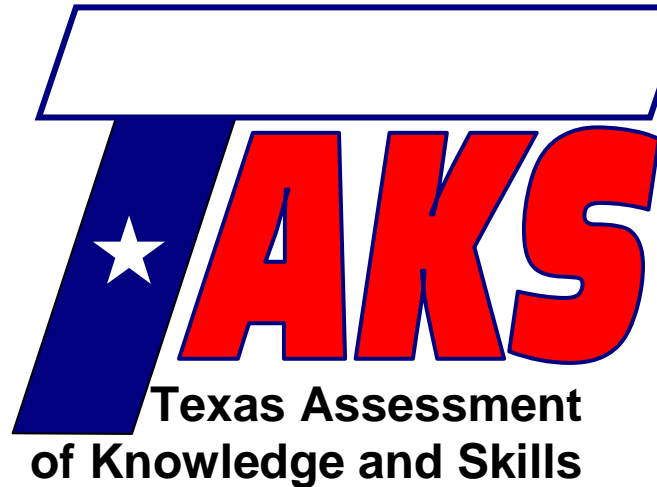


Student Name: _____

Date: _____

Contact Person Name: _____

Phone Number: _____



Exit Level Math Review

Lesson 23

Using Proportions

TAKS Objective 8 – Demonstrate an understanding of concepts and uses of measurement and similarity

Lesson Objectives:

- Use proportions to find the missing dimensions in similar figures
- Use proportions to find arc length
- Use proportions to find the area of a sector of a circle

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

- 1) Describe functional relationships in a variety of ways.
- 2) Demonstrate an understanding of the properties and attributes of functions.
- 3) Demonstrate an understanding of linear functions.
- 4) Formulate and use linear equations and inequalities.
- 5) Demonstrate an understanding of quadratic equations and other nonlinear functions.
- 6) Demonstrate an understanding of geometric relationships and spatial reasoning.
- 7) Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.
- 8) Demonstrate an understanding of concepts and uses of measurement and similarity.
- 9) Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.
- 10) Demonstrate an understanding of the mathematical processes and tools used in problem solving.

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TAKS Mathematics Chart



Length

Metric

1 kilometer = 1000 meters
1 meter = 100 centimeters
1 centimeter = 10 millimeters

Customary

1 mile = 1760 yards
1 mile = 5280 feet
1 yard = 3 feet
1 foot = 12 inches

Capacity and Volume

Metric

1 liter = 1000 milliliters

Customary

1 gallon = 4 quarts
1 gallon = 128 fluid ounces
1 quart = 2 pints
1 pint = 2 cups
1 cup = 8 fluid ounces

Mass and Weight

Metric

1 kilogram = 1000 grams
1 gram = 1000 milligrams

Customary

1 ton = 2000 pounds
1 pound = 16 ounces

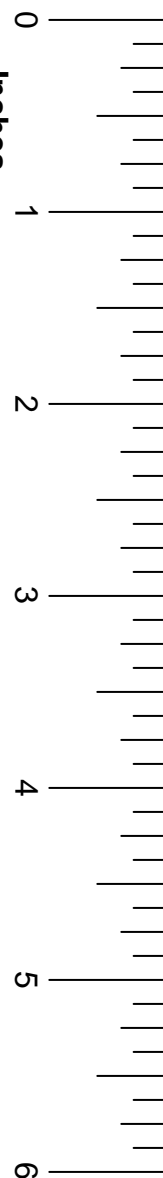
Time

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Inches



Proportions can be very useful when solving certain types of word problems.

A **proportion** is a set of equal ratios or fractions.

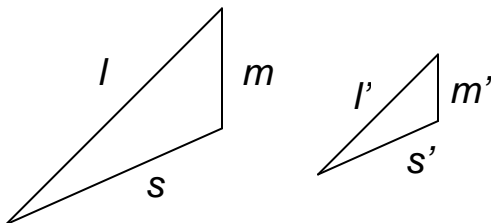
$$\frac{1}{2} = \frac{3}{6}$$

$$3 : 2 = 4.5 : 3$$

$$\frac{25}{40} = \frac{x}{50}$$

If two shapes are **similar**, their **corresponding sides** are proportional.

