Student Name: ______________________	**Contact Person Name:** ______________________
Date: ______________________	**Phone Number:** ______________________

TAKS

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 16

Two- and Three-dimensional Shapes

TAKS Objective 6 – Demonstrate an understanding of geometric relationships and spatial reasoning

Lesson Objectives:

- Find the measure of an angle using the lines and arcs of a circle
- Find the surface area and volume of composite solids
- Find the area and perimeter of composite shapes
- Find the area of a shaded region

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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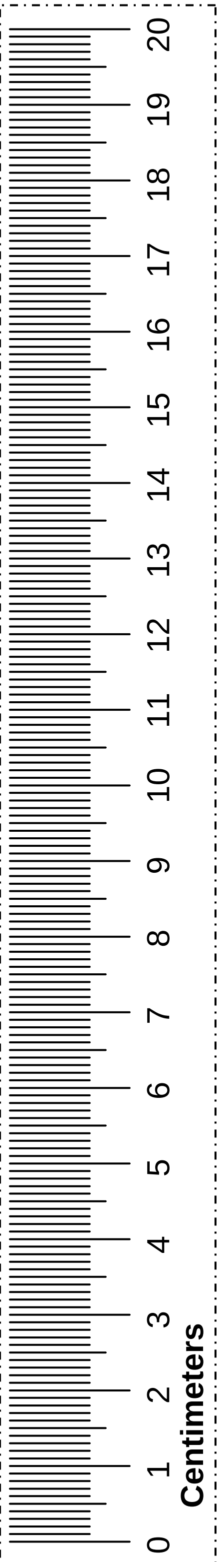
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

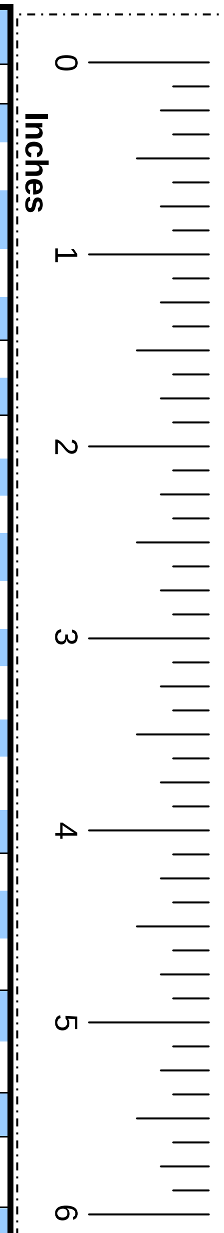
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
Triangles	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



The word "geometry" comes from the Latin for "earth measure." It involves analyzing the properties of shapes, such as the area, perimeter, and the angles they contain. The angle sums within the different two-dimensional shapes are as follows:

Shape	Angle Sum
Triangle	180°
Quadrilateral	360°
Pentagon	540°
Hexagon	720°
Heptagon	900°
Octagon	1080°
Nonagon	1260°
Decagon	1440°

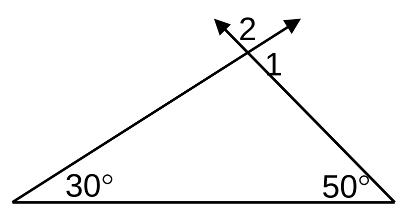
FACT

The angle sum for any shape can be found using the formula $(n-2)180°$, where n is the number of sides.

We can use this information to help us find the missing angles in a given shape.

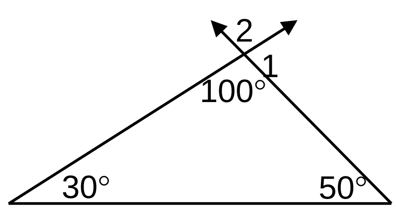
Example

What are the measures of angles $\angle 1$ and $\angle 2$?



Solution

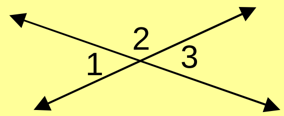
We can find the third interior angle of the triangle, because the three angles should sum to 180°. The two given angles are $30° + 50° = 80°$. Therefore, the third angle in the triangle is 100°, because $100° + 80° = 180°$. Write this in the triangle.



