Student Name: ______________________	Contact Person Name: ______________________
Date: ______________________	Phone Number: ______________________

TAKS

Texas Assessment
of Knowledge and Skills

Exit Level Math
Review

Lesson 14
Solving Quadratic Equations

TAKS Objective 5 – Demonstrate an understanding of quadratic equations and other nonlinear functions

Lesson Objectives:

- Solve a quadratic equation graphically and by using tables
- Factor a quadratic equation to solve it algebraically

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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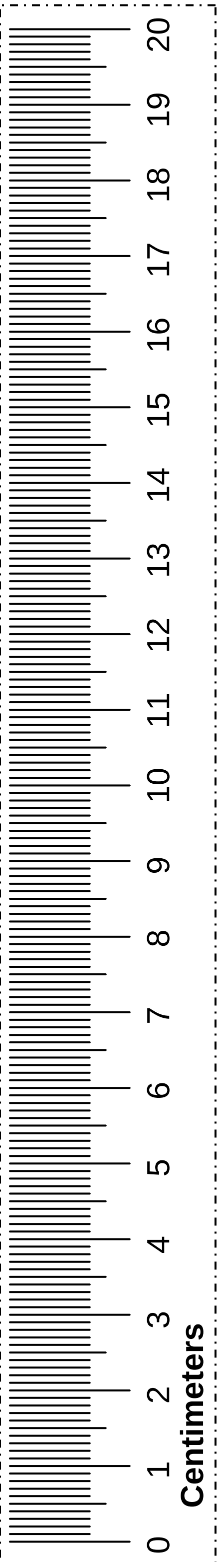
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

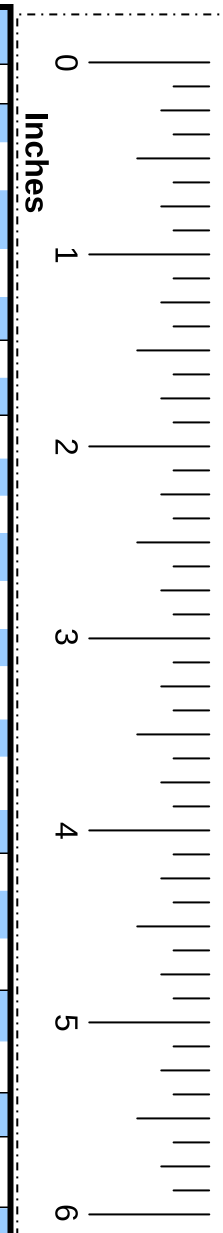
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
Triangles	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



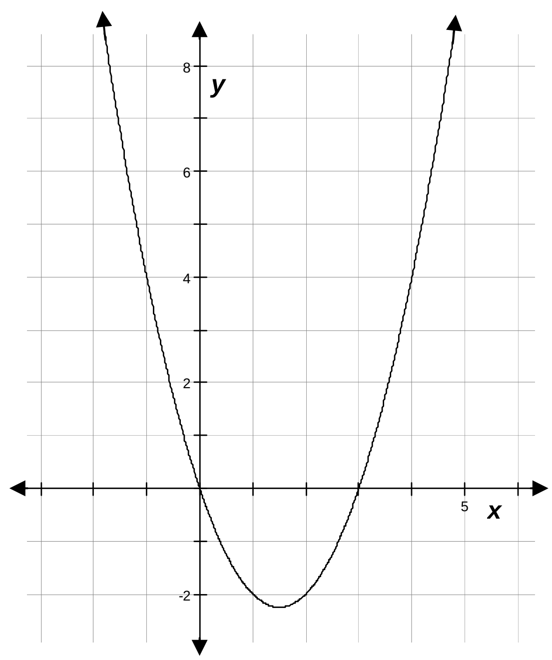
Think Back

A quadratic function is written in the form $y = ax^2 + bx + c$. The graph of a quadratic function is a parabola.

Quadratic equations can be solved graphically, algebraically, or by using a table of values. Solutions to a function such as $y = x^2 + 2x - 1$ consist of all the (x, y) ordered pairs that satisfy the equation. Graphically, this is every point that lies on the graph. However, you must also be able to find the zeros of a quadratic equation.

