| Student Name: | Contact Person Name: |
|---|---|
| Date: ______________________ | Phone Number: ______________ |

TAKS

Texas Assessment of Knowledge and Skills

# Exit Level Math Review

# Lesson 17

## Transformations and Tessellations

**TAKS Objective 6 –** Demonstrate an understanding of quadratic and other nonlinear functions

**Lesson Objectives:**

- Identify and perform one or more translations, rotations, or reflections within the plane
- Understand the properties of tessellations
- Describe a tessellation in terms of one or more transformation

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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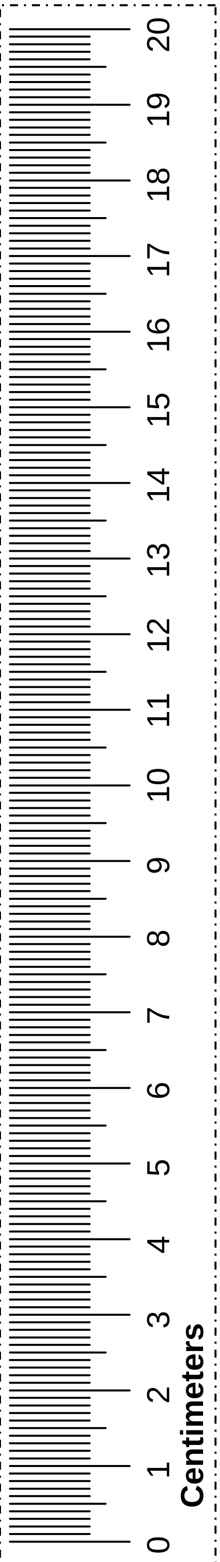
# *TAKS* Mathematics Chart




## Length

| Metric | Customary |
|---|---|
| 1 kilometer = 1000 meters | 1 mile = 1760 yards |
| 1 meter = 100 centimeters | 1 mile = 5280 feet |
| 1 centimeter = 10 millimeters | 1 yard = 3 feet |
| | 1 foot = 12 inches |

## Capacity and Volume

| Metric | Customary |
|---|---|
| 1 liter = 1000 milliliters | 1 gallon = 4 quarts |
| | 1 gallon = 128 fluid ounces |
| | 1 quart = 2 pints |
| | 1 pint = 2 cups |
| | 1 cup = 8 fluid ounces |

## Mass and Weight

| Metric | Customary |
|---|---|
| 1 kilogram = 1000 grams | 1 ton = 2000 pounds |
| 1 gram = 1000 milligrams | 1 pound = 16 ounces |

## Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

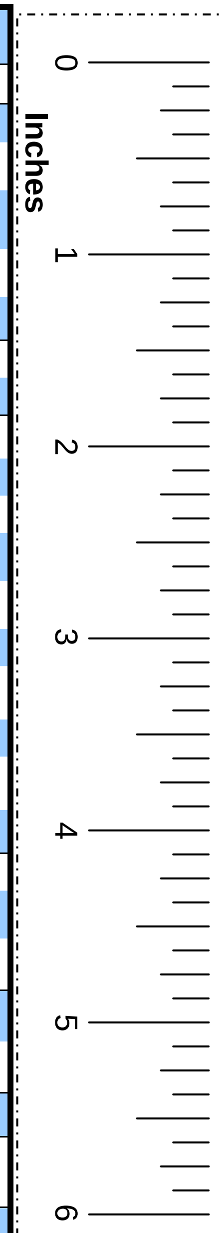
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

# *TAKS* Mathematics Chart

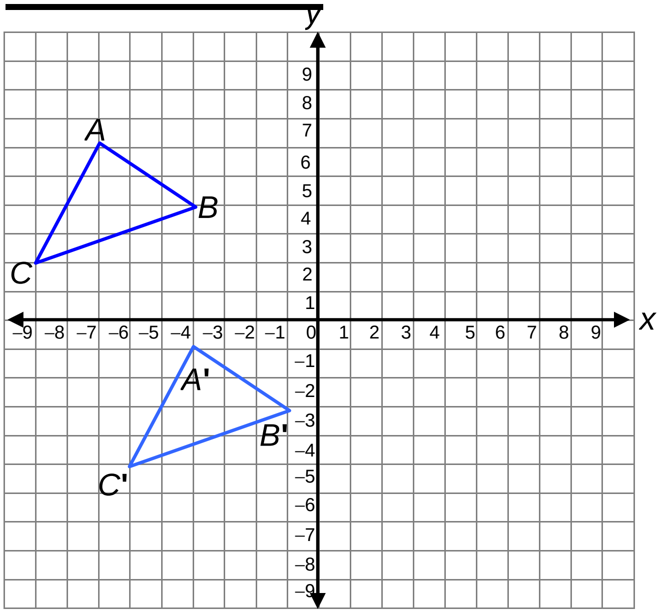
| | | |
|---|---|---|
| **Perimeter** | Rectangle | $P = 2l + 2w$ or $P = 2(l + w)$ |
| **Circumference** | Circle | $C = 2\pi r$ or $C = \pi d$ |
| **Area** | Rectangle | $A = lw$ or $A = bh$ |
| | Triangle | $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$ |
| | Trapezoid | $A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$ |
| | Regular polygon | $A = \frac{1}{2}aP$ |
| | Circle | $A = \pi r^2$ |
| ***P*** represents the perimeter of the base of a three-dimensional figure. | | |
| ***B*** represents the area of the base of a three-dimensional figure. | | |
| **Surface Area** | Cube (total) | $S = 6s^2$ |
| | Prism (lateral) | $S = Ph$ |
| | Prism (total) | $S = Ph + 2B$ |
| | Pyramid (lateral) | $S = \frac{1}{2}Pl$ |
| | Pyramid (total) | $S = \frac{1}{2}Pl + B$ |
| | Cylinder (lateral) | $S = 2\pi rh$ |
| | Cylinder (total) | $S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$ |
| | Cone (lateral) | $S = \pi rl$ |
| | Cone (total) | $S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$ |
| | Sphere | $S = 4\pi r^2$ |
| **Volume** | Prism or Cylinder | $V = Bh$ |
| | Pyramid or Cone | $V = \frac{1}{3}Bh$ |
| | Sphere | $V = \frac{4}{3}\pi r^3$ |
| **Special Right** | 30°, 60°, 90° | $x,\ x\sqrt{3},\ 2x$ |
| **Triangles** | 45°, 45°, 90° | $x,\ x,\ x\sqrt{2}$ |
| **Pythagorean Theorem** | | $a^2 + b^2 = c^2$ |
| **Distance Formula** | | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ |
| **Slope of a Line** | | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| **Midpoint Formula** | | $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ |
| **Quadratic Formula** | | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| **Slope-Intercept Form of an Equation** | | $y = mx + b$ |
| **Point-Slope Form of an Equation** | | $y - y_1 = m(x - x_1)$ |
| **Standard Form of an Equation** | | $Ax + By = C$ |
| **Simple Interest Formula** | | $I = prt$ |



A **transformation** is the movement of an object on the *x*- and *y*-axes.

You are responsible for knowing three types of transformations.

# Translations




A **translation** slides an object up, down, left, or right.

Here, triangle *ABC* has been translated 7 units down, and 3 units right to create triangle *A'B'C'*.

