Student Name: ______________________	Contact Person Name: ______________________
Date: ______________________	Phone Number: ______________________

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 15

Using the Quadratic Formula & Learning the Laws of Exponents

TAKS Objective 5 – Demonstrate an understanding of quadratic equations and other nonlinear functions

Lesson Objectives:

- Solve quadratic equations using the quadratic formula
- Discover the rules of exponents
- Use rules of exponents to simplify nonlinear expressions

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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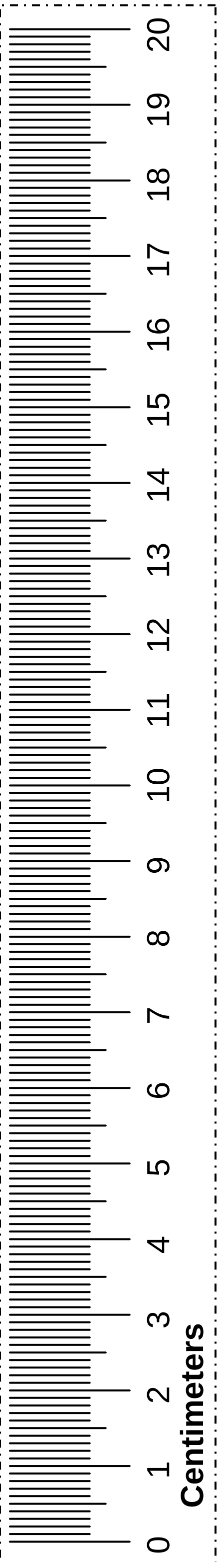
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

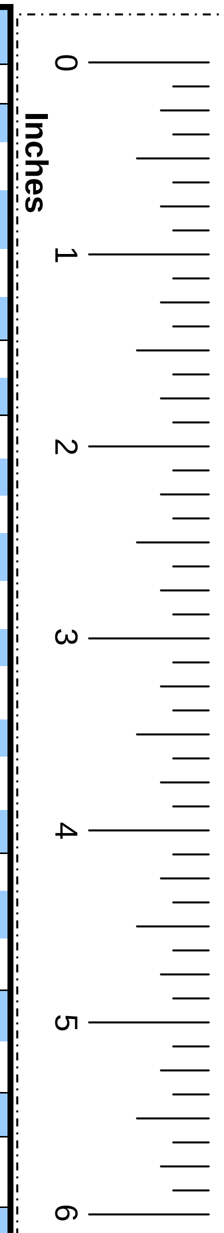
1 day = 24 hours

1 hour = 60 minutes

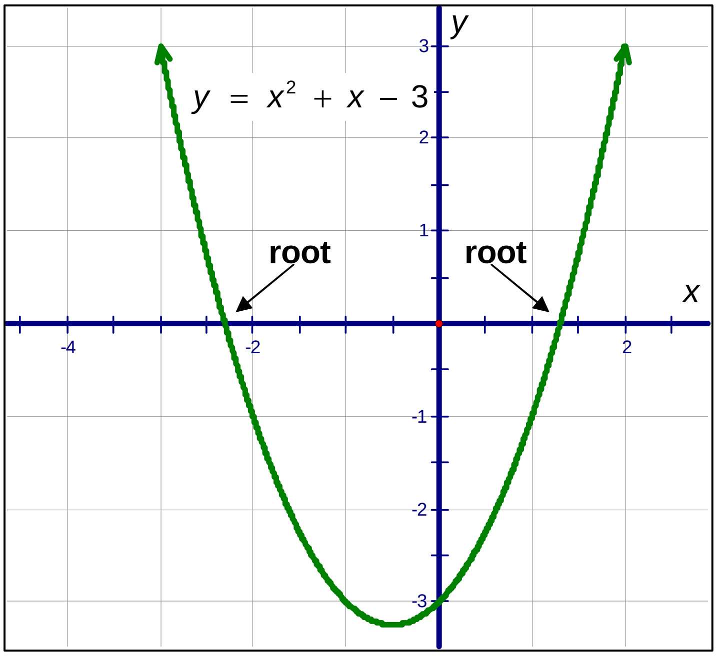
1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
Triangles	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Not every quadratic equation can be factored. We cannot factor $x^2 + x - 3 = 0$. This does not mean the function has no roots. Clearly, from the graph, we see that the equation has two roots.



