Student Name:	Contact Person Name:
______________________	______________________
Date: ______________________	Phone Number: ______________

Texas Assessment
of Knowledge and Skills

Exit Level Math
Review

Lesson 13
Properties of Quadratic Graphs

TAKS Objective 5 – Demonstrate an understanding of quadratic and other nonlinear functions

Lesson Objectives:

- Identify a quadratic graph's roots, zeros, *x*-intercepts, y- intercepts, vertex, and axis of symmetry
- Interpret and describe the effects of changes in the parameters of quadratic functions

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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Developed by the National PASS Center under the leadership of the National PASS Coordinating Committee with funding from the Region 20 Education Service Center, San Antonio, Texas, as part of the Mathematics Achievement = Success (MAS) Migrant Education Program Consortium Incentive project.

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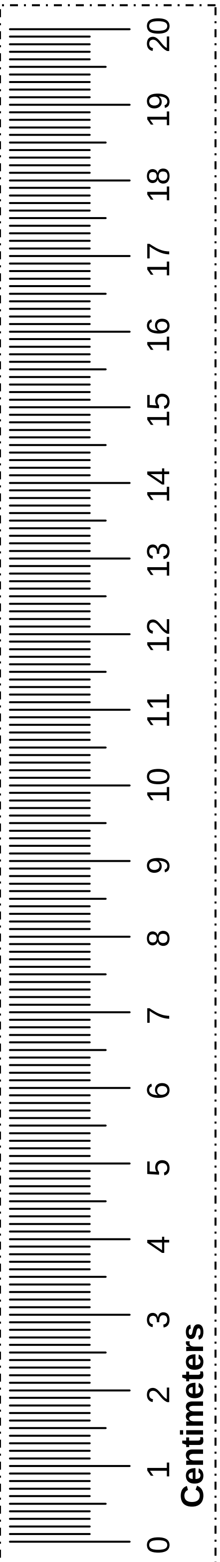


TAKS Mathematics Chart

Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

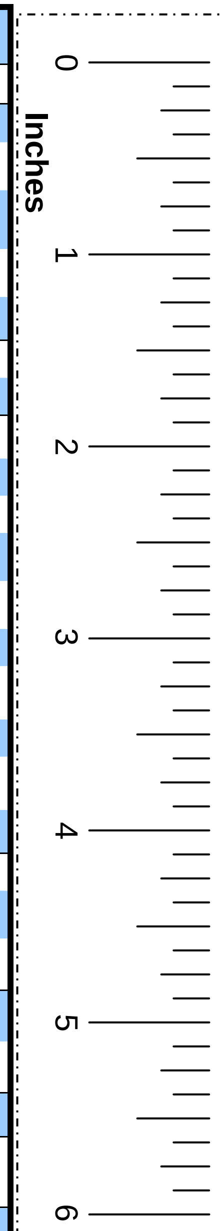
1 day = 24 hours

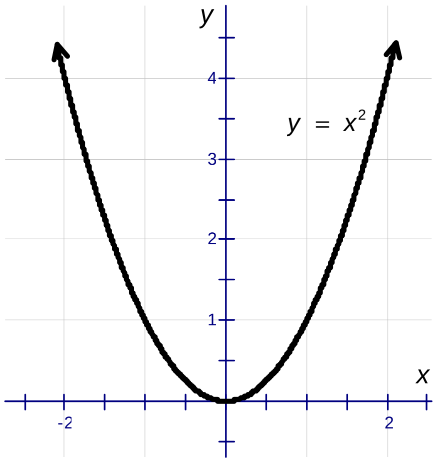
1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, \ x\sqrt{3}, \ 2x$
Triangles	45°, 45°, 90°	$x, \ x, \ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