**FACT**

*Before any transformation occurs, an object is called the* ***pre-image****. The object after a transformation is called the* ***image****. Above, triangle ABC is the pre-image. Triangle A'B'C' is the image.*

*Also, A' is read "A-prime."*

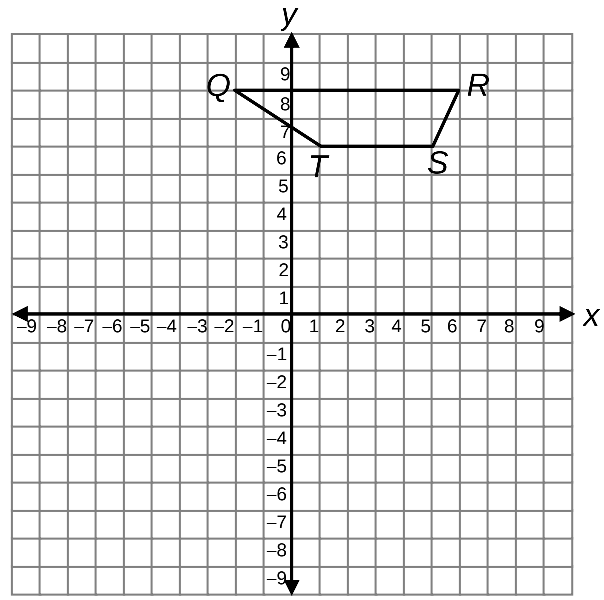
**FACT**

***To translate:***

- ***Up –*** add to the *y*-coordinate
- ***Down –*** subtract from the *y*-coordinate
- ***Left –*** subtract from the *x*-coordinate
- ***Right –*** add to the *x*-coordinate.

## *Example*

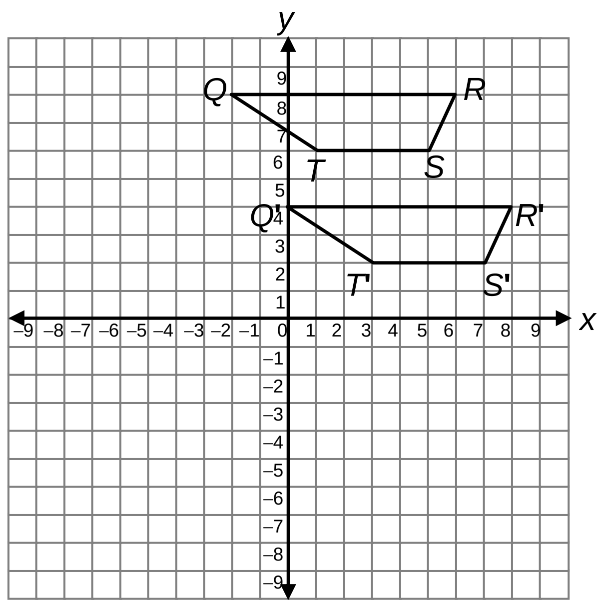
Find the image of quadrilateral *QRST* translated 2 units right and 4 units down.




### *Solution*

### Method 1: a graphic approach

Move each point 2 to the right, and 4 down. Label the image *Q'R'S'T'*.



**Method 2: an approach using the coordinates**

*Step 1*: Write each coordinate of *QRST*.

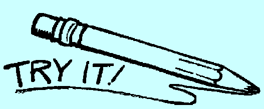
*Q*(–2, 8) *R*(6, 8) *S*(5, 6) *T*(1, 6)

*Step 2*: Since we are asked to translate 2 units right, add 2 to each *x*-coordinate. To translate down 4 units, subtract 4 from each *y*-coordinate.

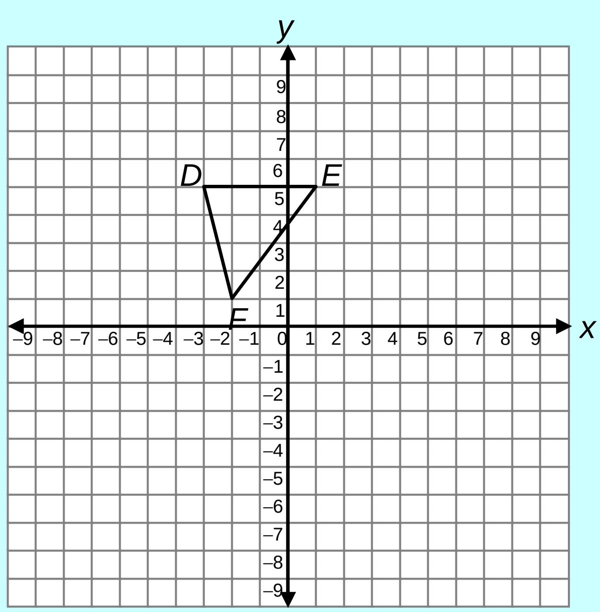
*Q*(–2, 8) *R*(6, 8) *S*(5, 6) *T*(1, 6)

*Q*'(0, 4) *R*'(8, 4) *S*'(7, 2) *T*'(3, 2)

This question does not specify whether to write coordinates or to sketch the graph of the image. Either answer format is correct.

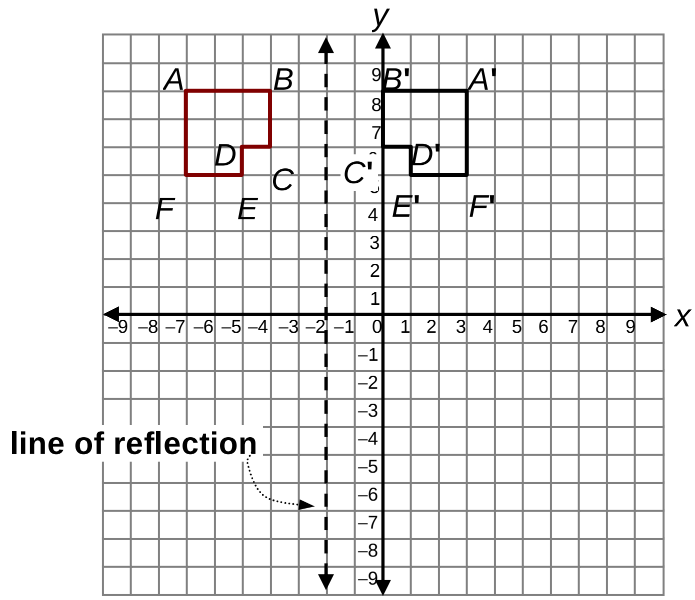



**1)** Graph the image of triangle *DEF* after a translation 2 units to the left and 1 unit up.



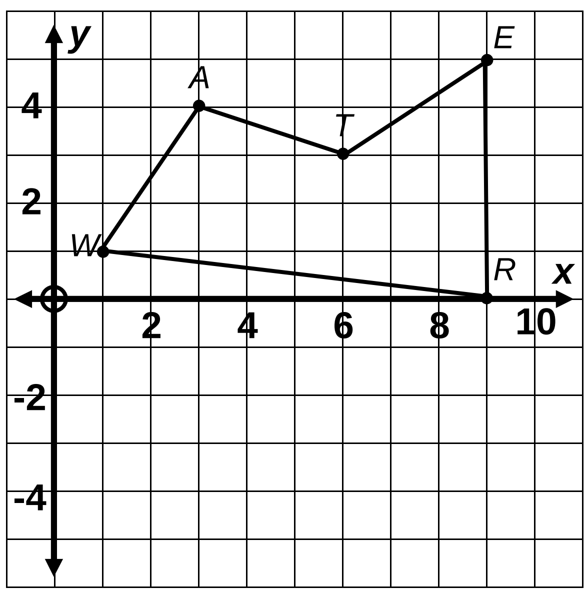
# Reflections

A **reflection** flips an object over a line. This line is called the **line of reflection**.




