Student Name: ______________________________

Date: ______________________________

Contact Person Name: ______________________________

Phone Number: ____________________

TAKS

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 18

Pythagorean Triples & Special Right Triangles

TAKS Objective 6 – Demonstrate an understanding of geometric relationships and spatial reasoning

Lesson Objectives:

- Use the Pythagorean theorem to verify Pythagorean triples
- Find a missing side of a right triangle using multiples of Pythagorean triples
- Solve problems involving special right triangles ($30° - 60° - 90°$ and $45° - 45° - 90°$)

Authors:

Tim Wilson, B.A.
Jason March, B.A., M.S.Ed

Editor:

Linda Shanks

Graphics:

Tim Wilson
Jason March

The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

National PASS Center
Geneseo Migrant Center
3 Mt. Morris – Leicester Road
Leicester, NY 14481
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass

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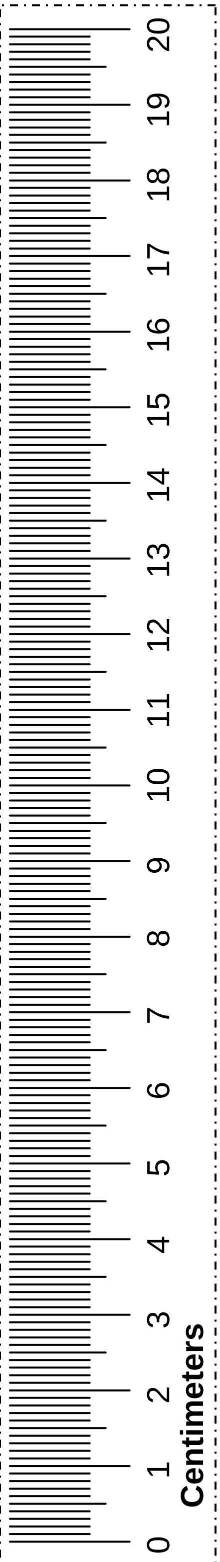
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

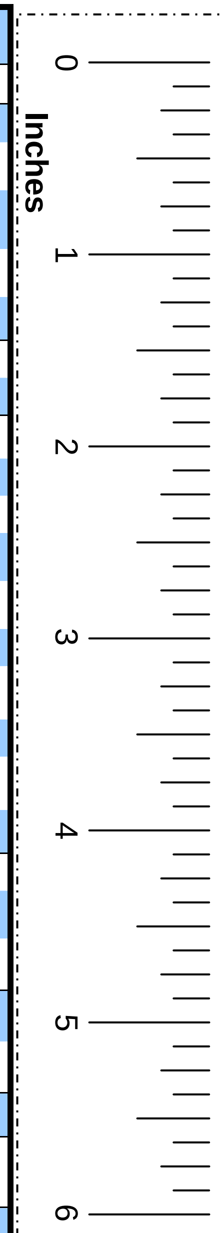
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

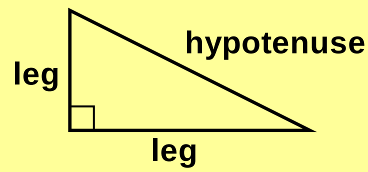
TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
Triangles	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

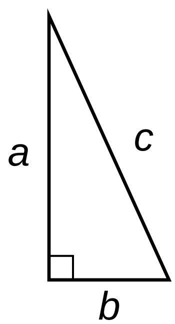


FACT

*In a right triangle, the sides touching the right angle are called **legs**. The side opposite the right angle is the **hypotenuse**.*

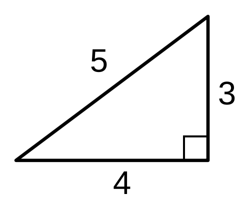


The **Pythagorean Theorem**, $a^2 + b^2 = c^2$, relates the sides of <u>right</u> triangles.



$$a^2 + b^2 = c^2$$

a and b are the lengths of the legs, and c is the length of the hypotenuse.