$\angle 1$ is supplementary to the angle we just found, so $\angle 1 = 80°$. Also, $\angle 2$ is a vertical angle to the $100°$ angle, so $\angle 2 = 100°$.

FACT

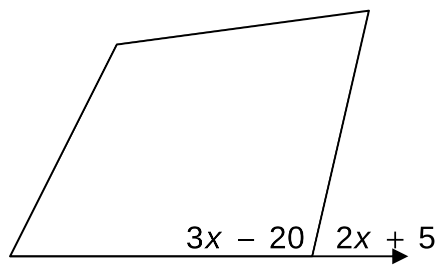
Supplementary angles are angles that add up to 180°. Vertical angles are formed by two intersecting lines and are equal to each other.



$\angle 1$ and $\angle 2$ are supplementary, and $\angle 1$ and $\angle 3$ are vertical angles.

Example

Find the value of x.



Solution

The two angles marked in the quadrilateral form a straight line. Thus, they are supplementary and sum to 180°.

$$3x - 20 + 2x + 5 = 180$$

Solve for x.

$$3x - 20 + 2x + 5 = 180$$

$$5x - 15 = 180$$

$$\underline{+15} \quad \underline{+15}$$

$$\frac{5x}{5} = \frac{195}{5}$$

$$x = 39$$

Sometimes you may be asked to find the measure of both angles. In that case, substitute the value in for x and see what you get for each angle.

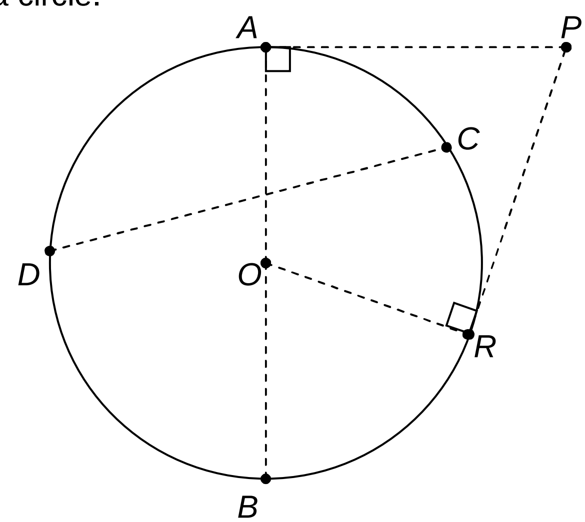
Circles are a very important shape in geometry. They have very unique properties, and they can affect shapes inscribed within them. Review the following diagram to become familiar with the different parts of a circle.

Radius: $\overline{OA}, \overline{OB}, \overline{OR}$

Diameter: $\overline{AB}$

Chord: $\overline{CD}, \overline{AB}$

Tangent: $\overline{PA}, \overline{PR}$



A **radius** is a line segment drawn from the center of the circle to the edge of the circle. In a circle, all radii are the same length $(OA = OB = OR)$.

A **diameter** is the distance across a circle that runs through the center of the circle.

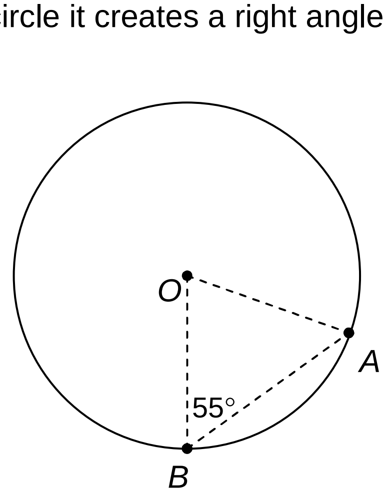
A **chord** is any line segment with both endpoints on the circle. Therefore, the diameter is also a chord.

A **tangent** is a line drawn outside the circle that intersects the circle at a single point. It touches the circle, but does not go through it. If two tangents are drawn from the same point, they are the same length $(PA = PR)$.