***Example***

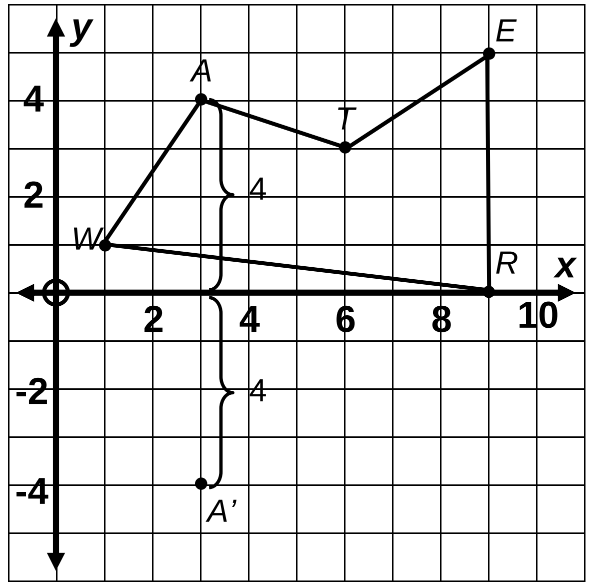
Reflect the polygon *WATER* over the *x*-axis.



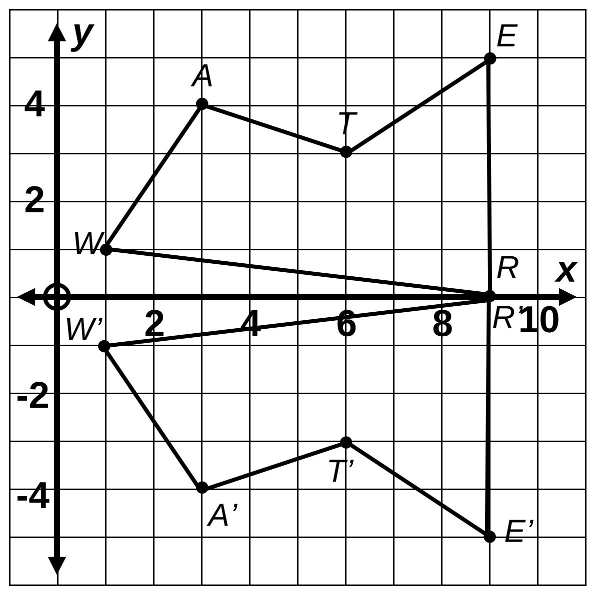

***Solution***

One way to solve this is graphically. Match each point's distance from the *x*-axis.

Start with point *A*. It is four units above the *x*-axis. To graph its reflected image, *A'* (said A-prime), we must travel four units below the *x*-axis. We do this for every point in the polygon *WATER*, to create its reflection, *W'A'T'E'R'*, over the *x*-axis.




Notice that *R* and *R'* are the same point. This is because *R* is on the axis of symmetry. Its reflection across the *x*-axis does not move from the original position.




In general, find each point's distance from the line of reflection and copy that distance to graph the reflection. A reflection is also called a <u>flip</u>, since the pre-image is flipped over a line.

If we observe the coordinates of *WATER* and *W'A'T'E'R'*, an interesting pattern arises.

## Reflection over the *x*-axis

| | | | | | |
|---|---|---|---|---|---|
| Pre-Image: | *W*(1,1) | *A*(3,4) | *T*(6,3) | *E*(9,5) | *R*(0,0) |
| Image: | *W'*(1,-1) | *A'*(3,-4) | *T'*(6,-3) | *E'*(9,-5) | *R'*(0,0) |

To reflect over the *x*-axis, **negate** each *y*-coordinate.

**Negate** means to switch the sign in front of a number.

-6 → 6      3 → -3      0 → 0

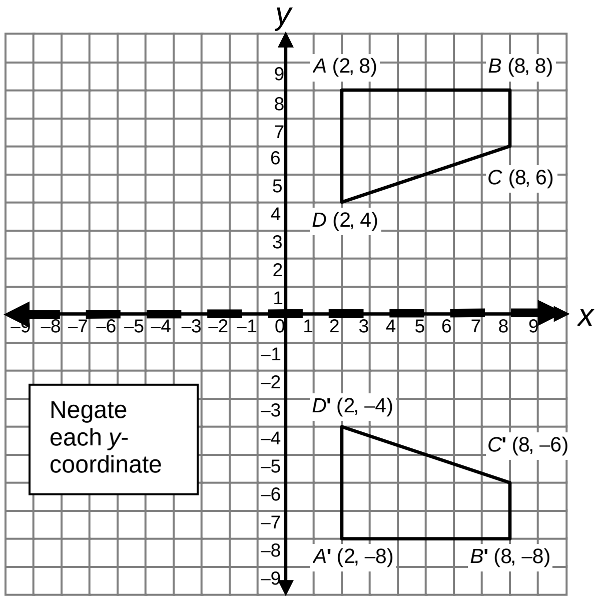
**FACT**

***If the line of reflection is:***

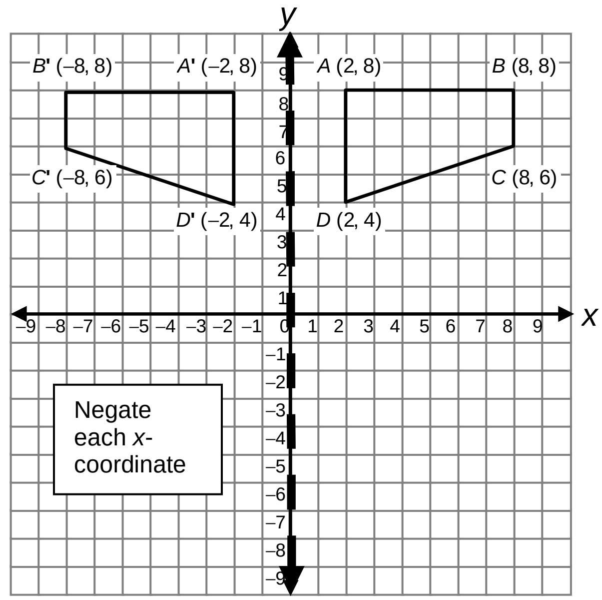
- ***the x-axis*****:** negate each *y*-coordinate
- ***the y-axis*****:** negate each *x*-coordinate
- ***the line y = x*****:** switch the *x*- and *y*-coordinates
- ***the line y = -x*****:** switch and negate the *x*- and *y*-coordinates

Observe these facts in the examples on the next page.