Example

The given graph is of the function $y = x^2 - 3x$. Using the graph, determine the solution set of the equation $x^2 - 3x = 0$.



A $\{1,2\}$

B $\{0,3\}$

C $\{1,5\}$

D not here

Solution

The graph shows $y = x^2 - 3x$, and we want to know where $x^2 - 3x = 0$. In other words, we are looking for the points on the graph where $y = 0$. This is where the graph intersects the x-axis. We need to find the zeros of the graph. The zeros of the graph are $x = 0, 3$. Thus, the answer is choice **B**.

Note the answers are given using set notation { }. On the test, you may see {4, 7} or you may see $x = 4, x = 7$. They mean the same thing.

If we substitute these values in for *x* algebraically, the equation is true.

$$x = 0, 3$$

$$0 = x^2 - 3x \qquad 0 = x^2 - 3x$$

$$0 = (0)^2 - 3(0) \qquad 0 = (3)^2 - 3(3)$$

$$0 = 0 - 0 \qquad 0 = 9 - 9$$

We can solve a quadratic equation using a table of values as well.

Example

The table provided models the function $f(x) = 2x^2 - 4x - 6$. What is the solution set of the equation $2x^2 - 4x - 6 = 0$?

x	***y***
-2	10
-1	0
0	-6
1	-8
2	-6
3	0
4	10

A $\{-1, 3\}$

B $\{-6\}$

C $\{1, 3\}$

D not here

Solution

Again, we are looking for the zeros of the function. To find them, look to where $y = 0$. This occurs at $x = -1$, and $x = 3$. Thus, the answer is choice **A**.

Substitute these values in for *x* to confirm that these are the zeros of the equation.

$$x = -1 \qquad x = 3$$

$$0 = 2x^2 - 4x - 6 \qquad 0 = 2x^2 - 4x - 6$$

$$0 = 2(-1)^2 - 4(-1) - 6 \qquad 0 = 2(3)^2 - 4(3) - 6$$

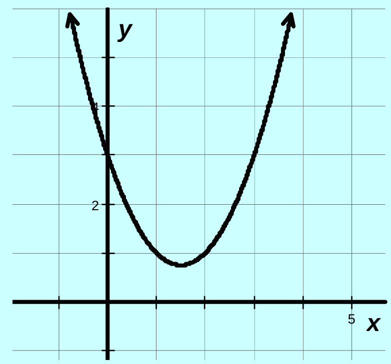
$$0 = 2(1) - 4(-1) - 6 \qquad 0 = 2(9) - 4(3) - 6$$

$$0 = 2 + 4 - 6 \qquad 0 = 18 - 12 - 6$$

Another way to solve these types of problems is by substituting the values of each answer choice for *x* and seeing if they make the equation true.

TRY IT!

1) The given graph is of the function $y = x^2 - 3x + 3$. Using the graph, determine the solution set of the equation $x^2 - 3x + 3 = 0$.



A $\{0, 3\}$

B $\{0\}$

C $\{2, 1\}$

D not here

2) The table provided models the function $f(x) = x^2 - 2x + 1$. What is the solution set to the equation $x^2 - 2x + 1 = 0$?

x	*y*
-2	9
-1	4
0	1
1	0
2	1
3	4
4	9

A $\{-1, 3\}$

B $\{1\}$

C $\{0, 1\}$

D not here

To solve a quadratic equation algebraically, we will need to **factor** the quadratic expression into two linear expressions.

A **factor** is one of the terms multiplied together in a product; if $a \bullet b = c$, then *a* and *b* are factors of *c*. A quadratic expression can be factored into two linear expressions.

Think of factoring as the opposite of multiplying expressions. If we wanted to multiply $x(x-3)$, we would distribute the x into the parentheses and get $x^2 - 3x$.

$$x(x-3) = x^2 - 3x$$
$$x^2 - 3x = x(x-3)$$

This means that the quadratic expression $x^2 - 3x$ can be factored into $x(x-3)$.

When we multiplied two binomials, we used the FOIL method.

$$\begin{aligned}(x-2)(x-3) &= x^2 \underbrace{- 3x - 2x}_{} + 6 \\ &= x^2 - 5x + 6 \\ x^2 - 5x + 6 &= (x-2)(x-3)\end{aligned}$$

This means that the quadratic expression $x^2 - 5x + 6$ can be factored into $(x-2)(x-3)$.