When a tangent intersects the radius (or diameter) of a circle it creates a right angle.

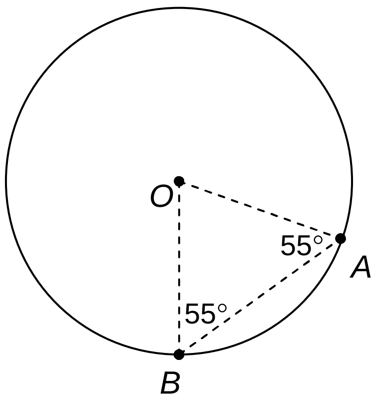
Example

Point *O* is the center of the circle. What is the measure of $\angle O$?



Solution

Since O is the center of the circle, we know that $OA = OB$ because they are both radii of the same circle. Therefore, ΔAOB is an isosceles triangle, and the base angles are equal. This means that $\angle A = \angle B = 55°$.



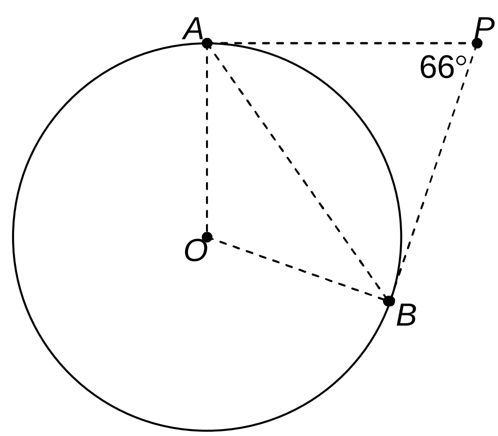
We now know two angles in the triangle, so we can find the third using the angle sum property.

$$\angle A + \angle B + \angle O = 180°$$
$$55° + 55° + \angle O = 180°$$
$$110° + \angle O = 180°$$
$$-110° \qquad\qquad -110°$$
$$\angle O = 70°$$

Since O is the center of the circle, $\angle O$ is a central angle. Therefore, the minor arc $\overset{\frown}{AB}$ is also 70°.

Example

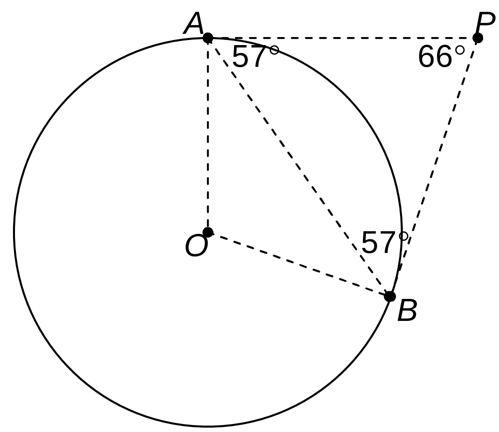
Segments $\overline{PA}$ and $\overline{PB}$ are tangents to circle O. The measure of $\angle P = 66°$. What is the measure of $\angle OAB$?



Solution

Since the two segments $\overline{PA}$ and $\overline{PB}$ are tangents drawn from the same point, they are the same length. This means that ΔAPB is an isosceles triangle, and the base angles are equal. Let x be the measure of one of these angles. Thus, $\angle PAB = \angle PBA = x$. Since we know $\angle P = 66°$, we can find x.

$$\angle PAB + \angle PBA + \angle P = 180^\circ$$
$$x + x + 66^\circ = 180^\circ$$
$$-66^\circ \quad -66^\circ$$
$$\frac{2x}{2} = \frac{114^\circ}{2}$$
$$x = 57^\circ$$

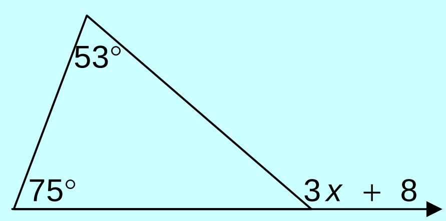


Fill in the angle dimensions of the triangle. Segment $\overline{AO}$ is a radius of the circle, so it is perpendicular to tangent $\overline{PA}$. This means that $\angle PAO = 90^\circ$. Since $\angle PAO = \angle OAB + \angle PAB$, we know $\angle OAB + \angle PAB = 90^\circ$. We know $\angle PAB = 57^\circ$, so we can solve for $\angle OAB$.

$$\angle OAB + 57^\circ = 90^\circ$$
$$-57^\circ \quad -57^\circ$$
$$\angle OAB = 33^\circ$$

TRY IT!