The set {3, 4, 5} is a **Pythagorean triple**.

$$a^2 + b^2 = c^2$$

$$(3)^2 + (4)^2 = (5)^2$$

$$9 + 16 = 25$$

$$25 = 25$$

A **Pythagorean triple** is a set of three whole numbers that satisfy the Pythagorean Theorem.

Example

Show that $\{5, 12, 13\}$ is a Pythagorean triple.

Solution

Always use the largest value as *c* in the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = 13^2$$

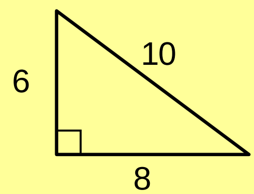
$$25 + 144 = 169$$

$$169 = 169$$

The numbers will equal at the end if it is a Pythagorean triple.

FACT

In a Pythagorean triple, it is assumed the largest number is the hypotenuse, since it is opposite the largest angle of the triangle.



Example

Show that $\{2, 2, 5\}$ is not a Pythagorean triple.

Solution

$$a^2 + b^2 = c^2$$

$$2^2 + 2^2 = 5^2$$

$$4 + 4 = 25$$

$$8 \neq 25$$

Side Note: Not only can no right triangle be represented with these side lengths, but no triangle at all can be. Try making a triangle with side lengths 2, 2, and 5 using toothpicks. It cannot be done! Do an internet search for “triangle inequality” to understand the reason why.

Showing that three numbers are a Pythagorean triple proves that a triangle with these side lengths will be a right triangle.

If we multiply each element of a Pythagorean triple, such as $\{3, 4, 5\}$, by another integer, for instance, 2, the result is another Pythagorean triple, $\{6, 8, 10\}$.

$$a^2 + b^2 = c^2$$
$$(6)^2 + (8)^2 = (10)^2$$
$$36 + 64 = 100$$
$$100 = 100$$

FACT

Multiplying a Pythagorean triple by any positive integer creates a new Pythagorean triple.

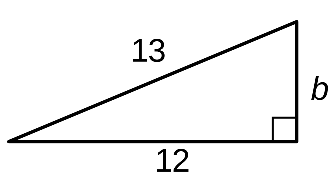
$\{3, 4, 5\} \xrightarrow{\times 3} \{9, 12, 15\}$

$\{5, 12, 13\} \xrightarrow{\times 2} \{10, 24, 26\}$

We can use this fact to solve for a missing side of certain right triangles.

Example

Find the length of side *b* in the right triangle below.

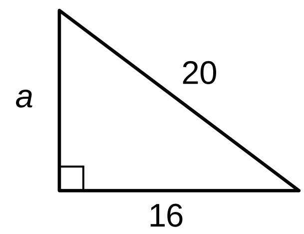


Solution

Use the Pythagorean triple $\{5, 12, 13\}$. The length of side *b* is 5 units.

Example

Find the length of side *a* in the right triangle below.



Solution

It is not obvious which Pythagorean triple these sides represent. Begin by dividing the given sides by their greatest common factor (GCF). A more obvious Pythagorean triple may be revealed this way.

{_, 16, 20}

$\div 4$

{_, 4, 5}

We see this is a $\{3, 4, 5\}$ Pythagorean triple. However, we are not done.

Since we <u>divided</u> by 4, we must now do the *opposite* and <u>multiply</u> by 4.

$\{3, 4, 5\}$

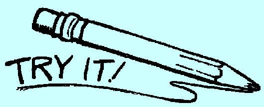
$\times 4$

$\{12, 16, 20\}$

Finally, side *a* is 12 units long.

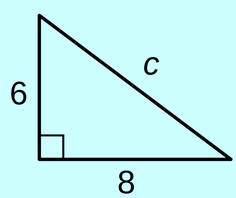
Problem Solving Tip

Somewhere on the test, create a reference list of Pythagorean triples. List the first three multiples of the two most common Pythagorean triples: $\{3, 4, 5\}$ and $\{5, 12, 13\}$. Check to see if the missing side of a right triangle corresponds to a Pythagorean triple before using the Pythagorean Theorem (a technique we will learn later).



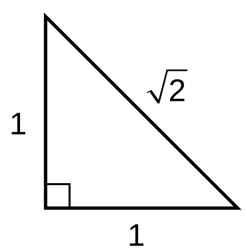
1) Verify that {10, 24, 26} is a Pythagorean triple.