To find the roots, we will use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Though it is helpful to, you do not need to memorize this formula; it is provided on the exam.

This formula can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. Observe below how to use this formula for the above equation.

$x^2 + x - 3 = 0$ Identify the values of *a*, *b*, and *c*.

$1x^2 + 1x - 3 = 0$

$a = 1 \quad b = 1 \quad c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-3)}}{2(1)}$$

Substitute *a*, *b*, and *c* into the quadratic formula.

$$x = \frac{-1 \pm \sqrt{1 + 12}}{2}$$

Simplify the top and bottom. There are two roots.

$$x = \frac{-1 \pm \sqrt{13}}{2}$$

They are $x = \frac{-1 + \sqrt{13}}{2}$ and $x = \frac{-1 - \sqrt{13}}{2}$.

Example

What is the solution set to the equation $2x^2 - 5x = 1$?

A $\{-2, 3\}$

B $\left\{\dfrac{5 + \sqrt{33}}{4}, \dfrac{5 - \sqrt{33}}{4}\right\}$

C $\left\{\dfrac{-5 + \sqrt{33}}{4}, \dfrac{-5 - \sqrt{33}}{4}\right\}$

D not here

Solution

To use the quadratic formula, a quadratic equation must always be in $ax^2 + bx + c = 0$ form. For this example, we do this by subtracting 1 from both sides.

$$\begin{array}{r} 2x^2 - 5x = 1 \\ \underline{-1} \quad \underline{-1} \\ 2x^2 - 5x - 1 = 0 \end{array}$$

Next, determine the values for *a*, *b*, and *c*.

$$\boxed{2}x^2 \boxed{-5}x \boxed{-1} = 0$$

$$a = 2$$
$$b = -5$$
$$c = -1$$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)}$$

Always substitute using parentheses. This helps avoid the common error of losing a negative sign.

Simplify using the correct order of operations (PEMDAS).

$$x = \frac{-(-5) \pm \sqrt{25 - 4(2)(-1)}}{2(2)}$$

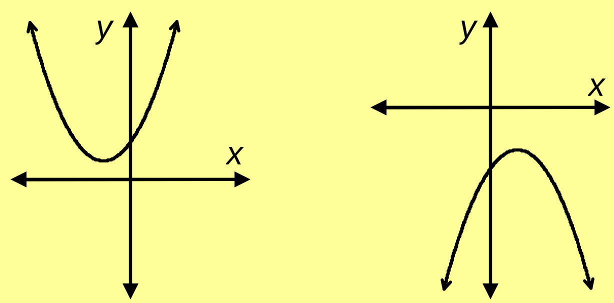
$$x = \frac{5 \pm \sqrt{25 + 8}}{4}$$

$$x = \frac{5 \pm \sqrt{33}}{4}$$

The two values for x are $x = \frac{5 + \sqrt{33}}{4}$ and $x = \frac{5 - \sqrt{33}}{4}$. The answer is choice **B**.

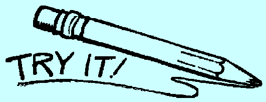
FACT

If the quadratic formula gives a negative number under the radical, there are no real roots. The graph of this function does not touch the x-axis.



Problem Solving Tip

When in doubt, use the quadratic formula. The roots of any quadratic equation can be found using the formula. So, if you're having trouble factoring, use the quadratic formula instead.



Use the quadratic formula to find the roots.

1) $x^2 + 3x - 1 = 0$

2) $-2x^2 - x + 3 = 0$

3) $3x^2 - 6x + 7 = 3$

4) $2x^2 - x - 4 = x^2$

Problem Solving Tip

To save time, calculate what is under the radical first.

If it is negative, we know there are no real solutions,

and there is no need to continue simplifying.

Next, we will further explore how to work with exponents.