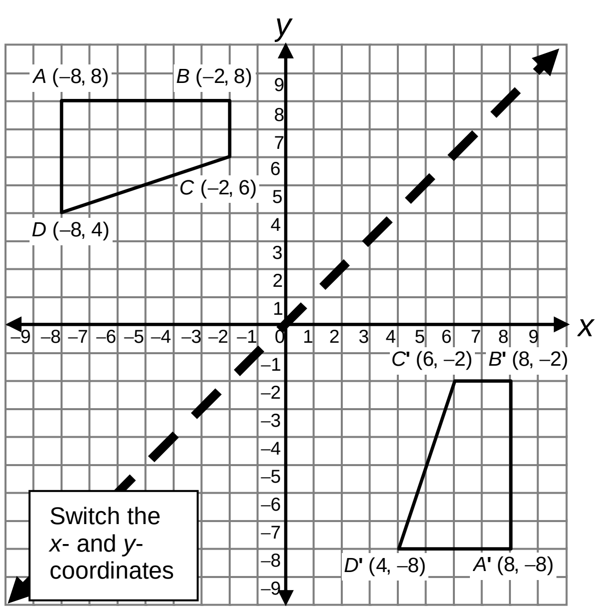
### reflection over the *x*-axis



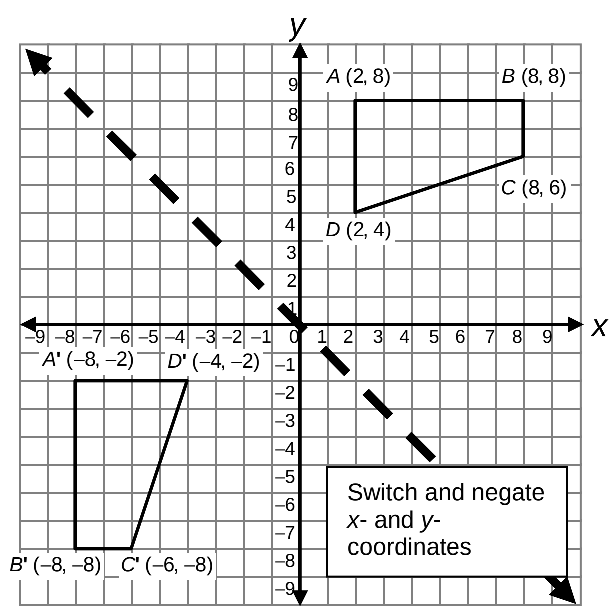

### reflection over the *y*-axis



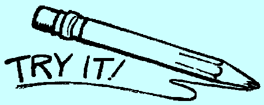

### reflection over the line $y = x$



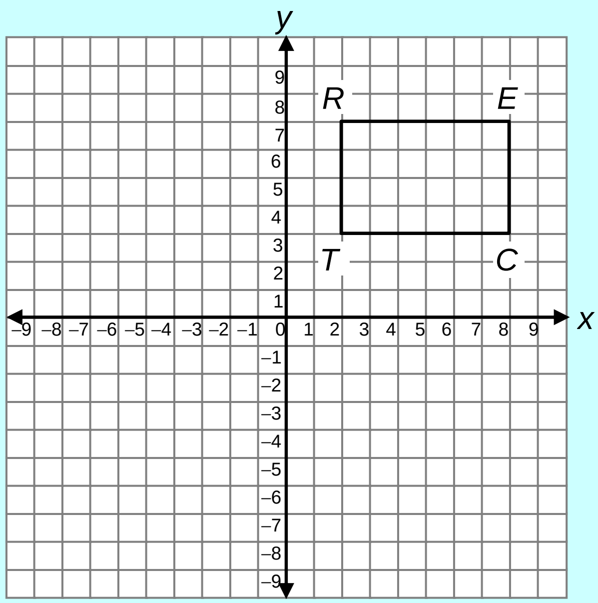

### reflection over the line $y = -x$



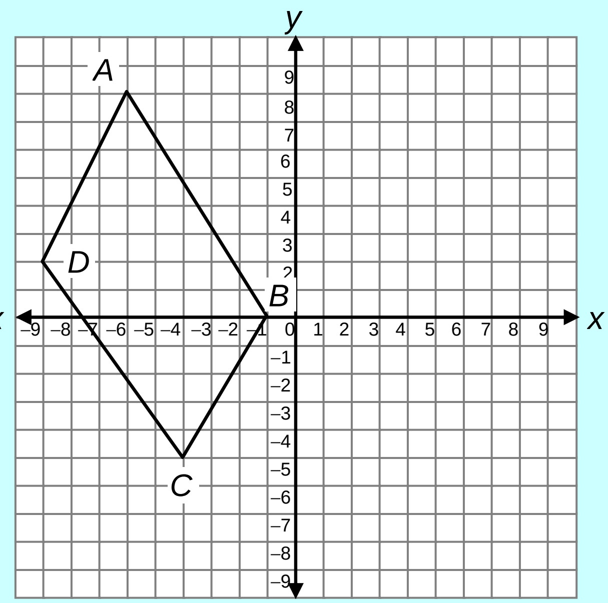

If reflecting over a line different from these four, it is easiest to use a graph. You can save time on the test if you remember these four rules however.



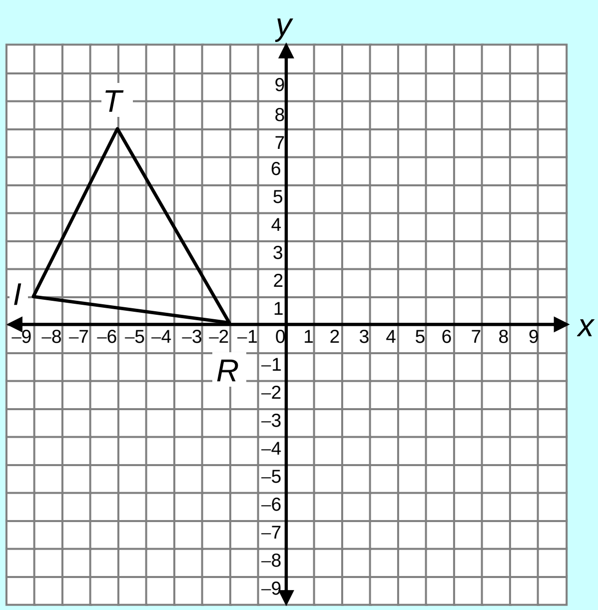

**2)** Graph *R'E'C'T'*, the image of *RECT* reflected over the line $x = 1$.




**3)** Write the coordinates of the polygon after a reflection over the *x*-axis.




**4)** Write the coordinates of the polygon after a reflection over the line $y = x$.

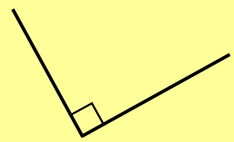


# Rotations

A **rotation** spins a figure about a point. This point is called the **center of rotation**.

**FACT**

*90° angles are formed by perpendicular lines or segments.*



***Think Back***

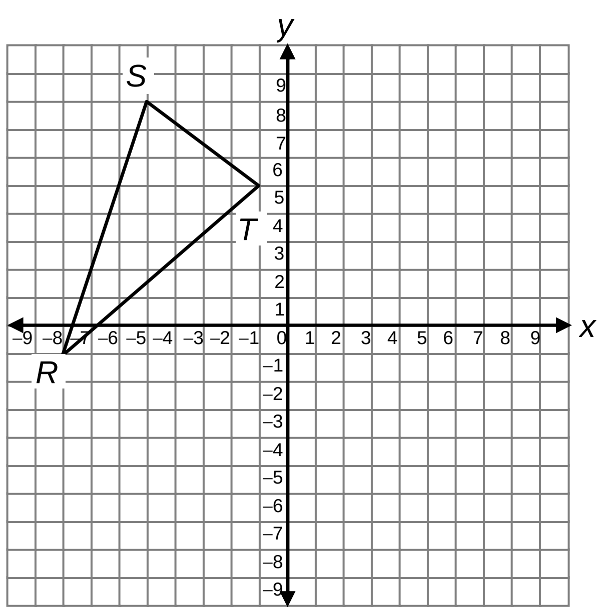
***Perpendicular slopes are negative reciprocals.***

$$\frac{a}{b} \to -\frac{b}{a}$$

Although figures can be rotated any number of degrees, you will only be responsible for rotating $90°$ about a point.

***Example***

Find the coordinates of *S*' if $\Delta RST$ is rotated $90°$ clockwise about point *T*.

