10d = 1.50$$

$$.05n + .10(20 - n) = 1.50$$

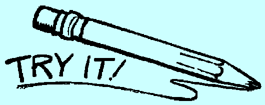
Solve for *n*.

$$\begin{aligned} .05n + .10(20 - n) &= 1.50 \\ .05n + 2 - .10n &= 1.50 \\ 2 - .05n &= 1.50 \\ \underline{-2} \qquad & \quad \underline{-2} \\ \frac{-.05n}{-.05} &= \frac{-.50}{-.05} \\ n &= 10 \end{aligned}$$

Thus, there are 10 nickels. To find the number of dimes, we use the equation $d = 20 - n$. We know *n* = 10, so substitute that into the equation.

$$d = 20 - n$$
$$d = 20 - 10$$
$$d = 10$$

Diego has 10 nickels and 10 dimes.




**4)** Two numbers sum to 30. If twice the first number minus triple the second is 10, which system of linear equations could be used to determine the first number, $x$, and the second number, $y$?

**A** $x + y = 10$
$3x - 2y = 30$

**B** $x + y = 30$
$3x - 2y = 10$

**C** $x + y = 10$
$2x - 3y = 30$

**D** $x + y = 30$
$2x - 3y = 10$

**5)** Jeff has $3.20 in nickels and quarters. If he has a total of 20 coins, how many of each coin does he have?

**A** 9 nickels and 11 quarters

**B** 10 nickels and 10 quarters

**C** 8 nickels and 12 quarters

**D** 12 nickels and 8 quarters

**6)** At a hot dog stand, the cost of a hot dog and a can of soda is $2. The cost for 3 hot dogs and 2 cans of soda is $5.25. Which pair of equations can be used to determine the cost of a hot dog, $h$, and the cost of a soda, $s$?

**A** $h + s = 2$
$3h - 2s = 5.25$

**B** $h + s = 2$
$3h + 2s = 5.25$

**C** $h + s = 2$
$2h + 2s = 5.25$

**D** $h + s = 2$
$2h + 3s = 5.25$

# Review

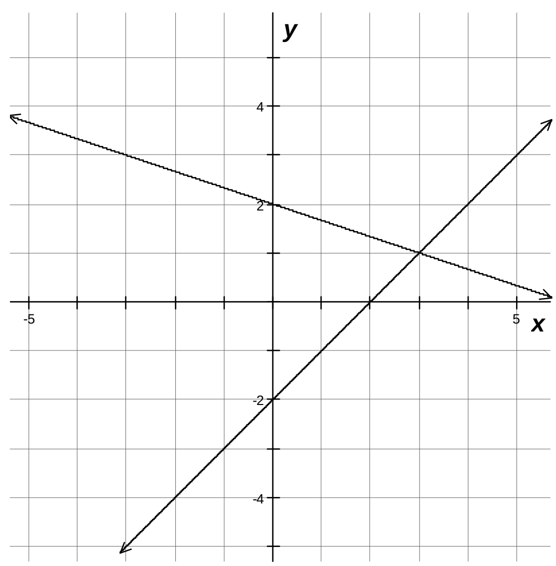
Know these concepts:

1. A system of linear equations has only one solution if the equations represent two distinct lines that intersect.
2. To determine the number of solutions a system of equations has, put both equations in $y = mx + b$ form.
3. A system of linear equations will have no solution if the lines are parallel (same slope and different *y*-intercepts).
4. A system of linear equations will have infinitely many solutions if they represent the same line (same slope and *y*-intercept).
5. To create a system of equations for a real-life situation, write the let-statements first. Then, analyze statements which represent the equations within the question.

## Practice Problems
**Lesson 12**

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 12.

**1)** The accompanying graph shows the equations $y = -\frac{1}{3}x + 2$ and $y = x - 2$. What is the solution to this system of equations?




**A** $(2, 0)$ **B** $(0, 2)$

**C** $(3, 1)$ **D** $(1, 3)$

**2)** How many solutions exist for the following system of equations?

$$y = 2x + 1$$
$$y = x - 1$$

**A** one
**B** two
**C** none
**D** infinitely many

**3)** How many solutions exist for the following system of equations?

$$3y = 2x + 1$$
$$3y + 2x = 1$$

**A** one
**B** two
**C** none
**D** infinitely many

**4)** Which is the best solution to the following system of linear equations?

$$2y + 10 = 3x$$
$$6x - 4y = 20$$

**A** $(1, -2)$
**B** $(-2, 2)$
**C** There is no solution
**D** There are infinitely many solutions

**5)** Wendy has \$1.15 in dimes and quarters. If she has 10 coins, which system of linear equations could be used to find the number of dimes, *d*, and the number of quarters, *q*, Wendy has?

**A** $d + q = 10$
$10d + 25q = 1.15$

**B** $d + q = 10$
$.10d + .25q = 1.15$

**C** $d + q = 10$
$.25d + .10q = 1.15$

**D** $d + q = 10$
$25d + 10q = 1.15$


**1)** C **2)** D **3)** $(4, -3)$ **4)** D **5)** A **6)** B

## NOTES

**End of Lesson 12**