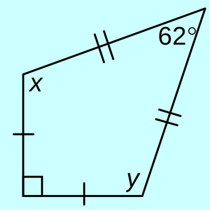
1) Find the value of x in the given triangle.



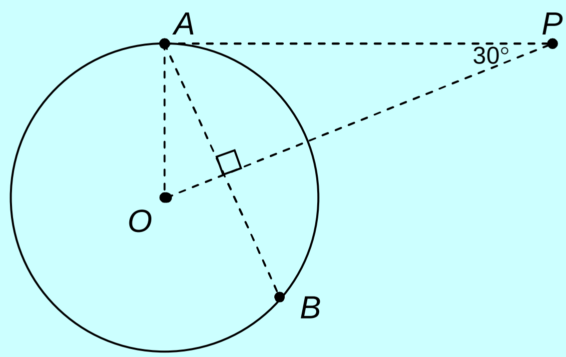
A $x = 17$ **B** $x = 40$

C $x = 52$ **D** $x = 8$

2) Find the measure of angles *x* and *y*.



3) Find the measure of $\angle AOP$.



On the exam, you will be asked to find the **perimeter** of a two-dimensional shape using provided formulas. The perimeter is the sum of all the side lengths of a shape. The perimeter of a circle is called the **circumference**.

Example

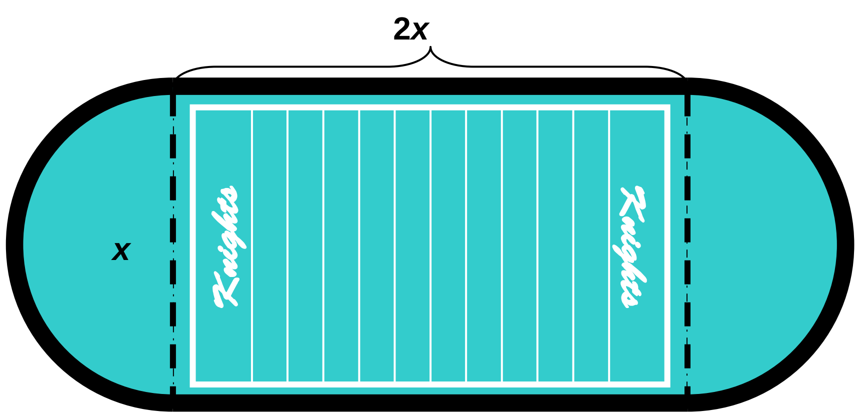
The high school track goes around the football field as shown below. Which formula best represents the distance around the track in terms of *x*?