In general, similar shapes are the same shape, but not always the same size.



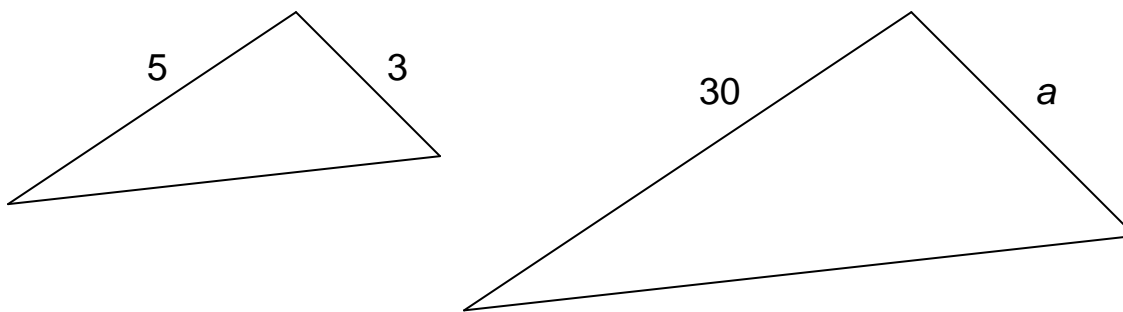
At left, the big triangle is similar to the little triangle. There are 3 pairs of corresponding, proportional sides.

- 1) l and l'
- 2) m and m'
- 3) s and s'

Since the ratios are equal in a proportion, we are able to solve for a variable if one is used. One goal of this lesson is to do so.

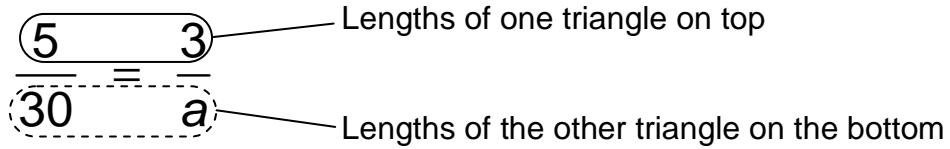
Example

The following triangles are similar. Find the length of the side labeled a .



Solution

We are told that the triangles are similar. That is our clue to make a proportion using the corresponding sides. Here is one way to set up the proportion:



To solve a proportion, we need to use the cross product. The cross product is the same as the product of the means and the extremes.

$$\frac{5}{30} = \frac{3}{a}$$

$$5a = 90$$

Next, solve the equation.

$$\frac{5a}{5} = \frac{90}{5}$$

$$a = 18$$

FACT

If two fractions are equal, their cross product is equal as well. We take the cross-product because it results in a linear equation, which we know how to solve.

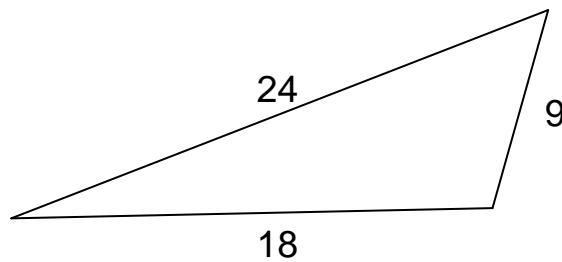
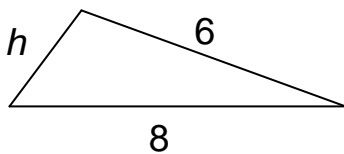
In other words, if $\frac{a}{b} = \frac{c}{d}$

Then $ad = cb$ as well.

Therefore, the length of the missing side is 18.

Example

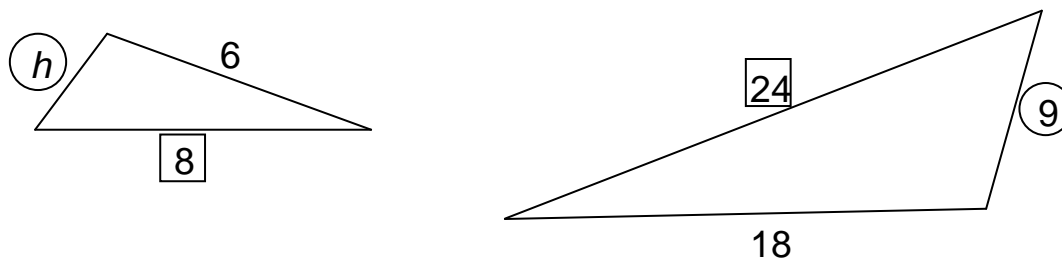
The following triangles are similar. Solve for the missing side, h .



