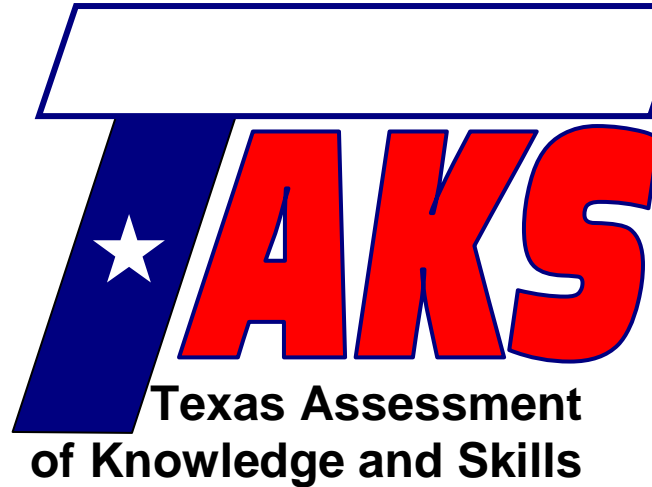


Student Name: _____

Date: _____

Contact Person Name: _____

Phone Number: _____



Exit Level Math Review

Lesson 21

Perspective

TAKS Objective 7 – Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes

Lesson Objectives:

- Solve isometric cube problems given front, side, and top views
- Find the surface area and volume of a 3-dimensional shape from its front, side, and top views

Authors:

Tim Wilson, B.A.
Jason March, B.A., M.S.Ed

Editor:

Linda Shanks

Graphics:

Tim Wilson
Jason March

The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

- 1) Describe functional relationships in a variety of ways.
- 2) Demonstrate an understanding of the properties and attributes of functions.
- 3) Demonstrate an understanding of linear functions.
- 4) Formulate and use linear equations and inequalities.
- 5) Demonstrate an understanding of quadratic equations and other nonlinear functions.
- 6) Demonstrate an understanding of geometric relationships and spatial reasoning.
- 7) Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.
- 8) Demonstrate an understanding of concepts and uses of measurement and similarity.
- 9) Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.
- 10) Demonstrate an understanding of the mathematical processes and tools used in problem solving.

National PASS Center
Geneseo Migrant Center
3 Mt. Morris – Leicester Road
Leicester, NY 14481
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass



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TAKS Mathematics Chart



Length

Metric

1 kilometer = 1000 meters
1 meter = 100 centimeters
1 centimeter = 10 millimeters

Customary

1 mile = 1760 yards
1 mile = 5280 feet
1 yard = 3 feet
1 foot = 12 inches

Capacity and Volume

Metric

1 liter = 1000 milliliters

Customary

1 gallon = 4 quarts
1 gallon = 128 fluid ounces
1 quart = 2 pints
1 pint = 2 cups
1 cup = 8 fluid ounces

Mass and Weight

Metric

1 kilogram = 1000 grams
1 gram = 1000 milligrams

Customary

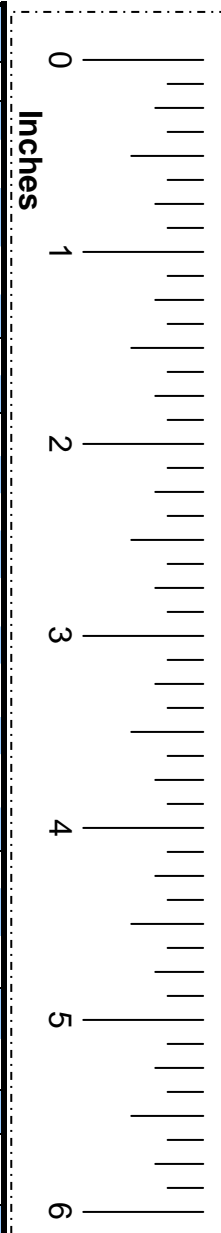
1 ton = 2000 pounds
1 pound = 16 ounces

Time

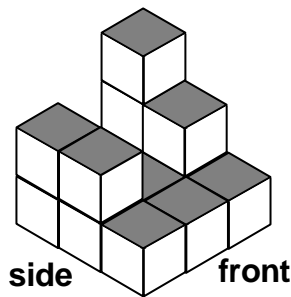
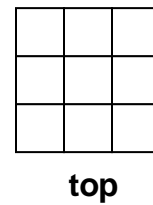
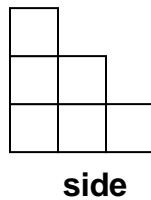
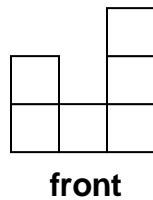
1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Representing 3-dimensional shapes on paper presents a unique problem, since the surface of paper is 2-dimensional. One way to accurately represent a 3-D figure is by showing how the shape looks from different perspectives. Usually, the top, side, and front views are shown.