When we factor a quadratic expression, we are really undoing the multiplication. There are three types of quadratic expressions that can be factored.

Think Back

FOIL

First, Outer, Inner, Last.

x^2 $\quad -4$

$$(x+1)(x-4)$$

x

$-4x$

$$= x^2 - 4x + x - 4$$

$$= x^2 - 3x - 4$$

1) Greatest common factor (GCF) binomial:

The GCF binomial has two terms with an *x*. The *x*-term can be factored out.

$$x^2 - 3x = x(x - 3)$$

Sometimes you can factor out the greatest common factor of each term.

$$2x^2 - 10x = 2x(x - 5)$$

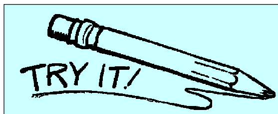
Example

Factor the quadratic expression $4x^2 - 6x$.

Solution

Both terms have an *x* in common, so we can factor out an *x*. Also, the coefficients (4 and 6) have a common factor of 2 $(4 = 2 \cdot 2 \text{ and } 6 = 2 \cdot 3)$.

$$4x^2 - 6x = 2x(2x - 3)$$



Factor the GCF from each quadratic.

3) $x^2 + 2x$

4) $3x^2 - 27x$

2) Difference of perfect squares binomial:

The difference of perfect squares binomial has two terms that are perfect squares. The factors of a difference of perfect squares are two binomials – one is the sum of the square roots, and the other is the difference of the square roots.

$$x^2 - 9 = (x+3)(x-3)$$

When we multiply (FOIL), the inner and outer products cancel out.

$$\underset{3x \quad -3x}{\overset{x^2 \quad -9}{(x+3)(x-3)}} = x^2 \cancel{-3x+3x} - 9 = x^2 - 9$$

Sometimes there may be a coefficient attached to the x^2-term.

Example

Factor the quadratic expression $4x^2 - 25$

Solution

Both terms are perfect squares. $\sqrt{4x^2} = 2x$ and $\sqrt{25} = 5$. The factors will be two binomials – one that is the sum of these terms and one that is the difference.

$$4x^2 - 25 = (2x+5)(2x-5)$$

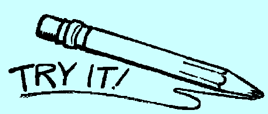
FACT

The square root of a product can be separated into the product of two square roots.

$$\sqrt{4x^2} = \sqrt{4} \cdot \sqrt{x^2}$$

However, the square root of a sum or difference cannot be separated into two square roots.

$$\sqrt{4 - x^2} \neq \sqrt{4} - \sqrt{x^2}$$



Factor each difference of perfect squares.

5) $x^2 - 9$

6) $4x^2 - 64$

3) A trinomial of the form $ax^2 + bx + c$, where $a, b, c \neq 0$:

A trinomial can be factored into two binomials. If the leading coefficient (a) is 1, there is a trick to factoring the trinomial. We know the first term in each of the binomials will be x.

$$x^2 - 2x - 15 = (x \qquad)(x \qquad)$$

Find two numbers whose sum is the coefficient of the middle term (-2) and whose product is the last term (-15). These will be the second terms in each of the binomials.

$$-2 = -5 + 3$$
$$-15 = -5 \cdot 3$$

$$x^2 - 2x - 15 = (x - 5)(x + 3)$$

> ***Think Back***
>
> ***Carry the sign in front of each term.***

Multiply (FOIL) the two binomials to check if you factored correctly.

$x^2 \quad -15$

$$(x - 5)(x + 3) = x^2 + 3x - 5x - 15 = x^2 - 2x - 15$$

$-5x \quad 3x$

Example

Factor the quadratic expression $x^2 - 3x + 2$.

Solution

The leading coefficient is 1, so the first term in each of the binomials will be x.

$$x^2 - 3x + 2 = (x \qquad)(x \qquad)$$

Find two numbers whose sum is -3 and whose product is 2.