A $P = 4x$

B $P = 6x$

C $P = (4 + 2\pi)x$

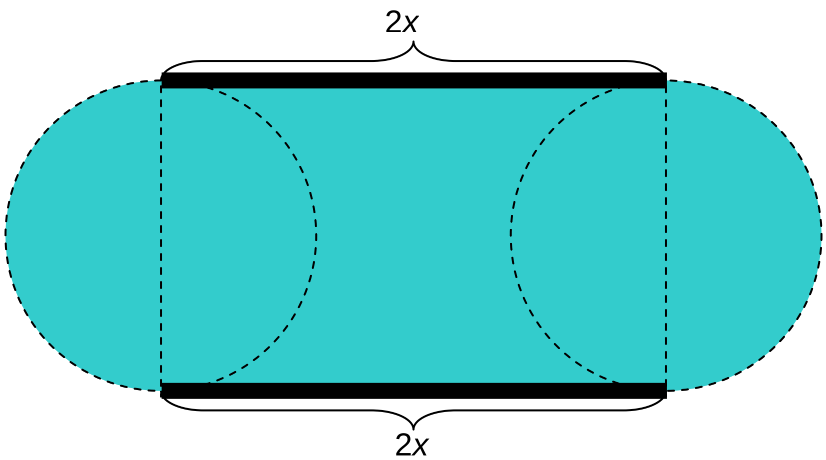
D $P = (4 + \pi)x$



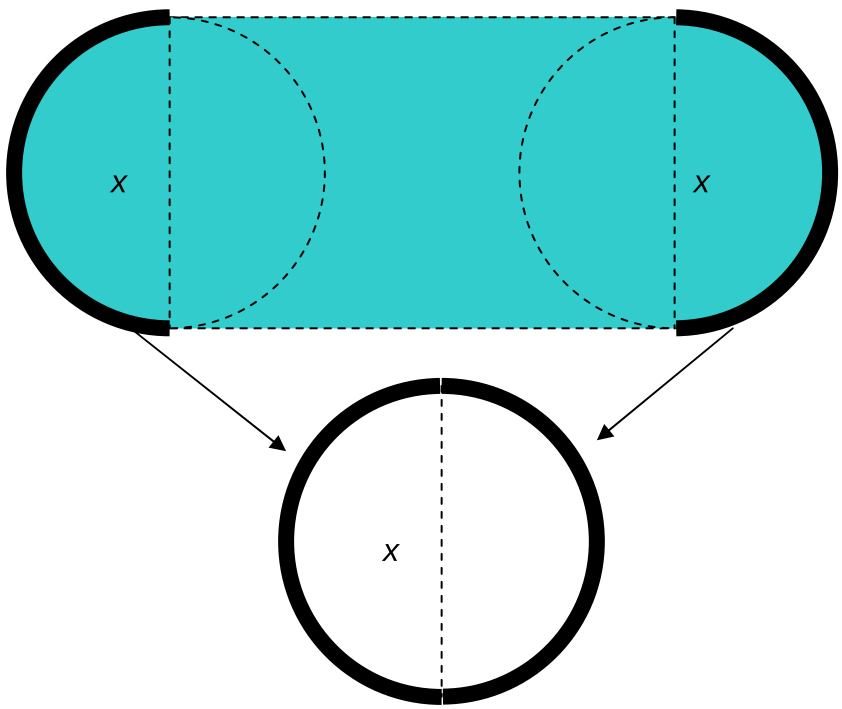
Solution

To solve this problem, we must figure out what shape we are looking at. As we can see the track is made of a rectangle and two semicircles.

The distance around the rectangular portion of the track is 4*x*.



Next, we will find the distance around the two semicircles. The two semicircles have the same diameter, so we can put them together to make a full circle.



The formula for circumference is

$$C = \pi d = \pi x$$

The total distance around the track should be the sum of all the sides we found.

$$P = 4x + \pi x$$

This is not the same as any of the formula choices. However, we can factor out an x, which gives the answer, choice **C**.

$$P = (4 + \pi)x$$

Similar to composite perimeter problems are shaded region problems.

Example

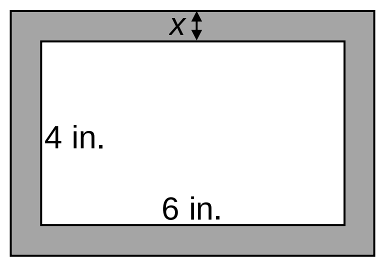
A picture frame is the same width, x, all the way around a 4 by 6 inch picture as shown below. Which formula can be used to determine the area of the picture frame?