Think Back

Exponents tell the number of factors.

$$x^2 = x \bullet x$$

If no exponent is shown, it is assumed to be one.

$$x = x^1$$

Exponents represent repeated multiplication. For example,

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$$

The same is true of a variable raised to an exponent.

$$a^5 = a \cdot a \cdot a \cdot a \cdot a$$

In this case, *a* is the **base**, and 5 is the **exponent**.

Exponent of zero

Use the following pattern to help you understand how to raise 2 to the zero power.

$2^4 = 16$

÷2

$2^3 = 8$

÷2

$2^2 = 4$

÷2

$2^1 = 2$

To decrease the exponent by 1, divide by the base, 2.

Divide by two once more. ÷2

$2^0 = 1$

RULE: Raising a nonzero base to an exponent of 0 is always 1.

$$x^0 = 1, \qquad x \neq 0$$

Negative exponents

We can understand negative exponents by continuing the previous pattern.

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

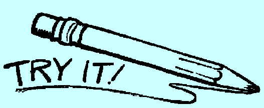
Continue dividing by 2.

$2^{-1} = \frac{1}{2} = \frac{1}{2^1}$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

RULE: $x^{-a} = \frac{1}{x^a}$



Simplify each expression. Use positive exponents.

5) 9^0

6) x^{-4}

7) 8^3

8) y^0

9) $\frac{1}{z^{-3}}$

10) 12^1

We will use these basic properties to form rules for multiplying, dividing, or raising exponents to a power.

Multiplying exponents

Examples

$x^3 \bullet x^2$

$= (x \bullet x \bullet x) \bullet (x \bullet x)$

$= xxxxx$

$= x^5$

$a^2 \bullet a^4$

$= (a \bullet a) \bullet (a \bullet a \bullet a \bullet a)$

$= aaaaaa$

$= a^6$

RULE: When multiplying in the same base, add the exponents.

$$x^a \bullet x^b = x^{(a+b)}$$

Dividing exponents

Examples

$$\frac{x^7}{x^3} = \frac{xxxxxxx}{xxx} = \frac{\cancel{xxx}xxxx}{\cancel{xxx}} = xxxx = x^4$$

$$\frac{v^4}{v} = \frac{v^4}{v^1} = \frac{vvvv}{v} = \frac{\cancel{v}vvv}{\cancel{v}} = vvv = v^3$$

$$\frac{a^2}{a^6} = \frac{aa}{aaaaaa} = \frac{\cancel{aa}}{\cancel{aa}aaaa} = \frac{1}{a^4} = a^{-4}$$

RULE: When dividing in the same base, subtract the exponents.

$$\frac{x^a}{x^b} = x^{(a-b)}$$

Raising exponents to a power

Examples

$\left(x^2\right)^3$

$= \left(x^2\right) \cdot \left(x^2\right) \cdot \left(x^2\right)$

$= \left(xx\right) \cdot \left(xx\right) \cdot \left(xx\right)$

$= xxxxxx$

$= x^6$

$\left(a^5\right)^3$

$= \left(a^5\right)\left(a^5\right)\left(a^5\right)$

$= \left(aaaaa\right)\left(aaaaa\right)\left(aaaaa\right)$

$= aaaaaaaaaaaaaaa$

$= a^{15}$

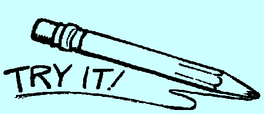
RULE: When raising to a power, multiply the exponents.

$$\left(x^a\right)^b = x^{ab}$$

Problem Solving Tip

Create a simple example to remind yourself of the rule.

$a \cdot a = a^2 \Rightarrow$ **add exponents**



Simplify each expression.

11) $x^2 \cdot x^8$

12) $\dfrac{a^9}{a^5}$

13) $\left(k^7\right)^2$

Each rule of exponents can be used in a single expression.

Example

Which expression is equivalent to $\dfrac{-(4x^2y)^2(3xy^2)}{6xy^3}$?