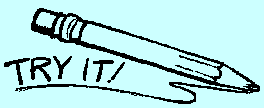
$$-3 = -2 + -1$$
$$-2 = -2 \cdot -1$$

$$x^2 - 3x + 2 = (x - 2)(x - 1)$$

Always multiply (FOIL) the two binomials to check if you factored correctly.

Problem Solving Tip

To determine the signs within the parentheses, look at the *c*-term in the trinomial $ax^2 + bx + c$. If *c* is positive, the signs in the binomial will be the same $(x + \quad)(x + \quad)$ or $(x - \quad)(x - \quad)$. If *c* is negative, the signs will be different $(x + \quad)(x - \quad)$ or $(x - \quad)(x + \quad)$.



Factor each trinomial.

7) $x^2 + 3x + 2$

8) $x^2 - 12x + 35$

The leading coefficient will not always be 1.

Example

Factor the quadratic expression $2x^2 + 5x - 12$.

Solution

There are not two integers whose sum is 5 and product is -12. There are two methods for factoring a trinomial where the leading coefficient is not 1.

Method 1: Guess and Check

The expression $2x^2 + 5x - 12$ will factor into two binomials. The first two terms have to be factors of $2x^2$. Since two is a prime number, the only two factors that work are 2*x* and *x*. Set up your factors as follows:

$$2x^2 + 5x - 12 = (2x \qquad)(x \qquad)$$

Notice the last term is negative. Using our problem solving tip, we know the signs will be different. The second terms must multiply to 12. Possibilities are 2 and 6, 3 and 4, or 1 and 12. Put these values in as the second terms and multiply (FOIL) see if it matches the original trinomial.

$$(2x+6)(x-2) = 2x^2 - 4x + 6x - 12 = 2x^2 + 2x - 12$$

$$(2x-6)(x+2) = 2x^2 + 4x - 6x - 12 = 2x^2 - 2x - 12$$

To save time, find only what the middle term will be. To do this, calculate the inner and outer product (the OI part of FOIL) and add them. If this sum is 5x, you have factored correctly.

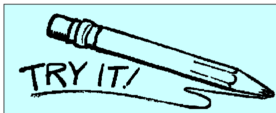
$$(2x+4)(x-3) \qquad \begin{array}{r} 4x \\ +\ -6x \\ \hline -2x \end{array}$$

$$(2x-4)(x+3) \qquad \begin{array}{r} -4x \\ +\ 6x \\ \hline 2x \end{array}$$

$$(2x+3)(x-4) \qquad \begin{array}{r} 3x \\ +\ -8x \\ \hline -5x \end{array}$$

$$(2x-3)(x+4) \qquad \begin{array}{r} 8x \\ +\ -3x \\ \hline 5x \end{array}$$

Stop once you get the correct trinomial. It is $(2x-3)(x+4)$.



Factor each trinomial.

9) $2x^2 + 4x + 2$

10) $4x^2 + 14x - 8$

Method 2: The *a-c* slide method $2x^2 + 5x - 12$

Step 1: Multiply *a*, the leading coefficient (2) and *c*, the last term (-12).

$$2 \cdot -12 = -24$$

Step 2: Find the factors of this number whose sum is *b* (5).

$$-24 = -3 \cdot 8$$

Step 3: Rewrite the expression breaking up the middle term into the sum of these numbers.

$$2x^2 + 5x - 12$$

$$2x^2 - 3x + 8x - 12$$

Step 4: Group the first two terms and the last two terms.

$$2x^2 - 3x + 8x - 12$$
$$\left(2x^2 - 3x\right) + \left(8x - 12\right)$$

Step 5: Factor the GCF out of each set of parentheses.

$$\left(2x^2 - 3x\right) + \left(8x - 12\right)$$
$$x\left(2x - 3\right) + 4\left(2x - 3\right)$$

Step 6: The expression in the parentheses is one of the factors. The other factor is made of the terms in front of the parentheses.

$$x\left(2x - 3\right) + 4\left(2x - 3\right)$$
$$= \left(x + 4\right)\left(2x - 3\right)$$

As expected, $2x^2 + 5x - 12 = \left(2x - 3\right)\left(x + 4\right) = \left(x + 4\right)\left(2x - 3\right)$.

Example

Factor $4x^2 - 7x - 2$ using the *a*-*c* slide method.