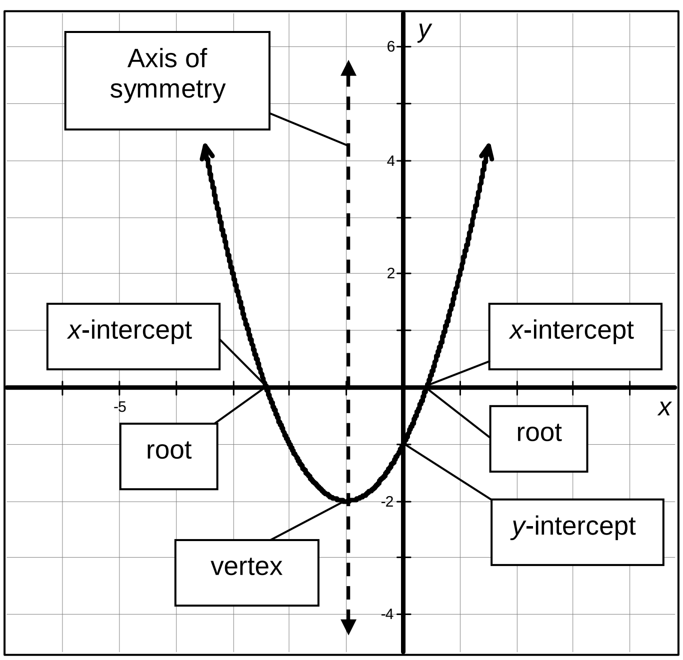


Recall the quadratic parent function, $y = x^2$. This is an example of a **quadratic function**.

A **quadratic function** is any function that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$. Its graph is a parabola.

Take note of all the features of the graph of a quadratic function.

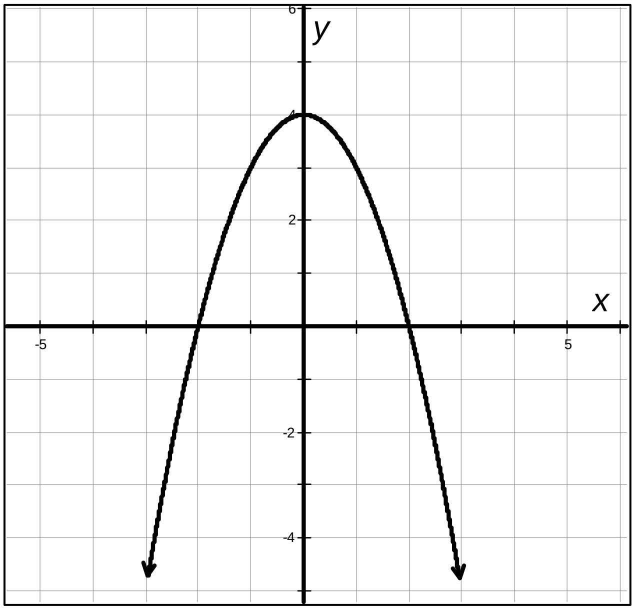
The example below is the graph of the function $y = x^2 + 2x - 1$.



- The **axis of symmetry** is the line that separates the graph of a quadratic function into two equal halves. (x = -1) It always passes through the vertex.
- ***x*-intercepts** are where the graph intersects the x-axis.
- **Roots**, also called **zeros**, are the solution to the quadratic equation $ax^2 + bx + c = 0$. The roots of a quadratic equation coincide with its x-intercepts.
- The **vertex** is the point of the quadratic function's minimum or maximum value.
- The ***y*-intercept** is where the graph intersects the y-axis.

Example

Fill in the missing information in the chart pertaining to the graph below.



Vertex	
Roots	
Zeros	
x-intercepts	
y-intercept	
Axis of symmetry	

Solution

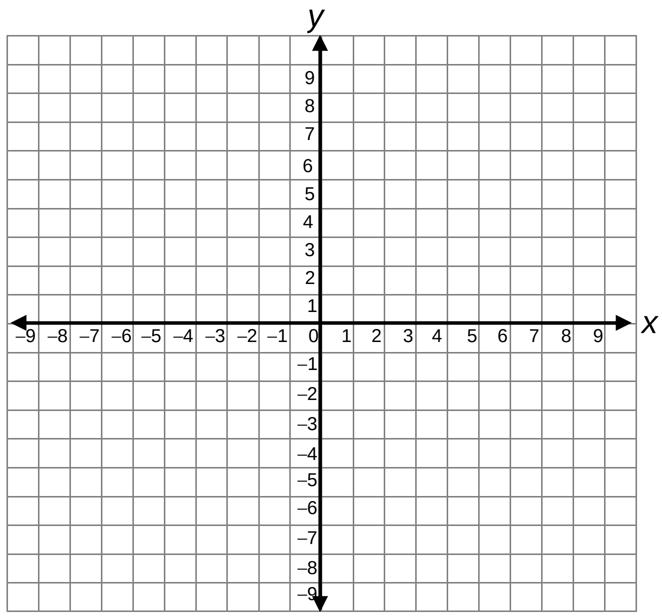
Vertex	$(0,4)$
Roots	$x = -2$ and $x = 2$
Zeros	$x = -2$ and $x = 2$
x-intercepts	$(-2,0)$ and $(2,0)$
y-intercept	$(0,4)$
Axis of symmetry	$x = 0$

FACT

Roots, zeros, and x-intercepts all fall on the exact same points of a quadratic graph. Roots and zeros are expressed as a single value (hence, they are interchangeable words). However, an x-intercept is written as an ordered pair $(x,0)$.