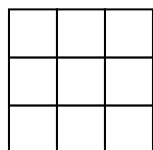
Problem Solving Tip

Take a minute to study the figures above and the relationship between them. Trust yourself that you will see how it fits.

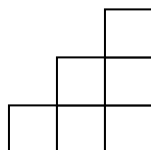
Sometimes, your brain needs to process something for more than a full minute to make sense of it.

Example

The front and right-side views of a figure made of identical cubes are shown below.

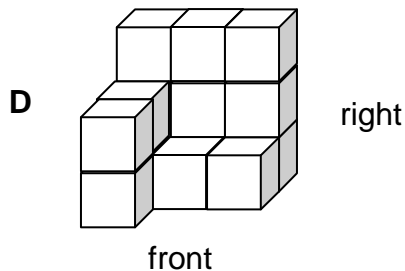
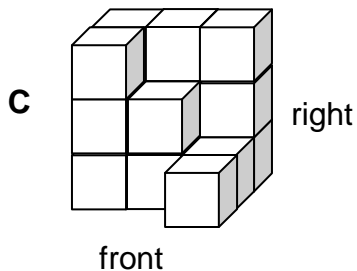
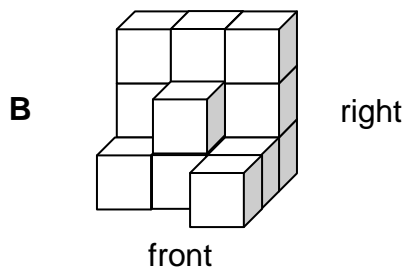
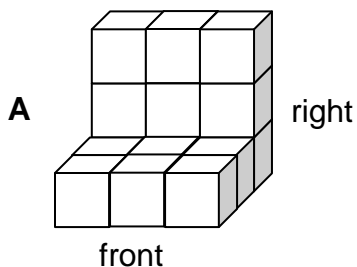


front view



right-side view

Which 3-dimensional figure is best represented by the two views above?

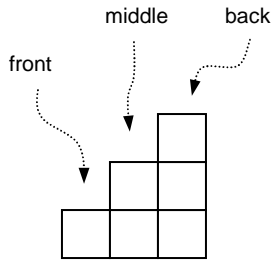


Problem Solving Tip

Before reading the solution, try to solve the problem on your own for at least two full minutes. Don't be discouraged if the answer is not obvious at first.

Solution

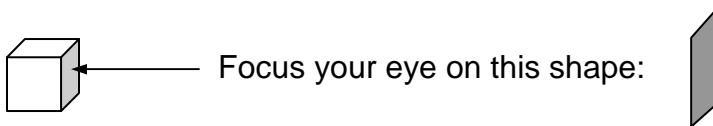
Consider the side-view.



right-side view

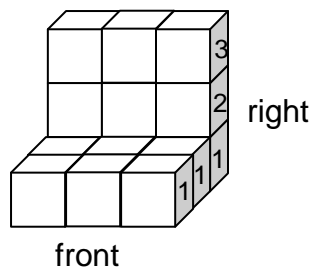
The front row will be 1 cube high, the middle row will be 2 cubes high, and the back will be 3 cubes high.

Next, observe what shape the right side of a cube looks like.



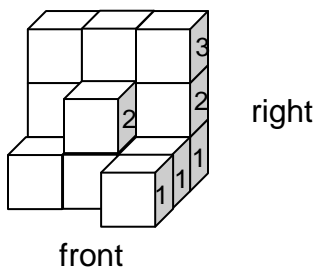
Focus your eye on this shape:

Focusing on this right-side-view shape, analyze each answer choice. The height from the front, to middle, to back row should be 1-2-3. Let's start with choice **A**.



This height is 1-1-3 from front to back. Choice **A** is not true.