- A** $h = 3$ **B** $h = 5$
- C** $h = 9$ **D** $h = 12$

Solution

First, determine two sets of corresponding sides. It requires some thinking to determine which sides correspond to each other. This is because the triangles are not in the same position. h and 9 are corresponding sides, because they are the shortest side on each triangle. 8 and 24 are corresponding sides, because they are lengths of the longest sides.



Next, set up a proportion, and cross multiply. Then, solve for h .

$$\begin{array}{c} \frac{h}{9} \quad \frac{8}{24} \\ \swarrow \quad \searrow \\ \searrow \quad \swarrow \\ \frac{24h}{24} = \frac{72}{24} \\ h = 3 \end{array}$$

The answer is choice **A**.

**FACT**

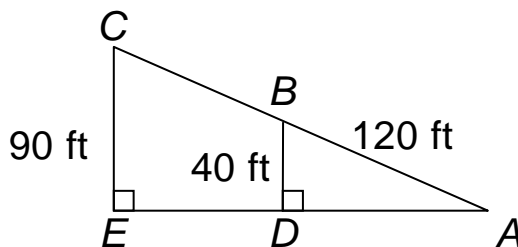
If two or more triangles have two congruent angles, they are similar triangles. As long as two angles have the same measure, the third angles will be the same, because the angle sum must be 180° . This is only true for triangles.

Similar triangles can be made by cutting a triangle with a line parallel to one of the sides.

Example

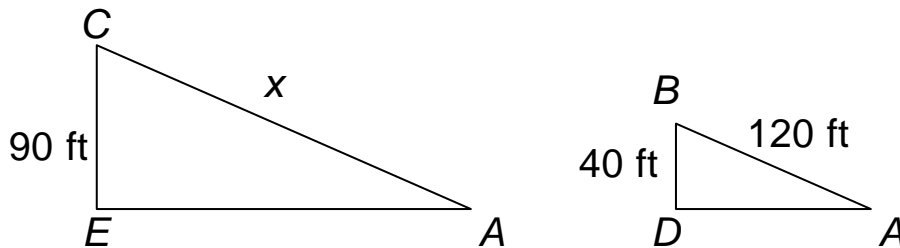
What is the distance from point B to point C ?

- A 270
- B 210
- C 150
- D 110



Solution

The similar triangles that are formed are $\triangle ABD$ and $\triangle ACE$. These triangles share angle A , and each has a right angle. Since they share two equal angles, they are similar triangles.



We are looking for the length of segment \overline{BC} , but we need to find segment \overline{AC} first since $AC - AB = BC$. To find \overline{AC} , we need to solve the following proportion:

$$\frac{40}{90} = \frac{120}{x}$$

$$\frac{40x}{40} = \frac{10,800}{40}$$

$$x = 270$$

FACT

A proportion can be set up in many ways. Here are other possible setups to solve this problem.

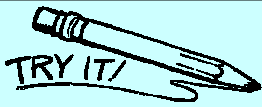
$$\frac{90}{40} = \frac{x}{120} \qquad \frac{90}{x} = \frac{40}{120} \qquad \frac{x}{90} = \frac{120}{40}$$

The length of \overline{AC} is 270 ft. Now we need to find the length of \overline{BC} .

$$AC - AB = BC$$

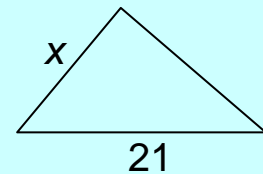
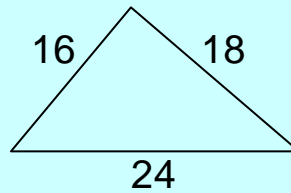
$$270 - 120 = 150 \text{ ft.}$$

The length of BC is 150 ft. The answer is choice **C**.