Example

The vertex of the graph of the quadratic function is $(1, 5)$. What are the zeros of this function if the point $(5, 0)$ lies on the graph?



A $x = 3$ and $x = -5$

B $x = 5$ and $x = -3$

C $x = 5$ and $x = 0$

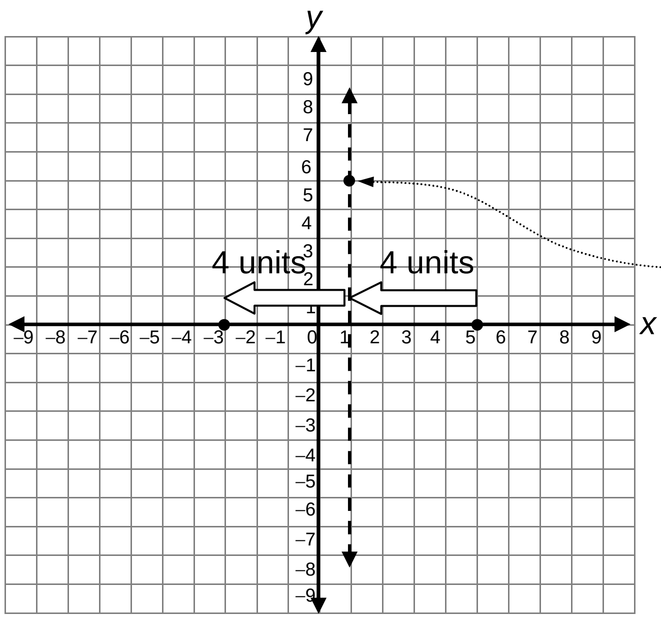
D Cannot be determined

Problem Solving Tip

Before reading the solution, try to solve the problem on your own. This particular problem is completely within your grasp. Take a minute to try to solve it without the explanation on the next page.

Solution

The fact you are given a blank set of axes should signal to you that graphing is useful to solve the problem.



Begin by graphing the given information – the vertex, $(1, 5)$, and the point on the graph, $(5, 0)$. Notice that $x = 5$ is a zero.

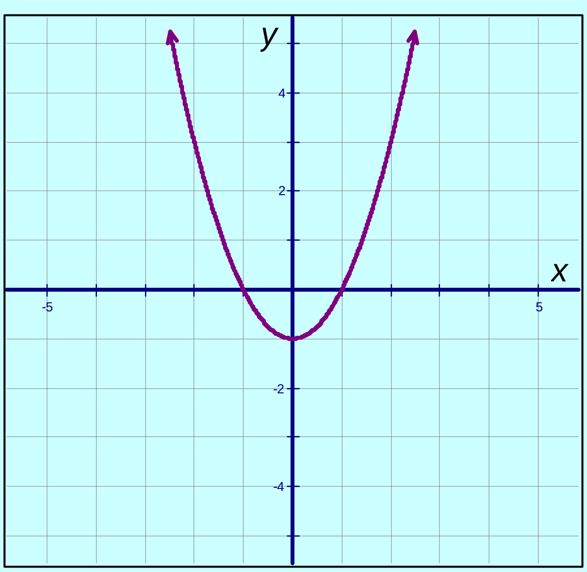
The axis of symmetry intersects the vertex. Sketch it.

Zeros are equidistant from the axis of symmetry. Count the number of units $(5, 0)$ is from the axis of symmetry and match that distance on the other side. This is the location of the second zero.

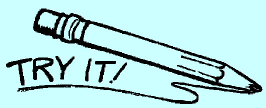
The answer is choice **B**.

TRY IT!