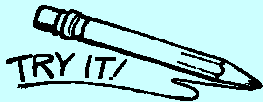
Next, check choice **B**.



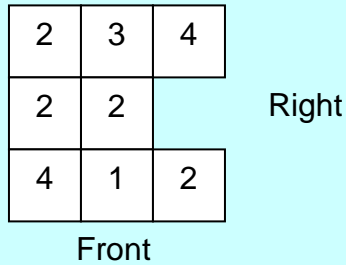
The right-side height from front to back is 1-2-3. Choice **B** may be correct.

After checking choices **C** and **D** (check them on your own right now), choice **B** is the only one with the correct right-side heights. Therefore, the answer is choice **B**.

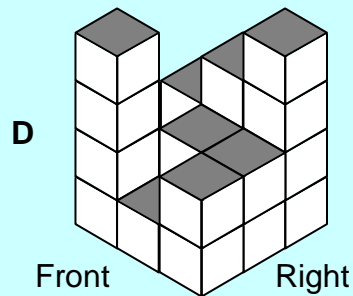
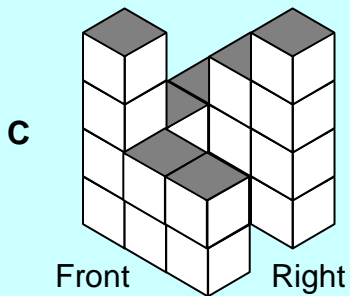
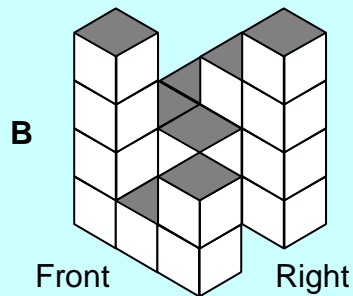
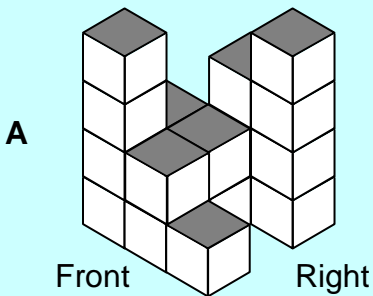
You may see a slightly different version of this problem. It will require the same type of analysis.



- 1) The diagram below shows the top view of a structure built with identical cubes, as well as the number of cubes in each column of the structure.



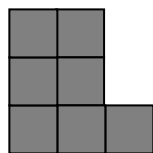
Which 3-dimensional view best represents the same structure?



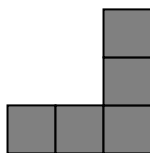
You may be asked to solve volume problems given perspective drawings.

Example

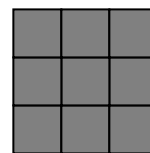
The front, side, and top views of a solid built from identical cubes are shown below.



Front



Side



Top

How many cubes were needed to construct this solid?

- A** 13 **B** 14
C 15 **D** 21

Solution

Method 1: a logical approach

The top view shows that there are at least 9 cubes. The front view shows that there are at least 2 towers of 3 cubes, for at least 4 additional cubes (do not double count ones on the bottom). The new minimum

total is 13 cubes. We cannot conclude there are exactly 13 cubes, as there could be multiple towers in the left and middle columns. From the side view though, we gather that the two towers shown in the front view are the only two towers. The structure has 13 blocks. The answer is choice **A**.

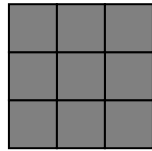
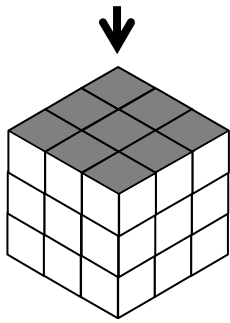
Problem Solving Tip

If after two full minutes spent on a test problem, you are making little progress, draw a star by the question number, set that question aside, and come back to it later if there is time.

Method 2: a picture approach

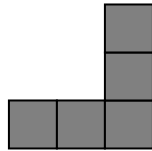
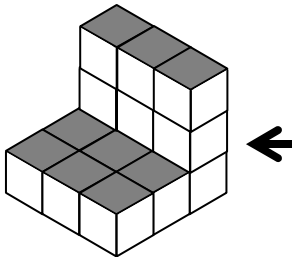
If you are artistic or you are a visual learner, drawing pictures may benefit you. One possible approach to doing this is below.