Solution

Step 1: Multiply $a = 4$ and $c = -2$.

$$4 \bullet -2 = -8$$

Step 2: Find two factors of -8 whose sum is $b = -7$.

$$-8 = -8 \bullet 1$$

Step 3: Rewrite the given expression, breaking the middle term into two terms with coefficients equal to the two factors found in step 2.

$$4x^2 - 7x - 2$$

$$4x^2 - 8x + x - 2$$

Step 4: Group the first two terms and the last two terms with parentheses.

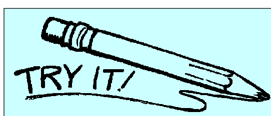
$$4x^2 - 8x + x - 2$$
$$\left(4x^2 - 8x\right) + \left(x - 2\right)$$

Step 5: Factor the GCF out of each set of parentheses.

$$\left(4x^2 - 8x\right) + \left(x - 2\right)$$
$$4x(x - 2) + (x - 2)$$

Step 6: Factor

$$4x(x - 2) + (x - 2)$$
$$(4x + 1)(x - 2)$$



Factor using the *a*-*c* slide method.

11) $3x^2 - 10x + 3$

Sometimes we may need to combine more than one factoring method to completely factor a quadratic expression.

Example

Factor the expression $3x^2 - 12$.

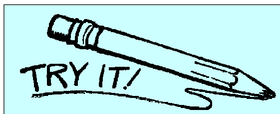
Solution

Notice that neither term is a perfect square and the second term does not have an x. However, both terms share a factor of 3.

$$3x^2 - 12 = 3\left(x^2 - 4\right)$$

Now the binomial in the parentheses is a difference of perfect squares.

$$3\left(x^2 - 4\right) = 3(x + 2)(x - 2)$$



Factor the following quadratic expressions fully.

12) $12x^2 - 18x$

13) $2x^2 - 72x$

14) $x^2 - 9x + 14$

15) $3x^2 + 21x + 30$

To solve a quadratic equation algebraically, we need to factor the quadratic expression. In the beginning of the lesson, notice how all the quadratic expressions were set equal to zero. This is necessary for solving a quadratic equation.

Example

What is the solution set to the equation $x^2 - 5x + 6 = 0$?

A $\{2,3\}$ **B** $\{-2,-3\}$

C $\{0\}$ **D** not here

Solution

Factor the quadratic expression. The leading coefficient is 1, so find two numbers whose sum is the middle coefficient (-5) and whose product is the last term (6).

$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$

One of these factors must be zero. We must set each of the linear factors equal to zero and solve for *x*.

$$(x-2)(x-3) = 0$$

$$\begin{array}{r|r} x - 2 = 0 & x - 3 = 0 \\ \underline{+2 \quad +2} & \underline{+3 \quad +3} \\ x = 2 & x = 3 \end{array}$$

The solution set is $\{2,3\}$, so the answer is choice **A**.

FACT

If the product of two factors is zero, then at least one of the factors must be zero. For example, $(x)(y) = 0$ *when either* $x = 0$, $y = 0$, *or both equal zero. Therefore, if* $(x-2)(x-3) = 0$, *then* $x - 2 = 0$ *or* $x - 3 = 0$.

Sometimes the quadratic expression may not be equal to 0.

Example

What is the solution set of the equation $2x^2 - 10x + 12 = 4$?

A $\{-2, 3\}$ **B** $\{1\}$

C $\{4\}$ **D** not here

Solution

Get the quadratic equation equal to zero. Subtract 4 from both sides.

$$\begin{array}{rcl} 2x^2 - 10x + 12 & = & 4 \\ -4 & & -4 \\ \hline 2x^2 - 10x + 8 & = & 0 \end{array}$$

Now that the quadratic expression is equal to zero, we can factor it.

Factor the GCF first.
$$2x^2 - 10x + 8 = 0$$
$$2\left(x^2 - 5x + 4\right) = 0$$

Factor the trinomial inside the parentheses.
$$2(x - 4)(x - 1) = 0$$

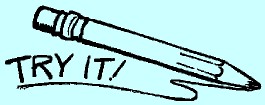
Set each factor equal to zero and solve for x.

$$2(x - 4)(x - 1) = 0$$

$$\cancel{2 \neq 0} \qquad \begin{array}{rcl} x - 4 & = & 0 \\ +4 & & +4 \\ \hline x & = & 4 \end{array} \qquad \begin{array}{rcl} x - 1 & = & 0 \\ +1 & & +1 \\ \hline x & = & 1 \end{array}$$

When we factor out a constant, we can ignore it as a solution.