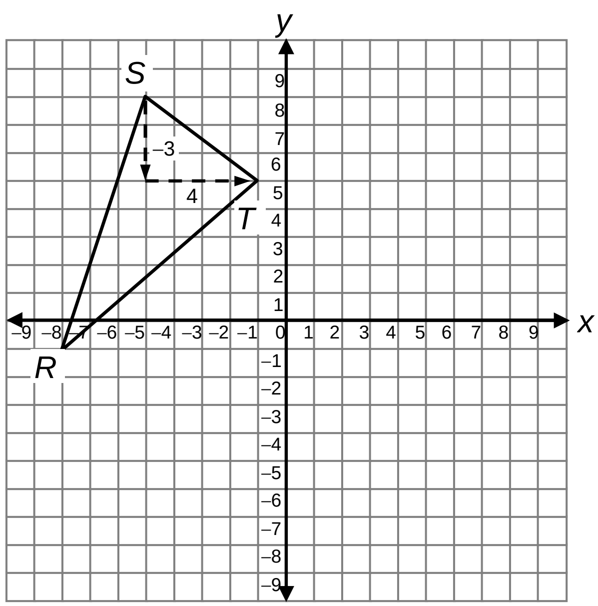


***Solution***

The question asks you to find only one point. Rotating every point of the figure will waste time.

Next, understand that the following method used to rotate $90^\circ$ is based on the fact that $90^\circ$ angles are formed by perpendicular lines. Perpendicular lines have slopes that are negative reciprocals.

Let's get started.

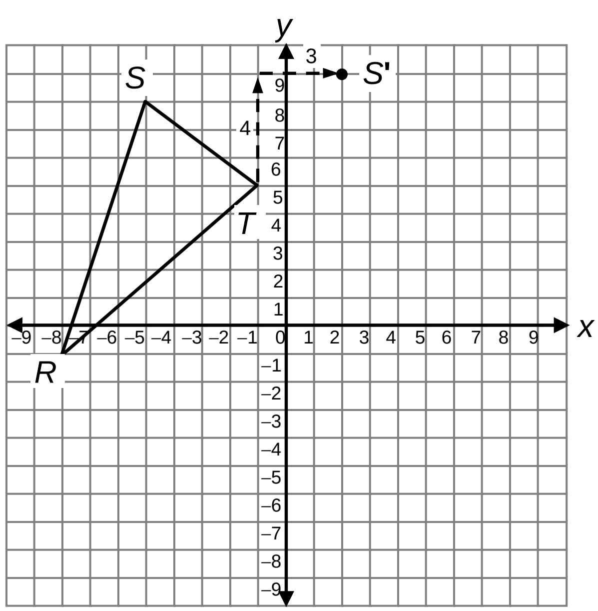



*Step 1*: Calculate the slope between the given point $S(-5, 8)$ and the center of rotation $T(-1, 5)$. Do not reduce the fraction.

$$m = \frac{\textbf{rise}}{\textbf{run}} = \frac{-3}{4} = -\frac{3}{4}$$

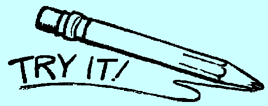
*Step 2*: Find the negative reciprocal of the slope found in step 1. Again, do not reduce the resulting fraction.

$$-\frac{3}{4} \rightarrow \frac{4}{3}$$

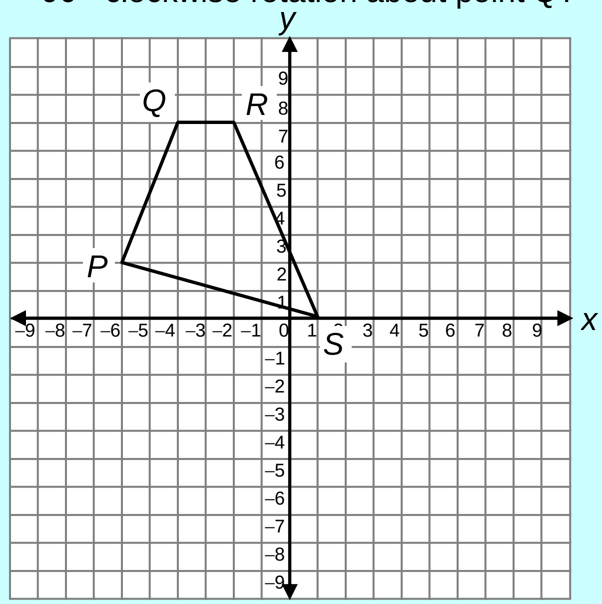



*Step 3*: From the center of rotation *T*, graph the slope found in step 2. (rise 4, run 3) Label this point *S*'.

The coordinates of *S*' are (2,9).




**5)** Which represents the coordinates of point *P*', the image of point *P* after a $90^\circ$ clockwise rotation about point *Q*?




**A** *P*' (–9, 9)

**B** *P*' (1, 9)

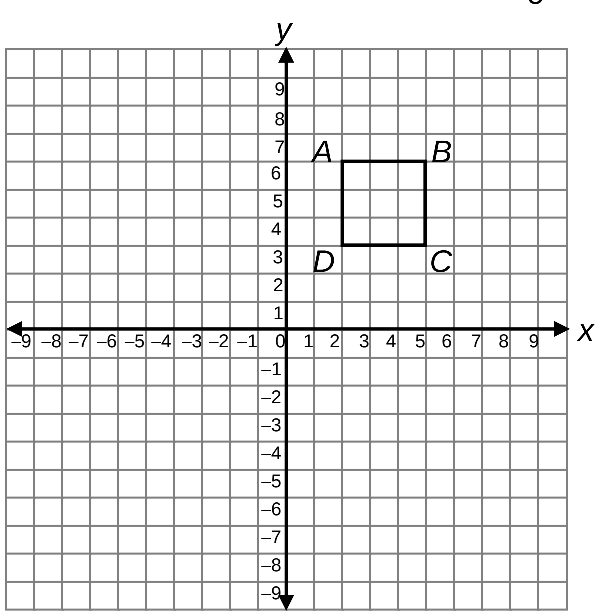
**C** *P*' (–1, 0)

**D** *P*' (2, 6)

On the test, you may be asked to graph or identify the image of a figure after one or more transformations.

***Example***

Which transformation creates an image with a vertex at the origin?




**A** Rotate *ABCD* $90^\circ$ around point *D*

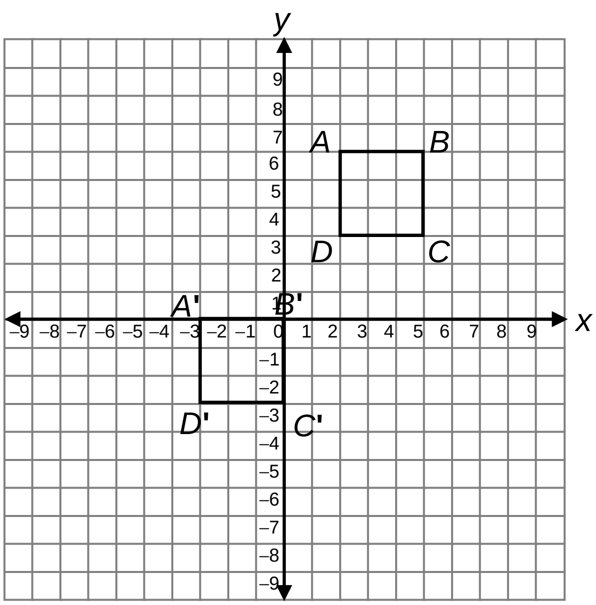
**B** Reflect *ABCD* across the line $x = 1$

**C** Reflect *ABCD* across the line $y = 3$

**D** Translate *ABCD* 5 units left and 6 units down

***Solution***

First, remember that the origin is the point $(0, 0)$. Next, think about how we need to transform *ABCD* in order for one of its vertices to be on the origin. The answer will be a transformation that maps *ABCD* both left and down. Immediately, we can eliminate choices **B** and **C**. Reflecting across the line $x = 1$ only moves *ABCD* to the left. Similarly, with $y = 3$ as the axis of symmetry, *ABCD* only moves down. This leaves choices **A** or **D** as possible answers. Perform the transformation that is easier to do – a translation.