TAKS Review



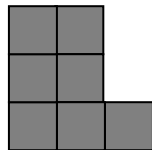
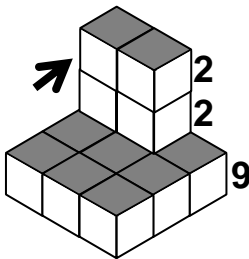
Top

Draw what you think the structure looks like from only the top view. The small arrows are provided only as a reference.



Side

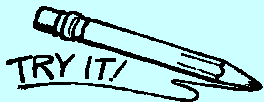
Next, redraw your first picture, according to the information given from either the side view or the front view.



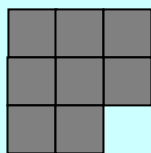
Front

Finally, redraw the current picture, given the final view. Count the number of blocks.

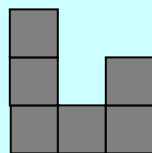
13 blocks: the answer is choice **A**.



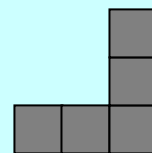
2) The top, front, and side view of a solid built from identical cubes is shown below.



Top



Front



Side

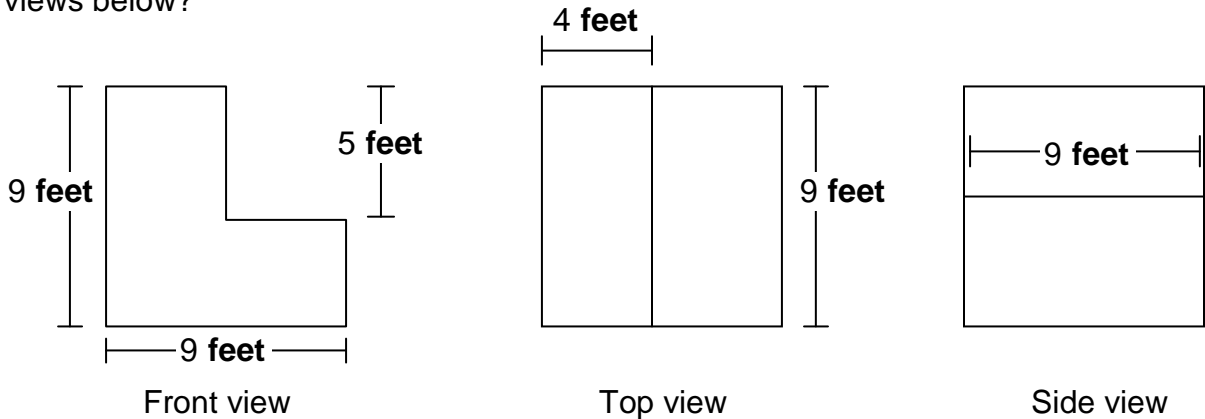
How many cubes are needed to construct this solid?

- A** 14
- B** 13
- C** 12
- D** 11

Perspective drawings are not always constructed from identical cubes.

Example

What is the volume of a 3-dimensional object with the dimensions shown in the 3 views below?



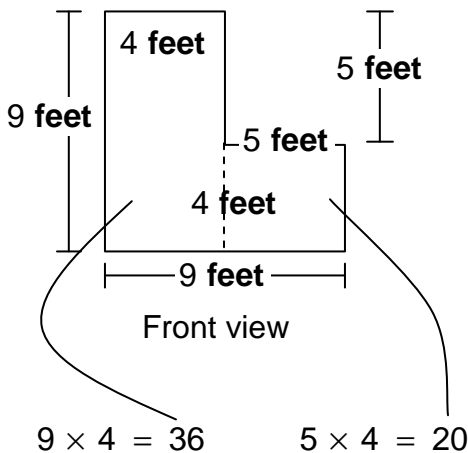
- A** 960 cu ft
- B** 120 cu ft
- C** 504 cu ft
- D** 512 cu ft

Solution

Method 1: use a formula

Study the perspective views until you understand this is a prism. Next, look on your formula sheet in the volume section. The volume of a prism is given to be $V = Bh$, where V means volume, B stands for the area of the base, and h is the height, or the distance between bases.