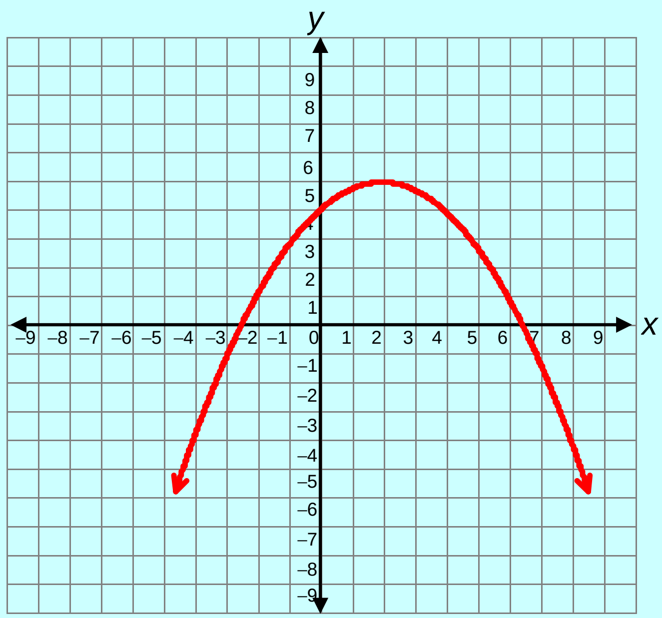
1) Fill in the chart that pertains to the graph below.



Vertex	
Roots	
Zeros	
x-intercepts	
y-intercept	
Axis of symmetry	



2) Which points best represent the roots of the quadratic equation below?



A $(6\frac{1}{2}, 0)$ and $(2, 5)$

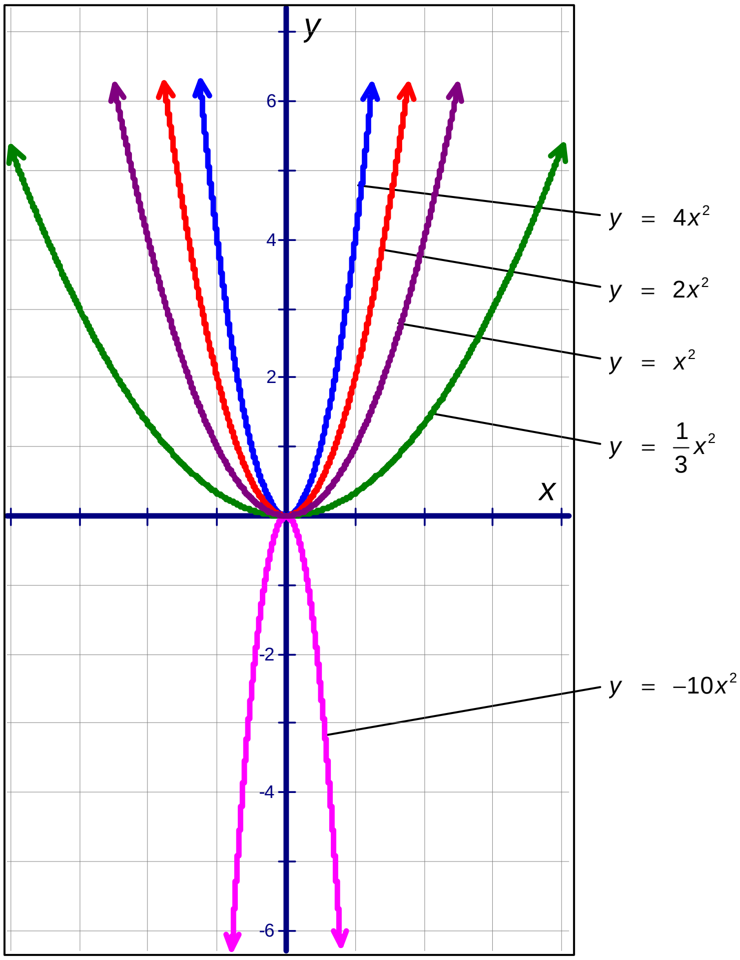
B $(2, 5)$ and $(-2\frac{1}{2}, 0)$

C $(-2\frac{1}{2}, 0)$ and $(6\frac{1}{2}, 0)$

D $(0, -2\frac{1}{2})$ and $(0, 6\frac{1}{2})$

When we change the *a*- or *c*-value of a quadratic function, its graph also changes in ways we can predict. Let us observe these changes and make generalizations about them on the next page.

Varying the *a*-value of $y = ax^2$



Of these six graphs, we have *a*-values of 4, 2, 1, $\frac{1}{3}$, and -10.

Observe:

- When $a > 0$, graphs open upward.
- When $a < 0$, graphs open downward.
- The narrowest graph is when $a = -10$. -10 has the greatest absolute value of the graphs shown.
- The widest graph is when $a = \frac{1}{3}$. $\frac{1}{3}$ has the least absolute value of the graphs shown.