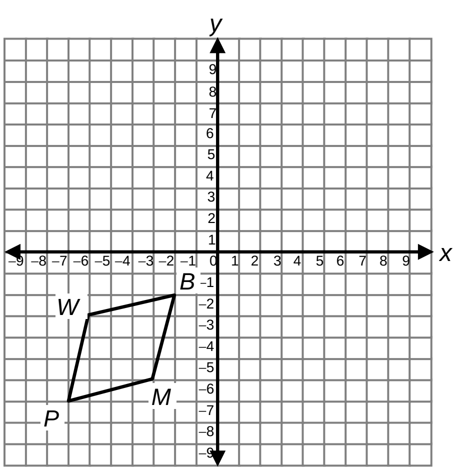



After a transformation 5 left and 6 down, we see point *B*' is on the origin.

The answer is choice **A**.

***Example***

If *WBMP* is reflected across the line $y = -x$ and then translated 4 units down to become parallelogram *W'B'M'P'*, what will be the coordinates of *M'*?




**A** (–6, –7)

**B** (6, –1)

**C** (6, 7)

**D** (6, 3)

***Solution***

There are two approaches to solve this problem.

**Method 1: coordinate approach**

Since we are asked to find *M*’, the image of *M*, it makes sense to only work with point *M*.

*Step 1*: reflect across the line $y = -x$

Do this by switching and negating the *x*- and *y*-coordinates of $M(-3, -6)$.

$$(-3, -6) \rightarrow (6, 3)$$

*Step 2*: translate 4 units down

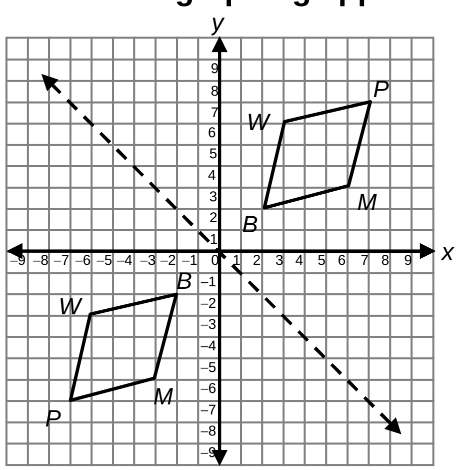
Do this by subtracting 4 from the *y*-coordinate of (6,2).

$$(6, 3) \rightarrow (6, -1)$$

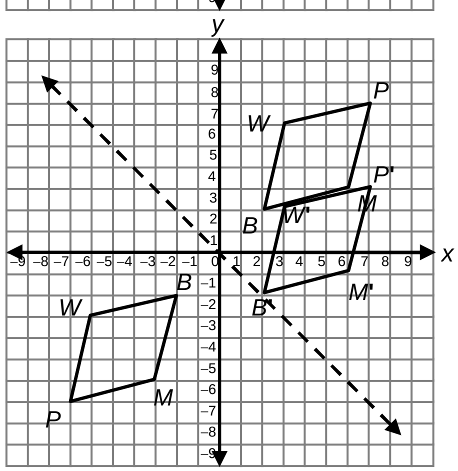
The answer is choice **B**.

If you feel confident in using this method, do so.

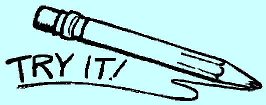
**Method 2: graphing approach**



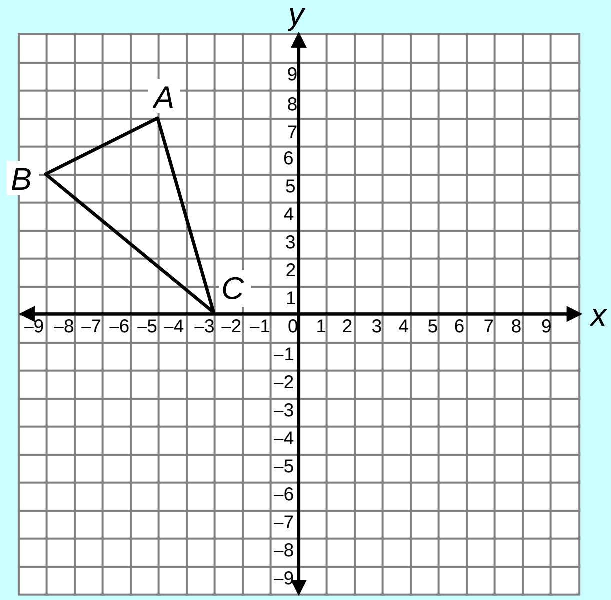

*Step 1*: Reflect over the line $y = -x$. Do this by counting the number of diagonal boxes to the line of reflection and matching that number on the other side for each point.




*Step 2*: Translate each point 4 units down. Read the coordinates of point *M’*. They are $(6, -1)$, choice **B**.




**6)** If $\Delta ABC$ is reflected across the line $y = x$, then reflected again across the $y$-axis, which will be the coordinates of $A'$?




**A** (–5, –7)

**B** (7, –5)

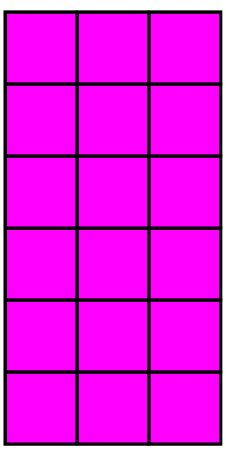
**C** (–7, –5)

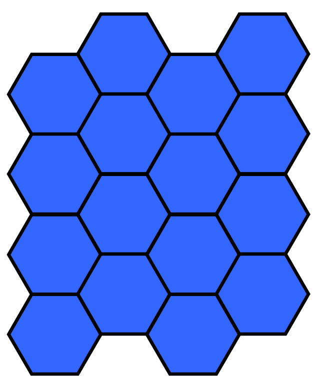
**D** (–7, 5)

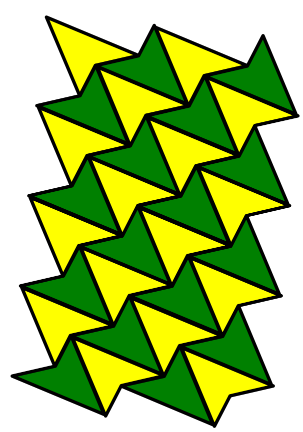
# Tessellations

A **tessellation** is a collection of figures that covers a plane. The figures connect with no overlaps and no gaps.

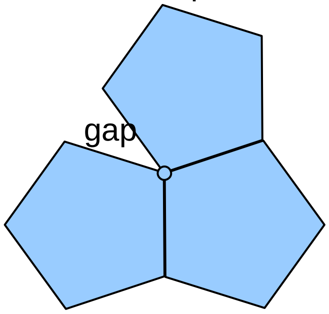
Squares and hexagons are among many polygons that can tessellate the plane.

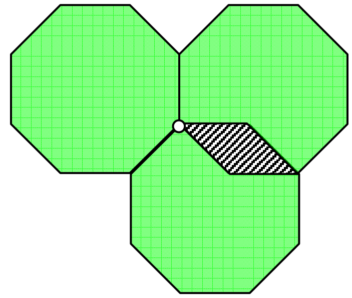






Not all shapes tessellate. For instance, regular pentagons and regular octagons do not tessellate the plane.