2) Find the length of side *c* in the right triangle below.



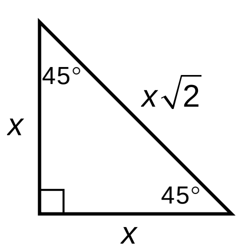
Special right triangles

Not every right triangle has sides that are Pythagorean triples.

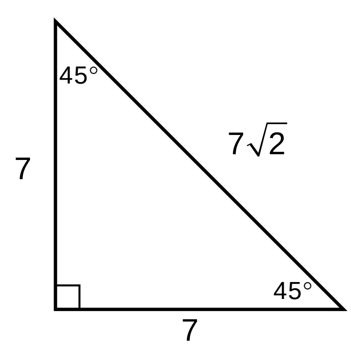


For two types of right triangles, their angles form sides that are in a special relationship.

1) 45°-45°-90° triangle

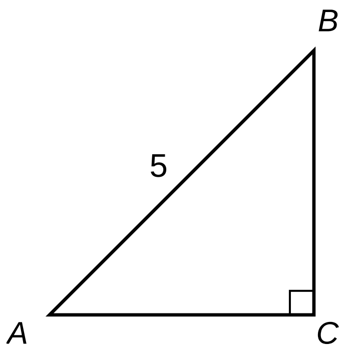


In a forty-five, forty-five, ninety right triangle, the sides opposite the 45° angles are equal. The side opposite 90° is equal to the length of a leg times $\sqrt{2}$.



Example

Which is closest to the length of side *AC* in the $45°\text{-}45°\text{-}90°$ triangle below.

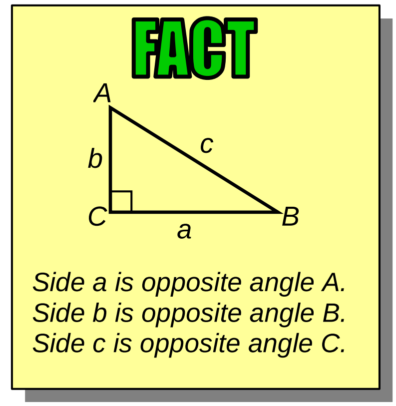


A 7.1 units

B 3.5 units

C 7.0 units

D 3.6 units



Solution

Method 1: numeric

This is a forty-five, forty-five, ninety special right triangle. The hypotenuse is the length of a leg times $\sqrt{2}$. Therefore, to solve for the length of leg *AC*, we will divide 5 by $\sqrt{2}$.

$$5 \div \sqrt{2} = 3.5355339 \approx 3.5 \text{ units}$$

Choice **B** is the answer.

Method 2: algebraic

On the reference table given to you on the exam, you will be given information regarding special right triangles as follows:

Special Right Triangles	$30°,\ 60°,\ 90°$	$x,\ x\sqrt{3},\ 2x$
	$45°,\ 45°,\ 90°$	$x,\ x,\ x\sqrt{2}$

Interpret this as: the sides opposite $45°$ are of length *x*. The side opposite $90°$ is of length $x\sqrt{2}$. If we connect this to the question, side length $5 = x\sqrt{2}$, and we wish to find $AC = x$. We will solve the equation $5 = x\sqrt{2}$ for *x* (shown at right). Again, choice **B** is the answer.

$$5 = x\sqrt{2}$$

$$\frac{5}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$$

$$\frac{5}{\sqrt{2}} = x$$

$$x = \frac{5}{\sqrt{2}} \approx 3.5$$

Example

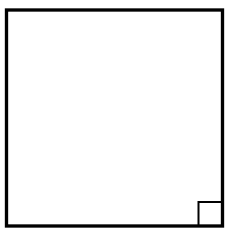
A fence around a square garden has a perimeter of 48 feet. Find the approximate length of the diagonal of this square garden.