A $A = 24x$

B $A = (4 + 2x)(6 + 2x) - 4(6)$

C $A = (4 + x)(6 + x) - 4(6)$

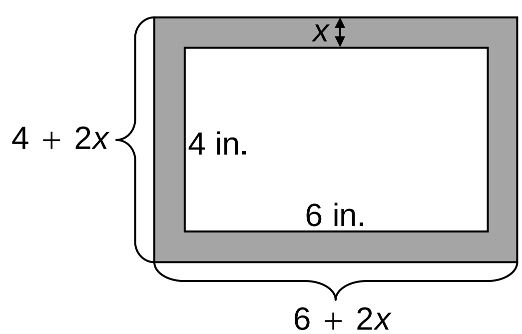
D not here



Solution

We need to find the area of the shaded region. The shaded region is given by the larger rectangle minus the smaller rectangle.

First, we will find the area of the larger rectangle. The sides of the larger rectangle are given by the dimensions of the photo plus the width of the frame. The frame is x inches thick on all four sides of the photo, so the dimensions of the larger rectangle are $4 + x + x = 4 + 2x$ and $6 + x + x = 6 + 2x$.



The area of a rectangle is the product of the length and the width. The area of the larger rectangle is $(4 + 2x)(6 + 2x)$. The area of the smaller rectangle is $(4)(6)$.

To find the area of the shaded region, subtract the area of the smaller rectangle from the larger rectangle.

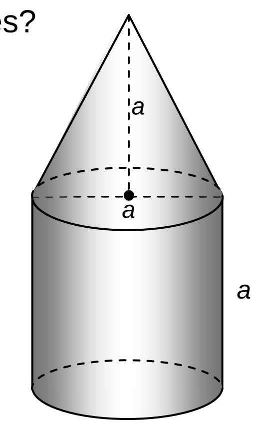
$$(4 + 2x)(6 + 2x) - 4(6)$$

Therefore, choice **B** is the answer.

You may be asked to find the volume of composite solids.
Area and volume formulas are provided on the exam. It is up to you to determine when to use the correct formula.

Example

A cone is placed on top of a cylinder as shown below. Which equation best represents V, the volume of the composite solid in cubic inches?



A $V = 3\pi a^3$

B $V = \dfrac{4\pi}{3} a^3$

C $V = \dfrac{4\pi}{6} a^3$

D $V = \dfrac{\pi}{3} a^3$

Solution

The volume formulas for each solid are provided on the formula sheet as:

$$\text{Volume of a cylinder: } V = Bh$$

$$\text{Volume of a cone: } V = \frac{1}{3}Bh$$

The letter B represents the area of the base and h is the height of each.

In the solid, both shapes share a <u>circular base</u>. The diameter of that base is represented by a. The radius is, therefore, half that $\left(r = \frac{1}{2}a\right)$. The height of the cylinder is also a, so the volume of the cylinder is given by the following.

$$V_{cylinder} = Bh = (\pi r^2)h = \pi\left(\frac{1}{2}a\right)^2 a$$

$$V_{cylinder} = \pi\frac{1}{4}a^3 = \frac{\pi}{4}a^3$$

Since the radius and height of the cone are the same as the cylinder, the volume for the cone is given by the following:

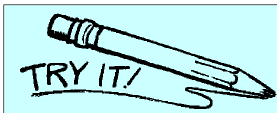
$$V_{cone} = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}a\right)^2 a$$

$$V_{cone} = \pi\frac{1}{12}a^3 = \frac{\pi}{12}a^3$$

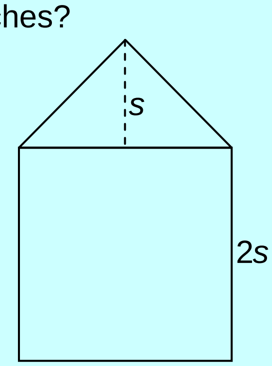
Add the volume of the cone and the cylinder to get the volume of the whole solid.

$$V = \frac{\pi}{4}a^3 + \frac{\pi}{12}a^3 = \frac{3\pi}{12}a^3 + \frac{\pi}{12}a^3 = \frac{4\pi}{12}a^3 = \frac{\pi}{3}a^3$$

Thus, choice **D** is the answer.



4) A triangle is placed on top of a square as shown below. Which equation best represents, A, the area of the composite figure in cubic inches?