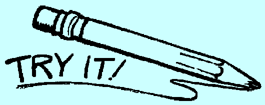
This may seem backward to you. Big absolute values create narrow graphs. Think of it this way: when we multiply the input by a large absolute valued number, it reaches large values at a fast rate, creating a narrow graph. When we multiply the input by a small absolute valued number, it takes longer for the output to reach the same large values. This creates a wide-looking graph.

Problem Solving Tip

To help remember:

When you feel <u>negative</u>, you frown.

When you feel <u>positive</u>, you smile.



3) How does the graph of $y = -\frac{3}{4}x^2$ differ from the graph of $y = \frac{4}{3}x^2$?

A The graph of $y = -\frac{3}{4}x^2$ opens downward and is wider than the graph of $y = \frac{4}{3}x^2$.

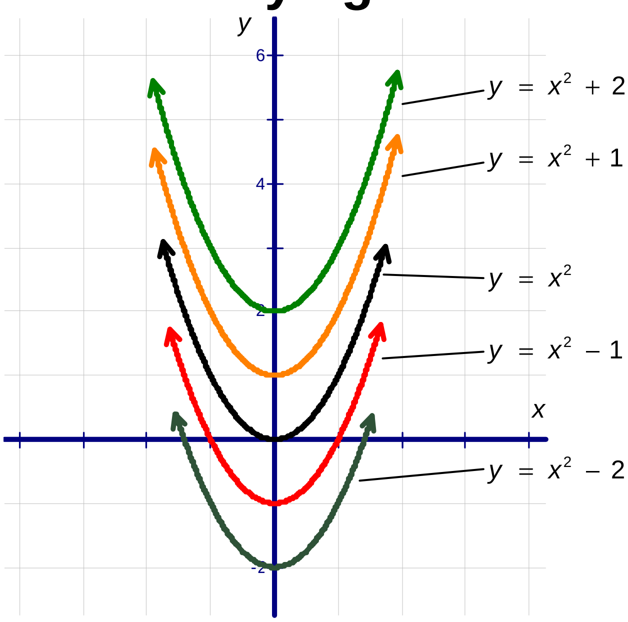
B The graph of $y = -\frac{3}{4}x^2$ opens upward and is wider than the graph of $y = \frac{4}{3}x^2$.

C The graph of $y = -\frac{3}{4}x^2$ opens upward and is narrower than the graph of $y = \frac{4}{3}x^2$.

D The graph of $y = -\frac{3}{4}x^2$ opens downward and is narrower than the graph of $y = \frac{4}{3}x^2$.

Changing the *c*-value alters the graph in a more obvious way.

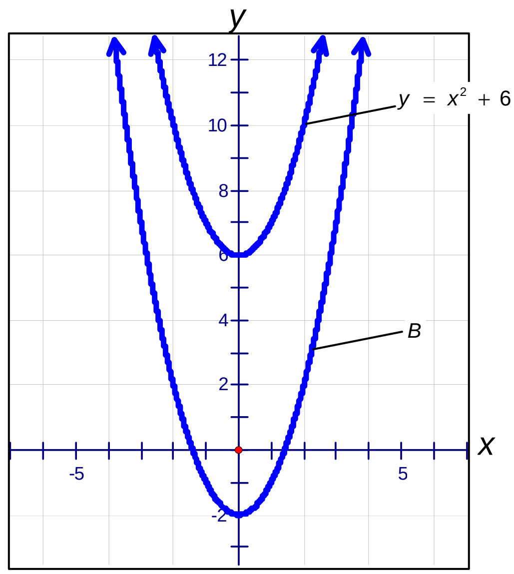
Varying the *c*-value of $y = x^2 + c$



This translates the graph up and down. For instance, the graph $y = x^2 + 2$ is four units higher than the graph of $y = x^2 - 2$. This is because the difference of their *c*-values is $2 - -2 = 4$.

Example

The graph of $y = x^2 + 6$ and its graph translated down 8 units are shown below.



What is the equation of graph B?

A $y = -x^2 - 2$

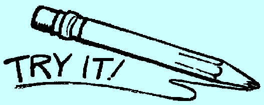
B $y = x^2 - 8$

C $y = x^2 - 2$

D $y = 8x^2 - 2$

Solution

A translation down 8 units means we must subtract 8 from the *c*-value of the original equation. $6 - 8 = -2$. Therefore, the equation of graph B is $y = x^2 - 2$ and the answer is choice **C.**



4) When the graph of $y = 19x^2 + 31$ is translated up 15 units, which of the following equations will best describe the resulting graph?