A 12 ft **B** 17 ft

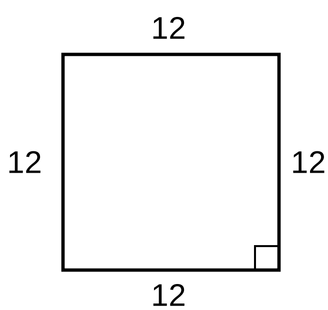
C 21 ft **D** 24 ft

Solution

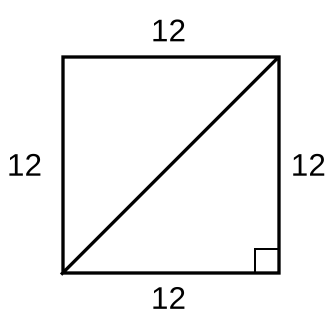
Draw a picture of a square fence to help visualize the problem.



We are told the perimeter is 48. Thus, each side of the fence must be $48 \div 4 = 12$. Write this on your picture.



Next, since we are asked to find the diagonal of this fence, draw the diagonal.



Notice that the legs of the right triangles formed are equal length. Therefore, this is a $45°\text{-}45°\text{-}90°$ triangle.

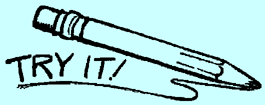
We know: $x = 12$

We wish to find: $x\sqrt{2}$

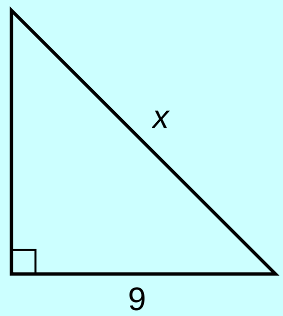
Plan: Multiply $x = 12$ by $\sqrt{2}$.

$12\sqrt{2} = 16.970563 \approx 17$

The answer is choice **B**.



3) Which is closest to the length of *x* in the $45°$-$45°$-$90°$ triangle below?



A 12

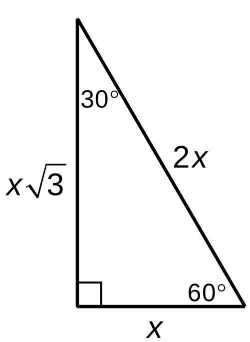
B 13

C 6

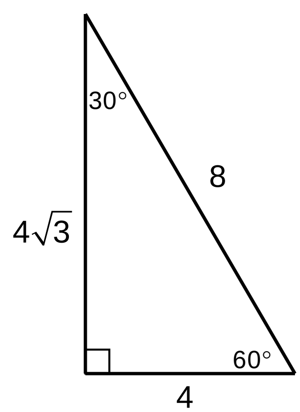
D 7

4) Hank swims the diagonal of a square pool with side-length 60 feet. What distance did Hank swim? (Round to the nearest whole foot.)

2) $30°$-$60°$-$90°$ triangle



In thirty, sixty, ninety right triangles, the hypotenuse is twice the length of the shorter leg. The length of the longer leg equals the length of the short leg times $\sqrt{3}$.



Example

ΔKMS has a right angle at M. The measure of $\angle MSK = 60^\circ$ and $KS = 17$ centimeters. Which is closest to the length of $\overline{KM}$?

A 9 cm

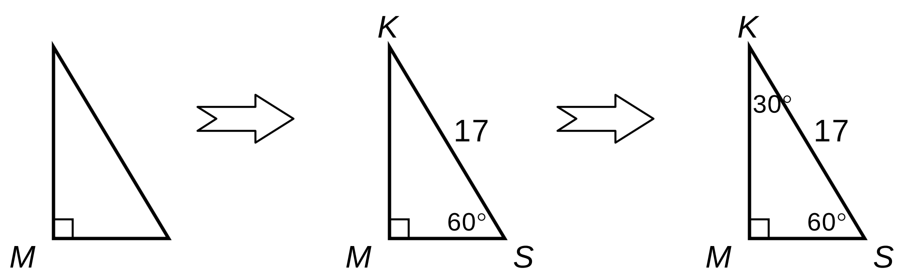
B 12 cm

C 10 cm

D 15 cm

Solution

We must draw a picture to set up the problem. One way to do this is shown below.