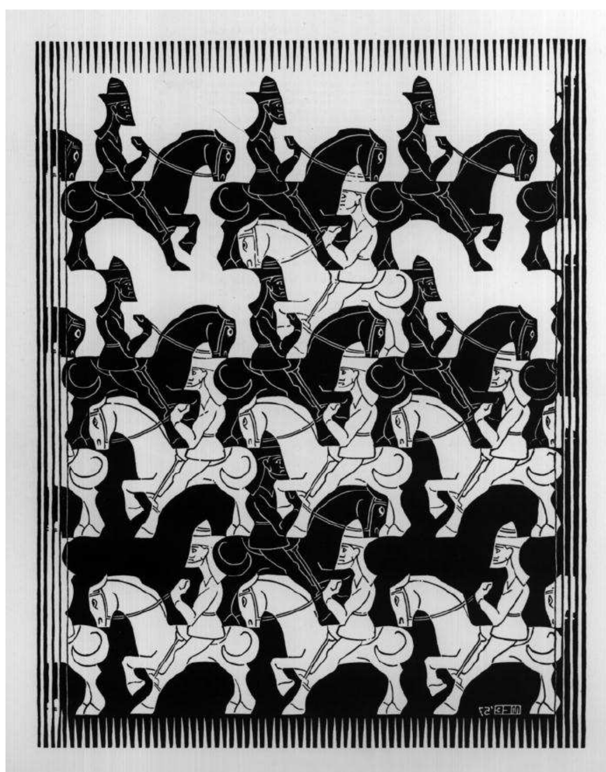



Of all the shapes that do tessellate the plane, what do they have in common?

**FACT**

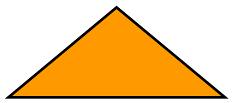
- *All quadrilaterals (four-sided figures) tessellate the plane; their angles add to* $360^\circ$.
- *In general, regular polygons (figures with all sides the same length) tessellate if each angle is a factor of* $360^\circ$.

This does not speak for irregular polygons however. For instance, artist M.C. Escher tessellated the plane with horsemen (right) and seahorses (below).

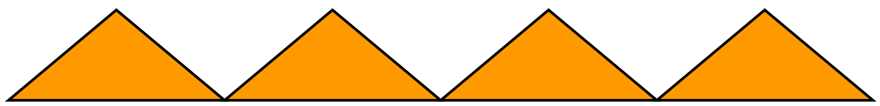


On the exam, you will need to describe a tessellation using transformations. Observe how transformations and tessellations are related.

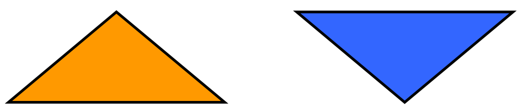
Consider this triangle.



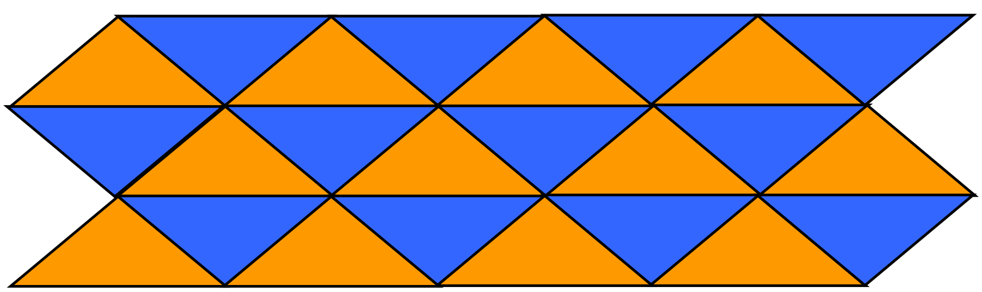
It cannot tessellate when placed side-by-side.



However, consider the original triangle, and a $180^\circ$ rotation of it.

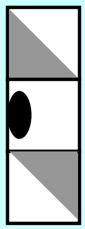


Put together, the triangle and its rotated image tessellate the plane.

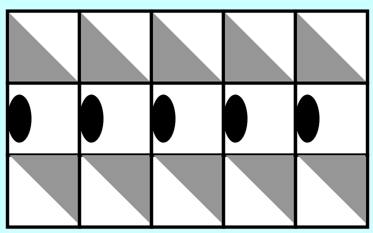


TRY IT!

**7)** Rory drew the following design.



He then used the design to create the following pattern below.



Which type of transformation did Rory use to create his pattern?

**A** Dilation

**B** Reflection

**C** Rotation

**D** Translation

# Review

**Know these concepts:**

**1.** Identify and graph transformations

- **a.** Translations slide objects
- **b.** Reflections flip objects
- **c.** Rotations turn objects

**2.** Perform translations and reflections using the coordinates of points

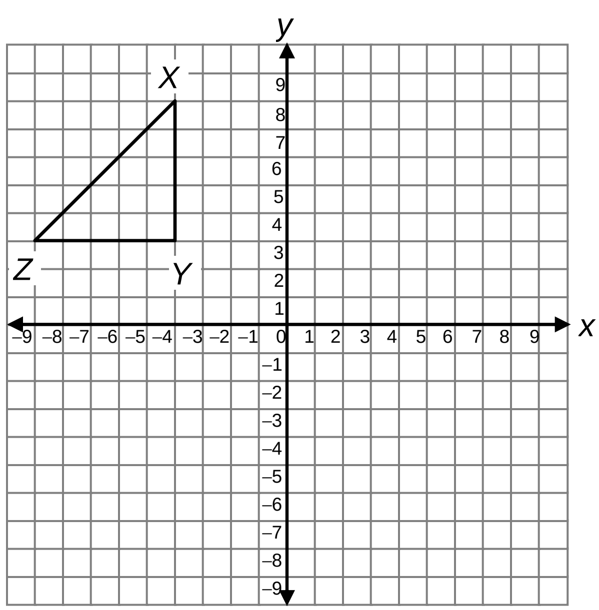
**3.** A tessellation is the tiling of shapes on the plane with no overlaps or gaps

- **a.** Tessellations can be described using transformations

## Practice Problems
**Lesson 17**

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 17.

**1)** If $\Delta XYZ$ is reflected over the line $y = x$, which of the following will be the coordinates of $Y'$?



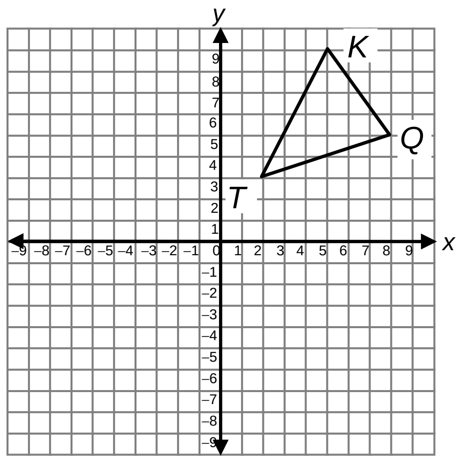

**A** (–4, –3)

**B** (3, –4)

**C** (4, 3)

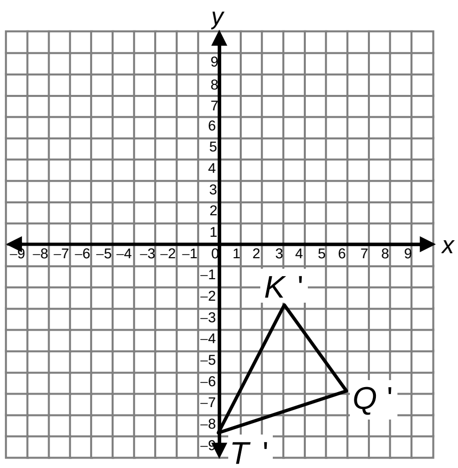
**D** (–9, 3)

**2)** $\Delta KQT$ is graphed on the grid.

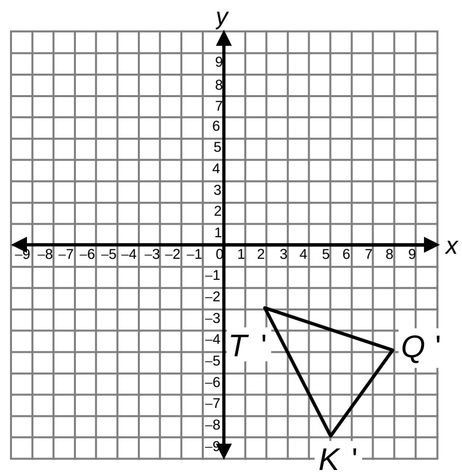



Which of the following best represents an image of $\Delta KQT$ translated 2 units to the left and reflected across the *x*-axis?