The solution set is $\{1, 4\}$, so the answer is choice **D**.



16) What are the zeros of the function $f(x) = -7(x+2)(x-4)$?

A -14 and 28 **B** 4 and -2

C -7 and 4 **D** 2 and -4

17) What is the solution set to the equation $x^2 - 8x + 6 = 6$?

A $\{2,6\}$ **B** $\{0,8\}$

C $\{-2,-6\}$ **D** not here

18) What is the solution set to the equation $x^2 - 5x - 2 = 2 - 5x$?

A $\{0,2\}$ **B** $\{-2,2\}$

C $\{0,8\}$ **D** $\{2,3\}$

Review

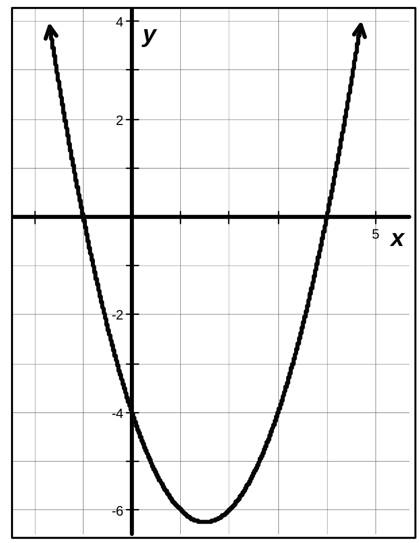
Know these concepts:

1. To solve a quadratic equation graphically, you need to see where the graph crosses the x-axis.
2. To solve a quadratic equation using a table, you need to find the rows where the y-value is zero.
3. There are three types of quadratic expressions that are factorable.
    a. GCF binomial
        i. Factor out the x and the common factor of the coefficients if possible
    b. Difference of perfect squares
        i. Factor into two binomials – one is the sum of the square roots of each term, the other is the difference of the square roots.
    c. A trinomial
        i. If the leading coefficient is 1, find two numbers whose sum is the middle coefficient and whose product is the last term.
        ii. If the leading coefficient is not 1, use the a-c slide method.
4. To solve a quadratic equation, make sure the quadratic expression is equal to zero.

Practice Problems
Lesson 14

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 14.

1) The given graph is of the function $y = x^2 - 3x - 4$. Using the graph, determine the solution set of the equation $x^2 - 3x - 4 = 0$.



A $\{1, 4\}$

B $\{-1, 4\}$

C $\{-4\}$

D not here

Factor the following quadratic expressions completely.

2) $x^2 + x - 20$ **3)** $4x^2 + 4x - 24$ **4)** $x^2 - y^2$

5) What is the solution set to the equation $6x^2 - 11x + 10 = 0$?

A $\{-1, -10\}$ **B** $\left\{-\frac{5}{3}, 1\right\}$

C $\left\{-\frac{2}{3}, \frac{5}{2}\right\}$ **D** not here

6) What is the solution set to the equation $2x^2 + 3x + 8 = 3x - 10$?

A $\{0, 2\}$ **B** $\{-3\}$

C $\{3\}$ **D** not here

ANSWERS TO TRY IT

1) **D**

2) **B**

3) $x(x+2)$

4) $3x(x-9)$

5) $(x+3)(x-3)$

6) $(2x+8)(2x-8)$

7) $(x+2)(x+1)$

8) $(x-7)(x-5)$

9) $(2x+2)(x+1)$

10) $2(2x-1)(x+4)$

11) $3x^2 - 10x + 3$

$= 3x^2 - x - 9x + 3$

$= (3x^2 - x) + (-9x + 3)$

$= x(3x - 1) - 3(3x - 1)$

$= (x - 3)(3x - 1)$

12) $6x(2x-3)$

13) $2x(x-36)$

14) $(x-7)(x-2)$

15) $3(x+5)(x+2)$

16) **B**

17) **B**

18) **B**

End of Lesson 14