Given: right angle at M.

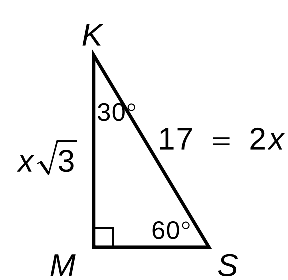
Given:
$\angle MSK = 60^\circ$
$KS = 17$

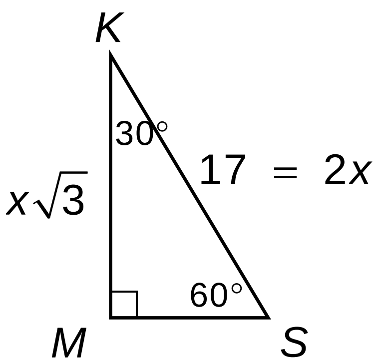
Use the fact the angles of a triangle total 180°
$180^\circ - 90^\circ - 60^\circ = 30^\circ$

We see that this is a special right triangle – a 30°-60°-90° triangle. Let's use the reference table provided on the test.

Special Right Triangles	$30^\circ, 60^\circ, 90^\circ$	$x,\ x\sqrt{3},\ 2x$
	$45^\circ, 45^\circ, 90^\circ$	$x,\ x,\ x\sqrt{2}$

On our picture, fill in the corresponding side length ratios.





We are given $2x$ and we wish to find $x\sqrt{3}$. Do this by first solving for x.

$$2x = 17$$
$$\frac{2x}{2} = \frac{17}{2}$$
$$x = 8.5$$

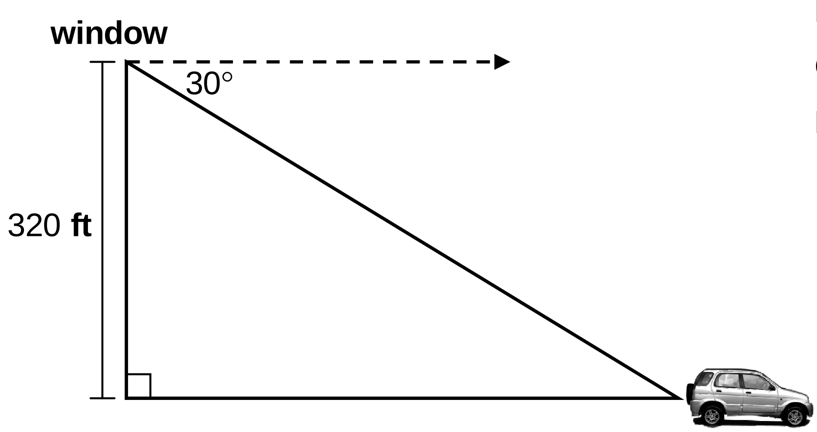
Now that we have x, we will find $x\sqrt{3}$ by multiplying: x times $\sqrt{3}$.

$$x = 8.5$$
$$x \cdot \sqrt{3} = 8.5 \cdot \sqrt{3}$$
$$x\sqrt{3} = 14.722432 \approx 15$$

The answer is choice **D**.

Example

Dylan is looking out a window of his apartment 320 feet above the ground. He sees a car at an angle of depression of $30°$. What is Dylan's approximate horizontal distance from the car at this point?