A $y = 34x^2 + 31$

B $y = 34x^2 + 46$

C $y = 19x^2 + 46$

D $y = 19x^2 + 16$

Review

Know these concepts:

1. A quadratic function can be written in the form $ax^2 + bx + c$.

2. The location and definition of the:

- **a.** axis of symmetry
- **b.** *x*-intercepts
- **c.** roots
- **d.** zeros
- **e.** vertex
- **f.** *y*-intercept

3. Varying the value of *a*.

- **a.** Changes the width of the graph
- **b.** If $a < 0$, the parabola opens downward.
- **c.** If $a > 0$, the parabola opens upward.

4. Varying the value of *c*

- **a.** Translates the graph up or down

Practice Problems
Lesson 13

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 13.

1) The vertex of the graph of a quadratic function is $(-2, 4)$. What are the zeros of this function if the point $(1, 0)$ lies on the graph?

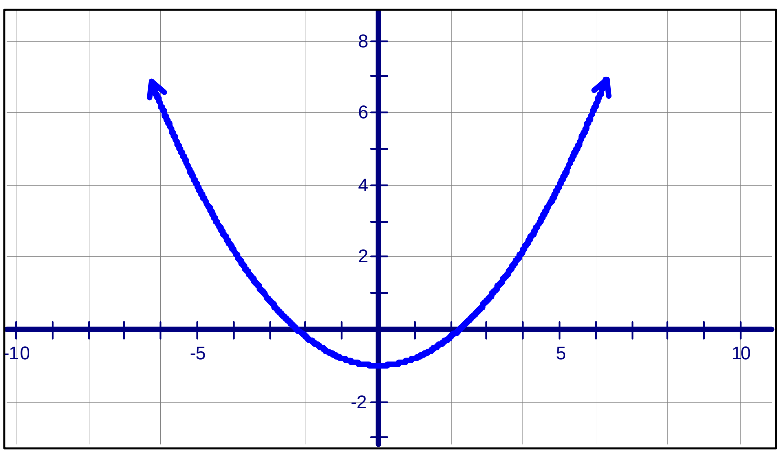
A $x = 5$ and $x = -1$

B $x = 1$ and $x = 0$

C $x = 1$ and $x = -5$

D Cannot be determined

2) The graph of a function of the form $y = ax^2 + c$ is shown below.



If the graph was translated only up or down to include the point (0,2), which of the following equations best represents the resulting graph?

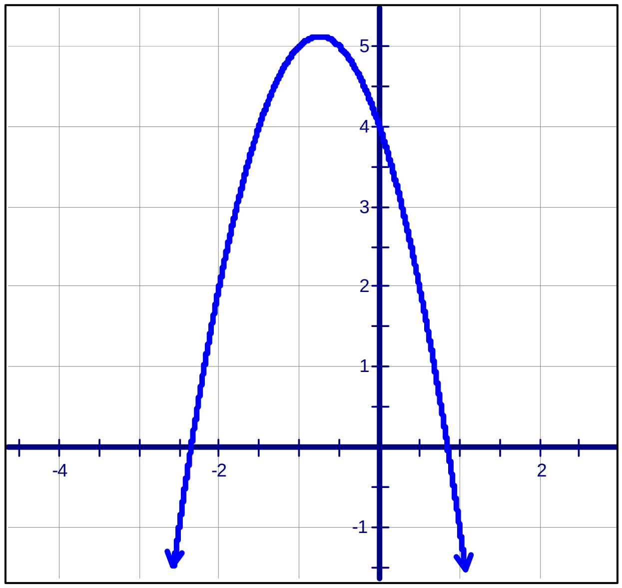
A $y = -\frac{1}{3}x^2 + 2$

B $y = \frac{1}{3}x^2 + 2$

C $y = \frac{1}{3}x^2 + 3$

D $y = -\frac{1}{3}x^2 + 3$

3) The graph of $f(x) = -2x^2 - 3x + 4$ is shown below



Which of the following statements appears to be true?

A The vertex is at $(-1, 5)$.

B The axis of symmetry is $x = -1$.

C One of the roots lies between $x = -3$ and $x = -2$.

D The *y*-intercept is $(4, 0)$.

ANSWERS TO TRY IT

1)

Vertex	(0,-1)
Roots	$x = 1$ and $x = -1$
Zeros	$x = 1$ and $x = -1$
x-intercepts	$(1, 0)$ and $(-1, 0)$
y-intercept	$(0, -1)$
Axis of symmetry	$x = 0$

2) **C**

3) **A**

4) **C**

End of Lesson 13