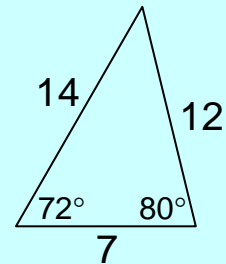
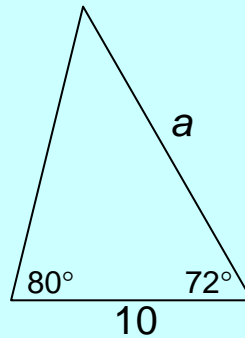
1) The following triangles are similar. Solve for the missing side, x .

- A $x = 16$
- B $x = 14$
- C $x = 18$
- D $x = 20$



2) Solve for the missing side, a .

- A $a = 10$
- B $a = 15$
- C $a = 20$
- D $a = 17\frac{1}{7}$

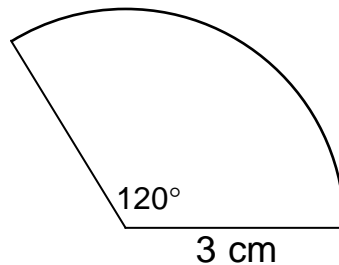


Proportions can be useful for problems related to circles.

Example

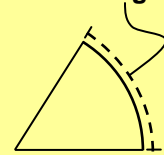
Find the length of the arc shown.

- A 18.84 cm
- B 6.28 cm
- C 9.42 cm
- D 3.14 cm



FACT

This distance is the **arc length**.



Solution

The length of an arc is related to the circumference of a circle. If there were a circle with a radius of 3, the circumference would be found using $C = 2\pi r$.

$$C = 2\pi r$$

$$C = 2\pi(3)$$

$$C = 6\pi$$

$$C = 18.84$$

Next, we need to find the arc length based on the portion of the circle that is shown. A whole circle has 360° in it. We can set up a proportion that compares the ratio of the angles to the ratio of the arc lengths.

$$\frac{\text{unknown arc length}}{\text{circumference of the circle}} = \frac{\text{measure of angle}}{360^\circ}$$

$$\frac{x}{18.84} = \frac{120}{360}$$

We can reduce the fraction on the right; then cross multiply to solve for x .

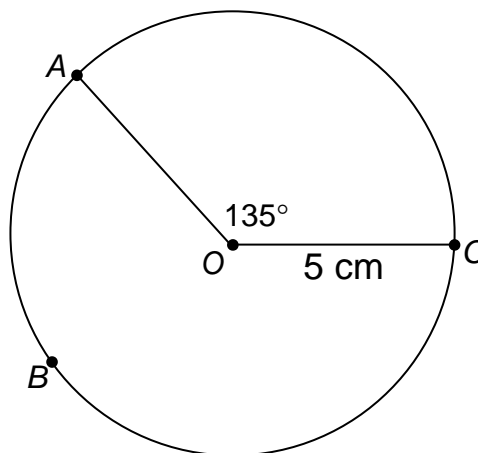
$$\begin{array}{r} \frac{x}{18.84} = \frac{1}{3} \\ \frac{3x}{3} = \frac{18.84}{3} \\ x = 6.28 \end{array}$$

Thus, the arc length is 6.28 cm, which means choice **B** is the answer.

Example

Find the length of major arc \widehat{ABC} .

- A 31.4 cm
- B 11.775 cm
- C 19.625 cm
- D 9.875 cm



Solution

First, find the circumference of the whole circle.

$$C = 2\pi r$$

$$C = 2\pi(5)$$

$$C = 10\pi$$

$$C = 31.4$$

Next, we need to find the arc length using a proportion. However, the arc we are looking for is a major arc, and we have been given the central angle of the minor arc. We could either (a) find the central angle of the major arc by subtracting the minor arc angle by 360° , or we could (b) find the length of the minor arc and subtract that from the total circumference. Both will result in the same answer.

$$\begin{aligned} &\text{method (a)} \\ \frac{x}{31.4} &= \frac{360 - 135}{360} \\ \frac{x}{31.4} &= \frac{225}{360} \\ \frac{x}{31.4} &= \frac{5}{8} \\ \frac{8x}{8} &= \frac{157}{8} \\ x &= 19.625 \end{aligned}$$