A 185 ft

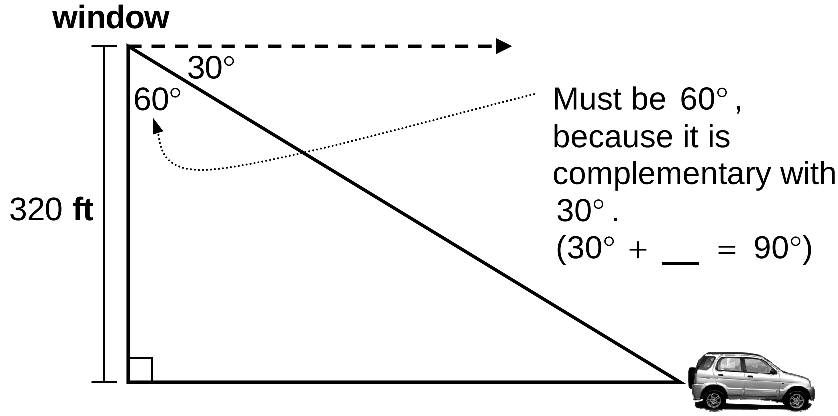
B 320 ft

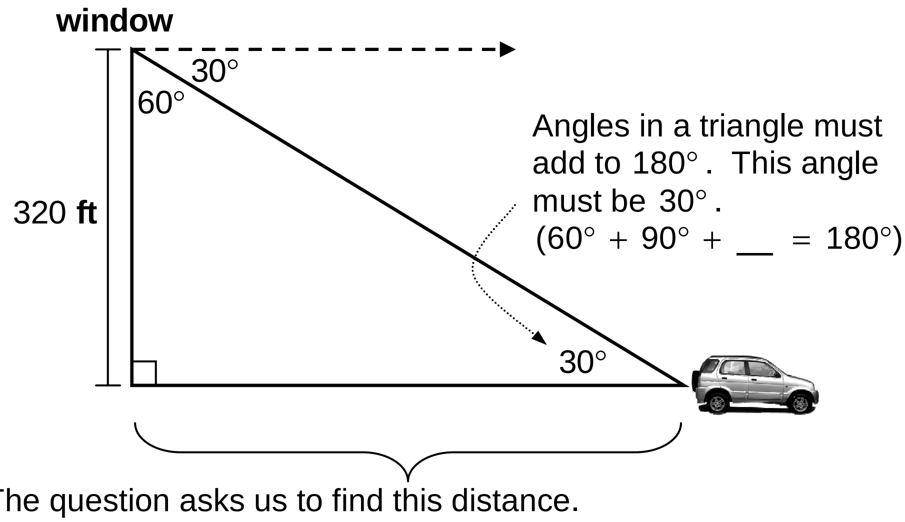
C 554 ft

D 640 ft

Solution

Start by filling in the angles of the triangle.





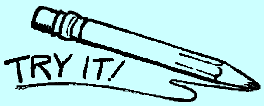
This is a thirty, sixty, ninety special right triangle.

Special Right Triangles	30°, 60°, 90°	x, $x\sqrt{3}$, $2x$
	45°, 45°, 90°	x, x, $x\sqrt{2}$

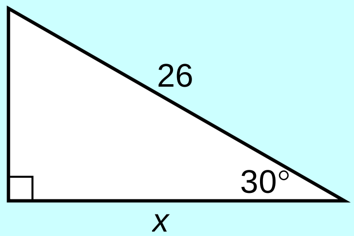
From the chart, $320 = x$ and we wish to find $x\sqrt{3}$. Multiply 320 by $\sqrt{3}$.

$$320 \bullet \sqrt{3} = 554.25626... \approx 554$$

The answer is choice **C**.



5) Which is closest to the length of x in the $30°$-$60°$-$90°$ triangle below?



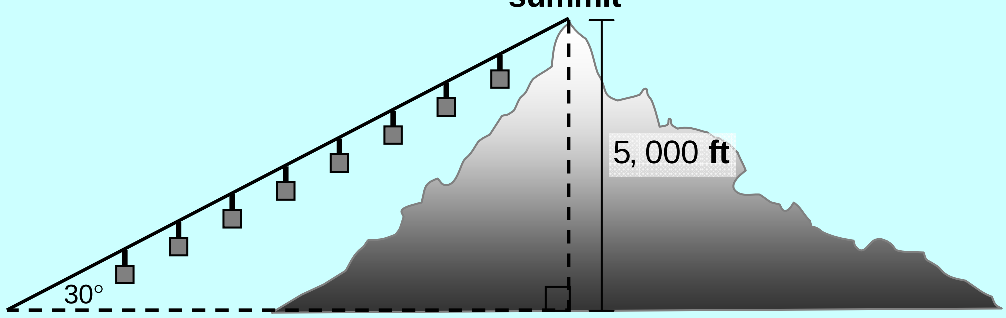
A 13

B 19

C 23

D 26

6) The cable cars of a ski lift rise 5,000 vertical feet from the base at a constant $30°$ angle of inclination.