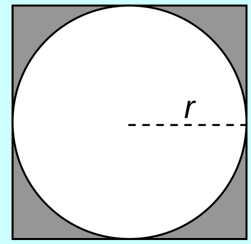
A $A = 5s^2$

B $A = \frac{5}{2}s^2$

C $A = 4s^2$

D $A = \frac{9}{2}s^2$

5) A circle is cut out of a square piece of paper as shown. Which formula can be used to determine how much extra paper was left over?



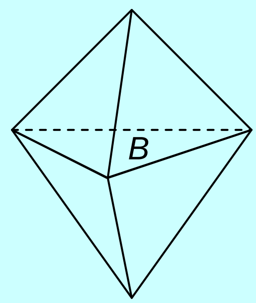
A $A = 4r^2$

B $A = (4 - \pi)r^2$

C $A = \pi r^2$

D $A = (2 - \pi)r^2$

6) Two triangular pyramids are put together to form a hexahedron as shown below. *B* represents the area of the triangular base, and the height of each is *h*. Which equation best represents, *V*, the volume of the composite solid in cubic inches?



A $V = 2B^2h$

B $V = \frac{2}{3}B^2h$

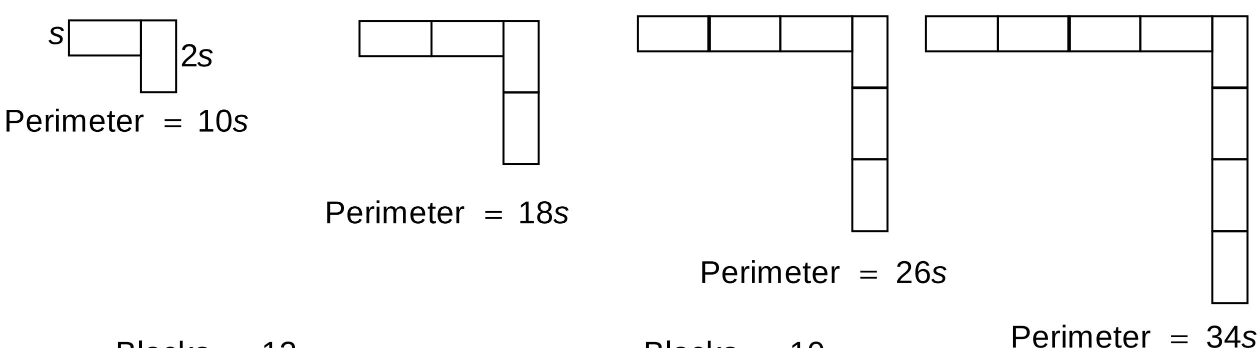
C $V = \frac{2}{3}Bh$

D $V = 2Bh$

You may be asked to develop a pattern for a set of shapes.

Example

Determine the number of blocks and the perimeter of the figure in the 6th step.



A Blocks = 12
Perimeter = 48s

B Blocks = 10
Perimeter = 42s

C Blocks = 12
Perimeter = 58s

D Blocks = 12
Perimeter = 50s

Solution

The best way to solve this type of problem is to create a table of each step and see what pattern develops.

Step	Blocks	Perimeter
1	2	10*s*
2	4	18*s*
3	6	26*s*
4	8	34*s*
5		
6		

The blocks increase by 2 each step. We will continue this pattern and fill in the remaining rows with 10 and 12 for the blocks.

The perimeter increases by 8*s* each step. We will assume it follows this pattern and fill in the remaining rows with 42*s* and 50*s*.

Step	Blocks	Perimeter
1	2	10*s*
2	4	18*s*
3	6	26*s*
4	8	34*s*
5	**10**	
6	**12**	

Step	Blocks	Perimeter
1	2	10*s*
2	4	18*s*
3	6	26*s*
4	8	34*s*
5	**10**	**42s**
6	**12**	**50s**

There are 12 blocks in the 6^{th} step and the perimeter is 50*s*. Thus, choice **D** is the answer.