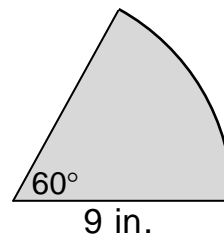
$$\begin{aligned} &\text{method (b)} \\ \frac{x}{31.4} &= \frac{135}{360} \\ \frac{x}{31.4} &= \frac{3}{8} \\ \frac{8x}{8} &= \frac{94.2}{8} \\ x &= 11.775 \\ 31.4 - 11.775 &= 19.625 \end{aligned}$$

Thus, the answer is choice **C**.

Example

Find the area of the sector of the circle shown.

- A 42.39 in.²
- B 4.71 in.²
- C 254.34 in.²
- D 28.26 in.²



Solution

This is very similar to the last problem. This time we need to find the area of a circle with the radius of 9.

$$A = \pi r^2$$

$$A = \pi(9)^2$$

$$A = 81\pi$$

$$A = 254.34$$

Next, we create a proportion using the inscribed angle and the area of the circle.

$$\frac{\text{unknown sector area}}{\text{area of the circle}} = \frac{\text{measure of angle}}{360^\circ}$$

$$\frac{x}{254.34} = \frac{60}{360}$$

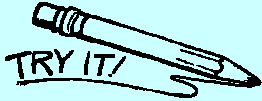
We can reduce the fraction on the right. Then, cross multiply to solve for x.

$$\frac{x}{254.34} = \frac{1}{6}$$

$$\frac{6x}{6} = \frac{254.34}{6}$$

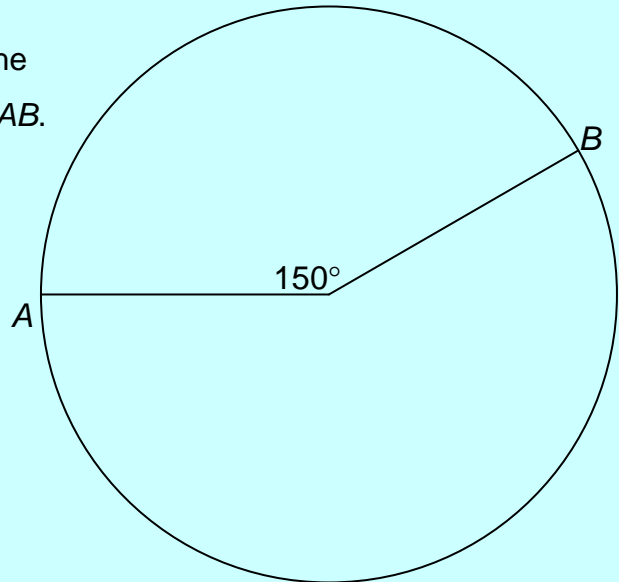
$$x = 42.39$$

Thus, the area of the sector is 42.39 in^2 . So, choice **A** is the answer.



- 3) Use your ruler to find the length of the radius. Then, find the length of arc AB .

- A 3.925 in.
- B 9.42 in.
- C .785 in.
- D 1.9625 in.

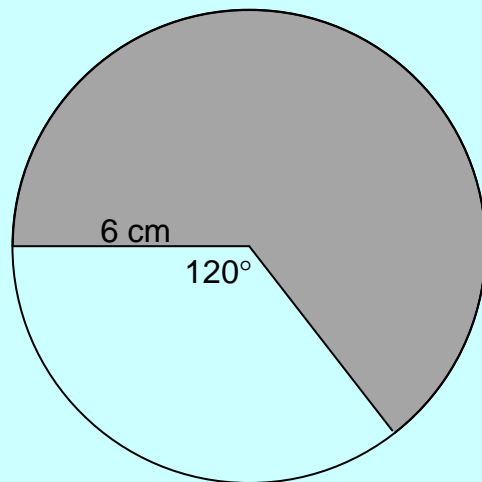


- 4) Diego orders a circular pizza that has a diameter of 12 inches. If the pizza is cut into 8 slices, what is the area of each slice?