What is the approximate straight-line distance that a cable car travels from the base to the summit of the mountain?

A 2,500 ft

B 2,900 ft

C 8,500 ft

D 10,000 ft

Review

Know these concepts:

1. A Pythagorean triple is a set of three numbers that satisfies the Pythagorean theorem, $a^2 + b^2 = c^2$.

  a. A multiple of any Pythagorean triple is also a Pythagorean triple.

  b. Pythagorean triples can be used to determine the missing side of some right triangles.

2. There are two types of special right triangles:

  a. $30°, 60°, 90°$

    i. Side lengths opposite these respective angles are in the ratio $x, x\sqrt{3}, 2x$

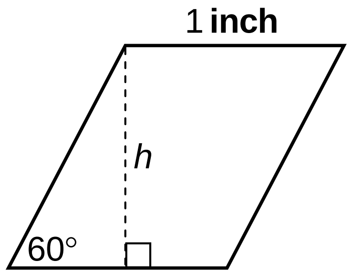
  b. $45°, 45°, 90°$

    i. Side lengths opposite these respective angles are in the ratio $x, x, x\sqrt{2}$

Practice Problems
Lesson 18

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 18.

1) A rhombus is shown below.



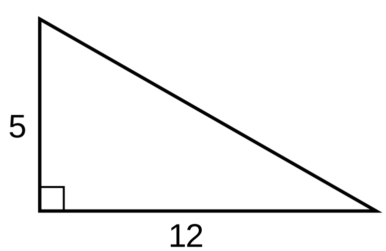
If the height, *h* intersects the base at its midpoint, which of these is closest to the height of the rhombus?

A 1 in. **C** 1.5 in.

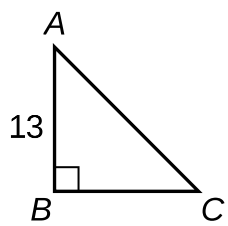
B 0.9 in. **D** 0.8 in.

2) Verify that {8,15,17} is a Pythagorean triple.

3) Using Pythagorean triples, fill in the missing side of the right triangle below.



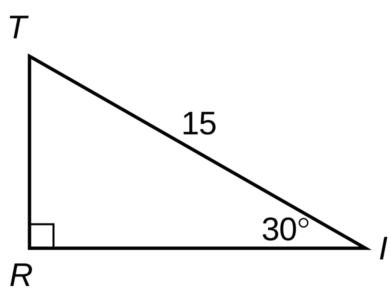
4) Which represents the length of side *BC* in the $45°,\ 45°,\ 90°$ triangle below?



A 13 **B** $13\sqrt{2}$

C $13\sqrt{3}$ **D** 26

5) Which is closest to the length of side *TR* in the $30°,\ 60°,\ 90°$ triangle below?



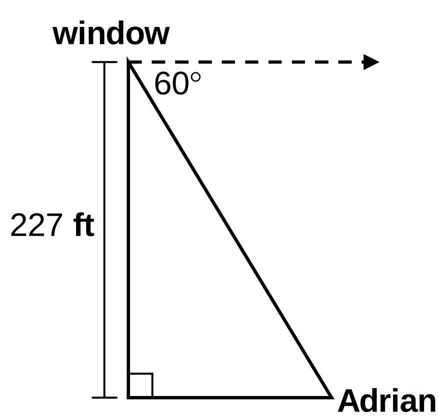
A 25.98

B 15.00

C 8.66

D 7.50

6) Rocky spots his friend, Adrian, from his bedroom window, 227 feet above ground. Adrian is at an angle of depression of $60°$. What is Rocky's approximate horizontal distance from Adrian at this point?



A 393 ft

B 131 ft

C 321 ft

D 113.5 ft

1)

$$a^2 + b^2 = c^2$$
$$10^2 + 24^2 = 26^2$$
$$100 + 576 = 676$$
$$676 = 676$$

2) 10

3) **B**

4) 85 feet

5) **C**

6) **D**

End of Lesson 18