A $-8x^4y$

B $8x^2y$

C $-2x^2$

D $2x^4y$

Solution

We must solve this directly, using order of operations. The rules of exponents can only be used with the same <u>same base</u>. (x^2y^2 cannot be combined.)

$$\frac{-(4x^2y)^2(3xy^2)}{6xy^3}$$

$$= \frac{-(4x^2y)^2(3xy^2)}{6xy^3}$$

Square each factor within the parentheses (multiply exponents).

$$= \frac{-(16x^4y^2)(3xy^2)}{6xy^3}$$

$$= \frac{-(16x^4y^2)(3xy^2)}{6xy^3}$$

Next, simplify the numerator by multiplying (add exponents of factors with the same base).

$$= \frac{-48x^5y^4}{6xy^3}$$

Finally, divide the top by the bottom of the fraction (subtract exponents).

$$= -8x^4y$$

The correct answer is choice **A**.

You may need to use the rules of exponents to solve situational problems.

Example

Which expression best describes the volume of a rectangular prism that has a height of $3x^4$ units and a square base with side lengths of $4x^4$

A $12x^8$ units^3

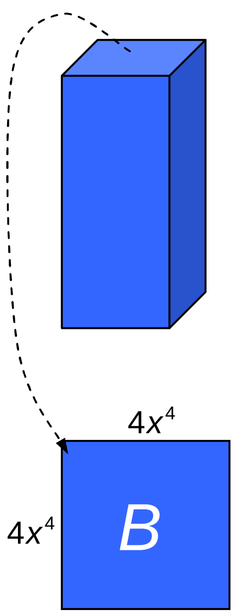
B $24x^8$ units^3

C $48x^{12}$ units^3

D $11x^{12}$ units^3

Solution

On the test, you are given the formula for the volume of a prism as $V = Bh$, where B is the area of the base, and h is the height of the prism. To help use this formula, we will draw a picture of a rectangular prism.



Be careful not to choose choice **A**. This is the most common error, found by multiplying the two values given.

First, we must find B, the area of the base. We are told it is a square. Two approaches to find this are:

$$\begin{aligned} B &= (\textbf{side length})^2 \\ &= \left(4x^4\right)^2 \\ &= \left(4x^4\right)^2 \\ &= 16x^8 \end{aligned}$$

and

$$\begin{aligned} B &= (\textbf{length}) \cdot (\textbf{width}) \\ &= \left(4x^4\right) \cdot \left(4x^4\right) \\ &= \left(4x^4\right) \cdot \left(4x^4\right) \\ &= 16x^8 \end{aligned}$$

Next, we multiply $B = 16x^8$ by the height, $h = 3x^4$

$$\begin{aligned} V &= Bh \\ &= \left(16x^8\right)\left(3x^4\right) \\ &= 48x^{12} \end{aligned}$$

The answer is choice **C**.

Example

The length of a rectangle is $4x^2yz^2$ units, and the rectangle's area is $12x^4y^3z^2$ square units. If $x \neq 0$, $y \neq 0$, and $z \neq 0$, which of the following best describes the width of the rectangle?

A $3x^2y^2$ units

B $3x^6y^4z^4$ units

C $8x^2y^2$ units

D $16x^6y^4z^4$ units

Solution

We must first understand how to set up the problem.

Given: Area and length

Find: Width

Useful formula: $A = lw$

Solve it for *w*, width.

$$A = lw$$
$$\frac{A}{l} = \frac{\cancel{l}w}{\cancel{l}}$$
$$\frac{A}{l} = w$$

Divide area over length. That is, simplify:

$\frac{12x^4y^3z^2}{4x^2yz^2}$ The factors line up nicely.

$= \frac{\overset{3}{\cancel{12}}x^4y^3z^2}{\cancel{4}x^2yz^2}$ Start by dividing 12 and 4.

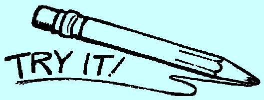
$= \frac{\overset{3}{\cancel{12}}x^{\overset{2}{\cancel{4}}}y^3z^2}{\cancel{4}x^{\cancel{2}}yz^2}$ Next, simplify, the *x*'s

$= \frac{\overset{3}{\cancel{12}}x^{\overset{2}{\cancel{4}}}y^{\overset{2}{\cancel{3}}}z^2}{\cancel{4}x^{\cancel{2}}\cancel{y}z^2}$ Simplify *y*'s

$= \frac{\overset{3}{\cancel{12}}x^{\overset{2}{\cancel{4}}}y^{\overset{2}{\cancel{3}}}\cancel{z^2}}{\cancel{4}x^{\cancel{2}}\cancel{y}\cancel{z^2}}$ Cancel the z^2 factors.