Step 1: Find B



The base is given by the front view. We find its area by breaking the shape into two rectangles. Use information from other views to fill in missing measurements.

$$B = 36 + 20 = 56$$

TAKS Review

Step 2: Find the volume.

$$V = Bh$$

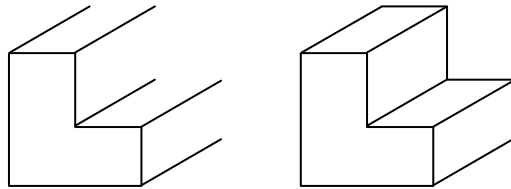
$$V = (56)(9)$$

$$V = 504$$

The answer is choice **C**.

Method 2: use a picture

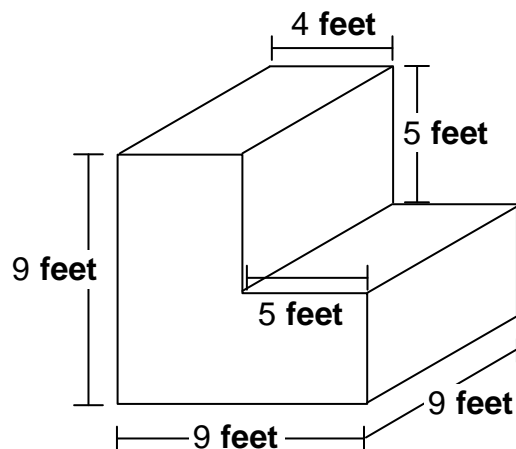
Study the perspective views until you understand this is a prism. Next, sketch a 3-D view of this object. To do this, draw equal-length parallel line segments from the vertices as follows.



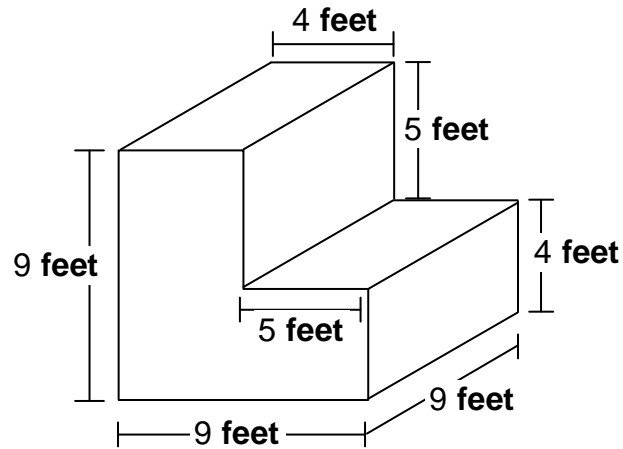
Connect the segments.

The connection lines should be parallel with those of the base.

Next, label your sketch with lengths taken from the given information.



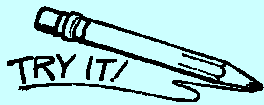
Using this given information, fill in the missing measurements.



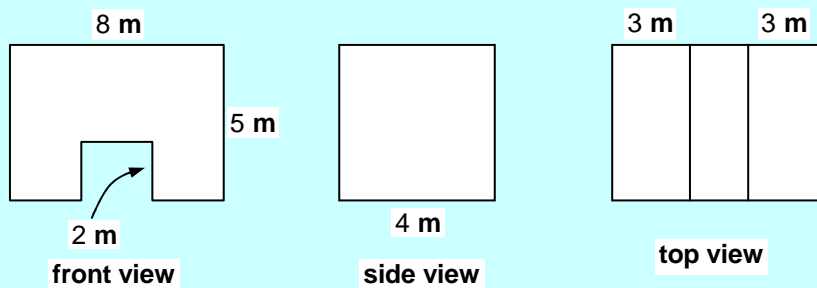
Next, calculate the volume in one of two ways:

1. Split the solid into two rectangular prisms and add their volumes.
2. Calculate the volume of the $9 \times 9 \times 9$ cube and subtract the area of the top right chunk missing from it.

In either method, the result will be the same as in Method 1. The answer is choice C.



- 3) Three views of a 3-D figure with the given dimensions are shown below.



What is the volume of this figure?