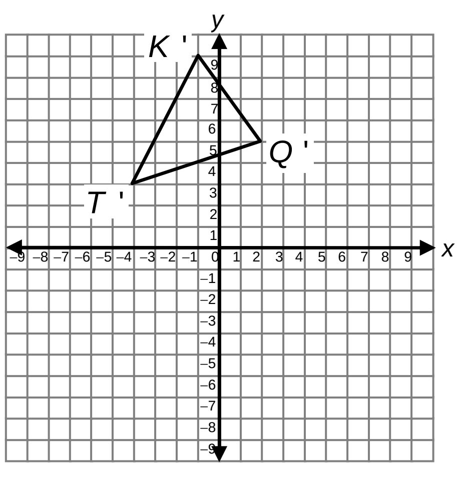
**A**



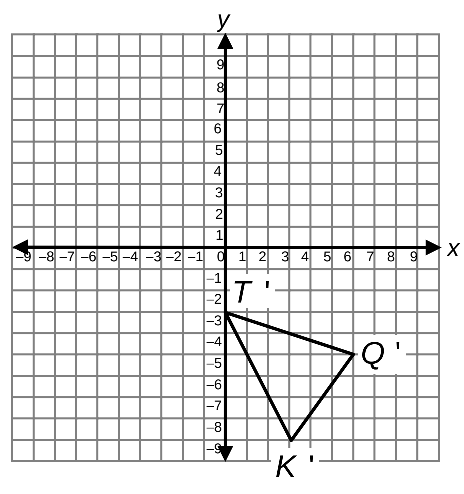

**B**



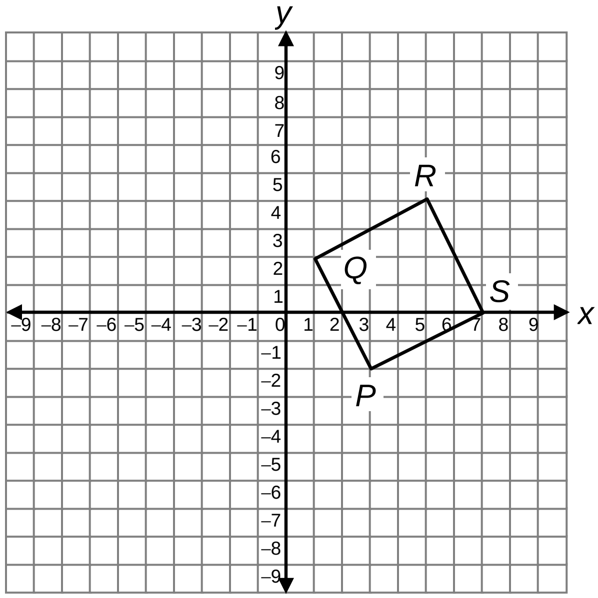

**C**




**D**



**3)** Which represents *P*', the image of *P* after a $90^\circ$ rotation counterclockwise about point *Q*?



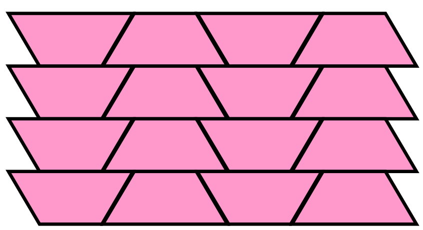

**A** *P*' (7, 0)

**B** *P*' (5, 4)

**C** *P*' (7, –4)

**D** *P*' (2, 6)

**4)** Which best describes the tessellation below?



**A** combining the isosceles trapezoid with its translation up and down

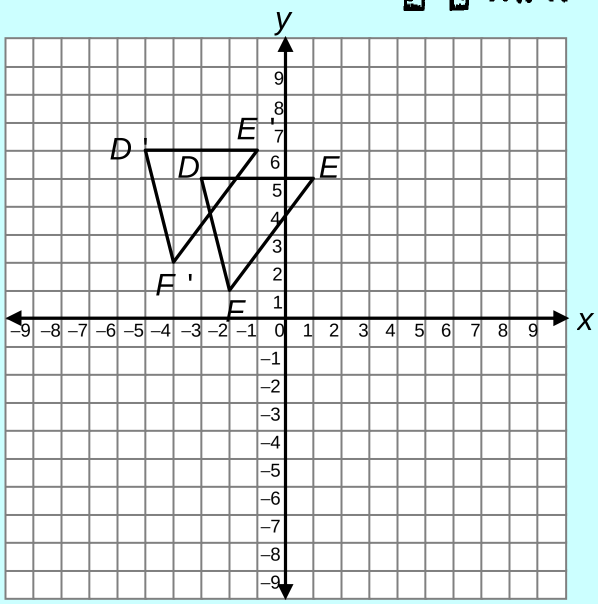
**B** combining the isosceles trapezoid with a $180^\circ$ rotation of the trapezoid

**C** combining the isosceles trapezoid with a $90^\circ$ rotation of the trapezoid

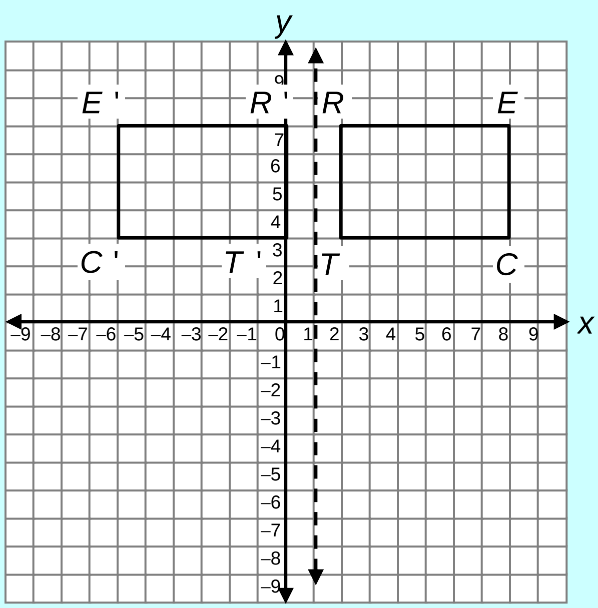
**D** combining the isosceles trapezoid with its translation left and right

## ANSWERS TO TRY IT

**1)**




**2)**




**3)** *A* '(–6, –8) *B* '(–1, 0) *C* '(–4, 5) *D* '(–9, –2)

**4)** *T* '(7, –6) *R* '(0, 2) *I* '(1, –9)

**5)** **A** **6)** **C** **7)** **D**

## NOTES

End of Lesson 17