- A 113.04 square inches
- B 18.84 square inches
- C 56.52 square inches
- D 14.14 square inches

- 5) Find the area of the shaded sector of the circle.

- A 75.40 cm^2
- B 11.304 cm^2
- C 37.86 cm^2
- D 6.28 cm^2



Review

Know these concepts:

1. Similar figures have corresponding sides in proportion to each other.
2. To find the missing side in a set of similar figures, set up a proportion with corresponding sides in the same fraction.
3. To find the length of an arc of a circle, set up the following proportion:

$$\frac{\text{unknown arc length}}{\text{circumference of the circle}} = \frac{\text{measure of angle}}{360^\circ}$$

4. To find the area of a sector of a circle, set up the following proportion:

$$\frac{\text{unknown sector area}}{\text{area of the circle}} = \frac{\text{measure of angle}}{360^\circ}$$



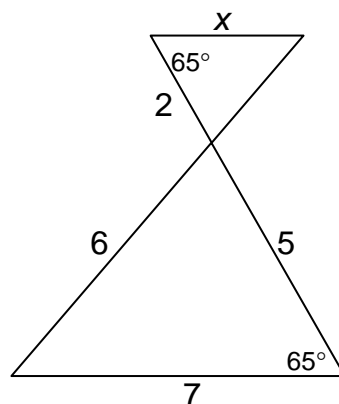
Practice Problems

Lesson 23

Directions: Write your answers in your math journal. Label this exercise
TAKS Review – Lesson 23.

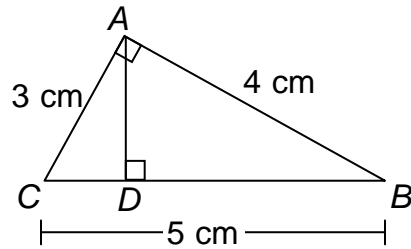
- 1) Find the value of x .

- A** $x = 2.8$
B $x = 2.\bar{3}$
C $x = 2.5$
D $x = 3$



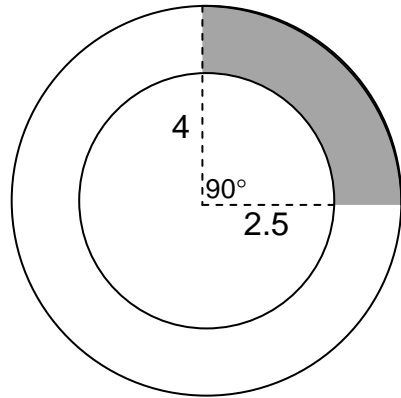
- 2) The length of segment AC is 3 cm, segment AB is 4 cm, and segment BC is 5 cm. Find the length of segment AD .

- A 2 cm
 B 2.4 cm
 C 2.8 cm
 D 3 cm



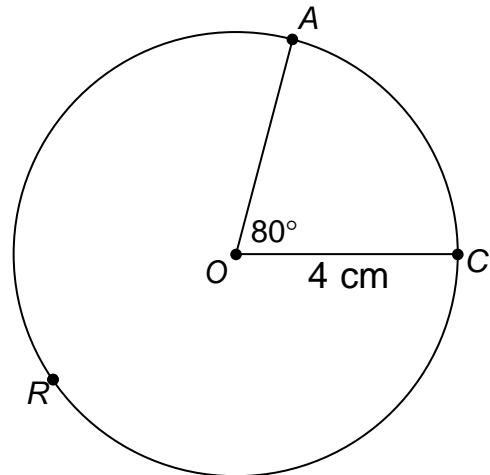
- 3) The radius of the smaller circle is 2.5 units, and the radius of the larger circle is 4 units. Find the area of the shaded region.

- A 50.24 square units
 B 19.624 square units
 C 30.615 square units
 D 7.66 square units



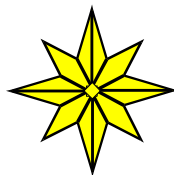
- 4) Find the length of major arc ARC in circle O .

- A 25.12 cm
 B 5.58 cm
 C 2.29 cm
 D 19.55 cm





- 1) B
- 2) C
- 3) A
- 4) D
- 5) A



End of Lesson 23