- A 144 m^3
- B 160 m^3
- C 36 m^3
- D 152 m^3

 **Review**

Know these concepts:

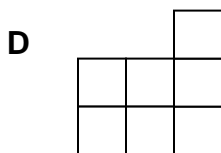
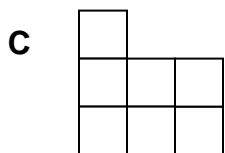
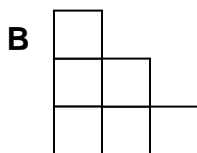
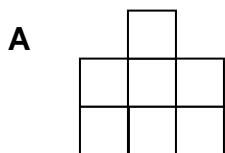
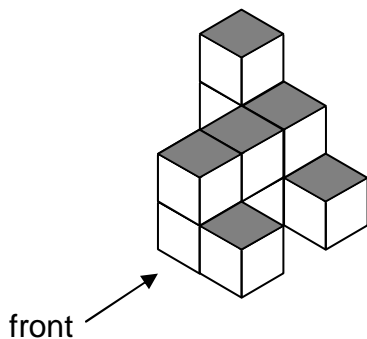
1. Perspective drawings represent 3-dimensional solids
2. Perspective drawings can be solved:
 - a. logically
 - b. by drawing a picture
 - c. using volume formulas



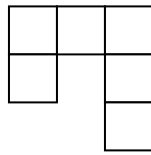
Practice Problems
Lesson 21

Directions: Write your answers in your math journal. Label this exercise
TAKS Review – Lesson 21.

- 1) Which represents the right-side view of the 3-dimensional figure below?

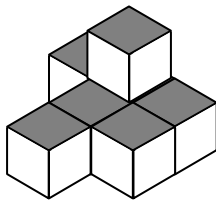


- 2) The top view of a structure built from identical cubes is below.

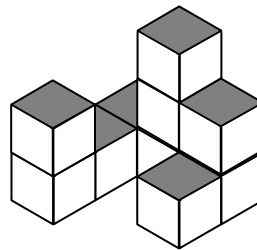


Which 3-dimensional figure has the same top view?

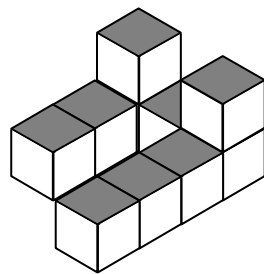
A



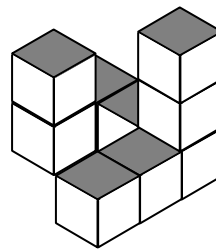
B



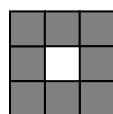
C



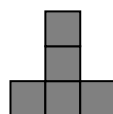
D



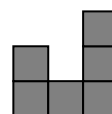
- 3) The top, front, and side view of a solid built from identical cubes is shown below.



Top



Front



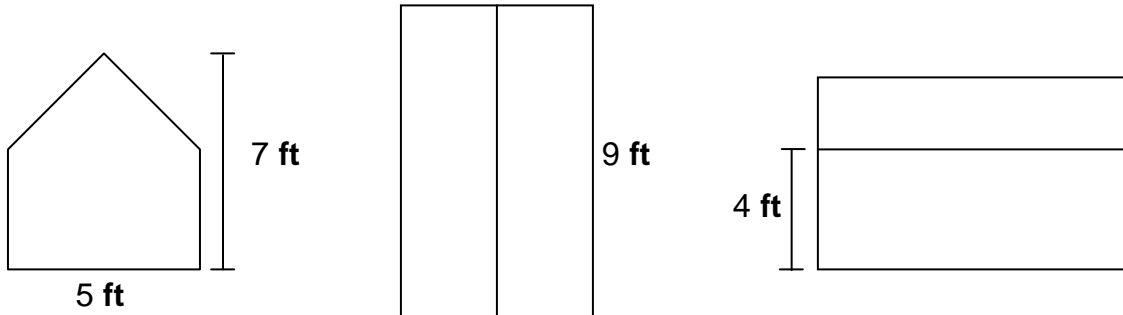
Side

How many cubes are in the 3-dimensional solid represented by these drawings?

- | | | | |
|----------|----|----------|----|
| A | 10 | B | 11 |
| C | 12 | D | 19 |

TAKS Review

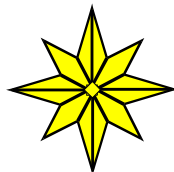
- 4) What is the volume of a 3-dimensional object with the dimensions shown in the 3 views below?



- A 247.5 cu ft B 315 cu ft
C 189 cu ft D 105.5 cu ft

**ANSWERS TO
TRY IT**

1) B 2) D 3) A



End of Lesson 21