Review

Know these concepts:

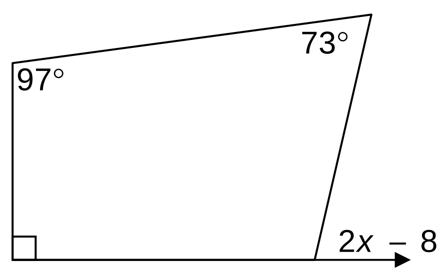
1. The angle sum in a triangle is 180°.
2. The formula for the angle sum of any polygon is $180(n - 2)$, where n is the number of sides
3. In a circle, all radii are the same length. Also, two tangents drawn from a single point outside the circle are the same length.
4. A tangent and a radius (or diameter) intersect to form a right angle.
5. Sometimes a figure is made of simpler shapes, and you must add their areas/volumes
6. Sometimes you must subtract the areas of two shapes to find the area of a shaded region.
7. To help extend a pattern, create a table.

Practice Problems

Lesson 16

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 16.

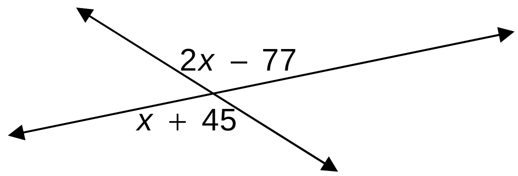
1) Find the value of the exterior angle.



A 44° **B** 80°

C 100° **D** 54°

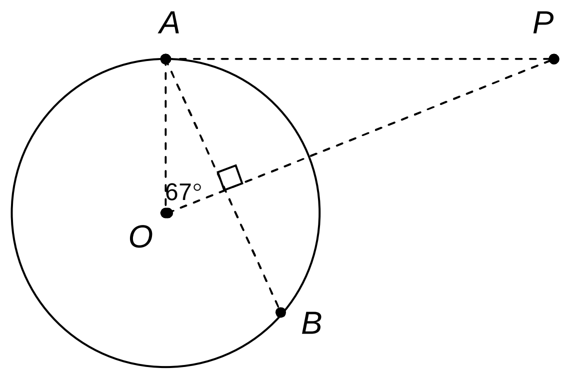
2) Find the value of x.



A $x = 122$ **B** $x = 66$

C $x = 142$ **D** $x = 41$

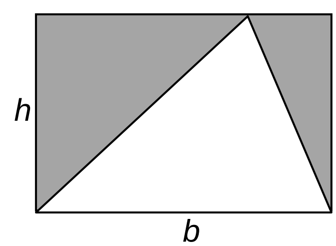
3) Find the measure of $\angle P$.



A 90° **B** 87°

C 113° **D** 23°

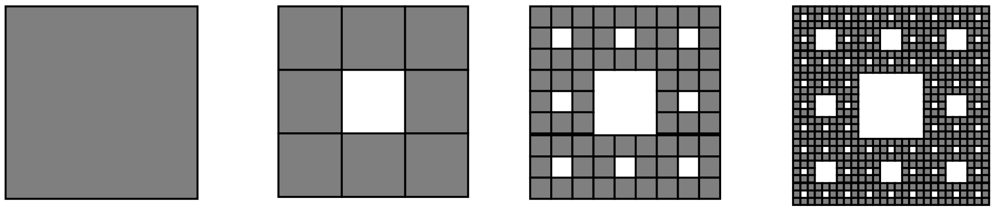
4) A triangle is cut out of a rectangular piece of paper as shown. Which formula can be used to determine how much extra paper was left over?



A $\frac{1}{2}bh$ **B** $1 - \frac{1}{2}bh$

C $bh\left(1 - 2\right)$ **D** $\frac{1}{2}bh - 1$

5) The first 4 stages of a certain fractal are shown below.



Stage 1 Stage 2 Stage 3 Stage 4

In each stage after the first, each square is divided into 9 squares, and then the middle square is removed. If the pattern continues, how many shaded square units will Stage 5 contain?

ANSWERS TO TRY IT

1) B

2) $x = y = 104°$

3) 60°

4) A

5) B

6) C

End of Lesson 16