$= 3x^2y^2$ Finally, write the simplified answer.

The answer is choice **A**.



Simplify each expression.

14) $$\frac{\left(8ab^{2}\right)\left(2a^{2}b\right)^{2}}{16a^{4}b}$$

15) $$\frac{16a^{7}b^{5}}{-(2ab)^{3}(a^{2}b)}$$

16) A triangle has a base of length $4ab^{3}$ units and a height of $7a^{4}b^{2}$ units. Which shows the area of the triangle?

A $28a^{5}b^{5}$

B $14a^{5}b^{5}$

C $\frac{7}{4}a^{3}b^{-1}$

D not here

Review

Know these concepts:

1. The quadratic formula can be used to solve <u>any</u> quadratic equation, even ones that can be factored.

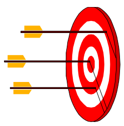
- **a.** To use it, the equation must be in $ax^{2}+bx+c=0$ form. Sometimes, algebraic methods must be used to reach this form.
- **b.** If the value within the radical is negative, the quadratic has no real roots – write "no real roots" if this is the case.

2. Rules of exponents

- **a.** $x^{0}=1$
- **b.** $x^{-a}=\frac{1}{x^{a}}$
- **c.** $x^{a}\bullet x^{b}=x^{(a+b)}$

d. $\dfrac{x^a}{x^b} = x^{(a-b)}$

e. $(x^a)^b = x^{ab}$



Practice Problems
Lesson 15

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 15.

Find the zeros using the quadratic formula. If no real zeros exist, write "no real solutions."

1) $4x^2 + x + 2 = 0$

2) $3x + 9 = -x^2 + x$

3) What is the solution set for the equation $4(3x - 2)^2 = 36$?

A $\{-\frac{11}{6}, \frac{11}{6}\}$

B $\{-\frac{11}{3}, \frac{11}{3}\}$

C $\{-\frac{1}{3}, \frac{5}{3}\}$

D $\{-\frac{3}{3}, \frac{4}{3}\}$

4) Which expression represents the simplified form of $\left(3m^{-2}n^4\right)\left(-4m^6n^{-7}\right)$?

A $-\dfrac{12m^4}{n^3}$

B $-\dfrac{1}{12m^4n^3}$

C $-\dfrac{m^4n^3}{12}$

D $-\dfrac{12n^3}{m^4}$

5) What is the simplified form of $\dfrac{\left(-6a^3b^5\right)\left(2a^2b^3\right)}{-18a^4b^8c^3}$?

A $-\dfrac{2a^2b}{3c^3}$

B $\dfrac{2a}{3c^3}$

C $\dfrac{2a^2b}{3bc^3}$

D $-\dfrac{2ab}{3c^3}$

ANSWERS TO TRY IT

1) $x = \dfrac{-3 \pm \sqrt{13}}{2}$

2) $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(3)}}{2(-2)}$

$x = \dfrac{1 \pm \sqrt{1 - -24}}{-4}$

$x = \dfrac{1 \pm \sqrt{1 + 24}}{-4}$

$x = \dfrac{1 \pm \sqrt{25}}{-4}$

$x = \dfrac{1 \pm 5}{-4}$

$x = \dfrac{1 + 5}{-4}$ **or** $x = \dfrac{1 - 5}{-4}$

$x = -\dfrac{6}{4} = -\dfrac{3}{2}$ **or** $x = \dfrac{-4}{-4} = 1$

3) no real roots

4) $x = \dfrac{1 \pm \sqrt{17}}{2}$

5) 1

6) $\dfrac{1}{x^4}$

7) 512

8) 1

9) z^3

10) 12

11) x^{10}

12) a^4

13) k^{14}

14) $2ab^3$

15) $2a^2b$

16) **B**

End of Lesson 15