Student Name:	Contact Person Name:
Date: ______	Phone Number: ______

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 24

Changing Figures and Using a Ruler

TAKS Objective 8 – Understand the concepts and uses of measurement and similarity

Lesson Objectives:

- Describe the effects of changes in the parameters of two- and three-dimensional figures
- Solve volume and surface area problems using a ruler

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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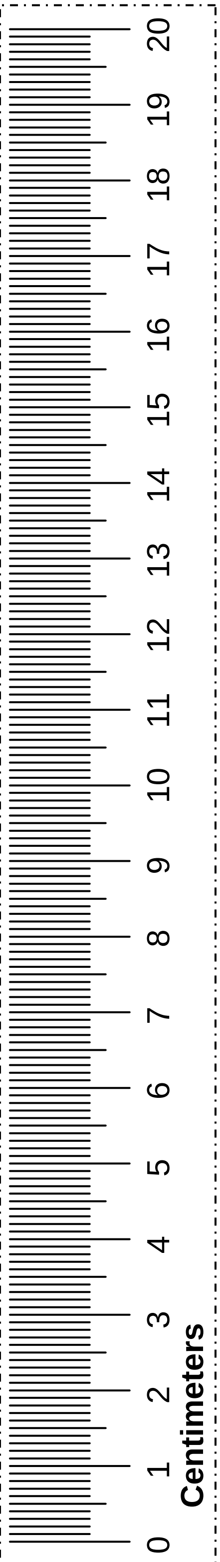
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

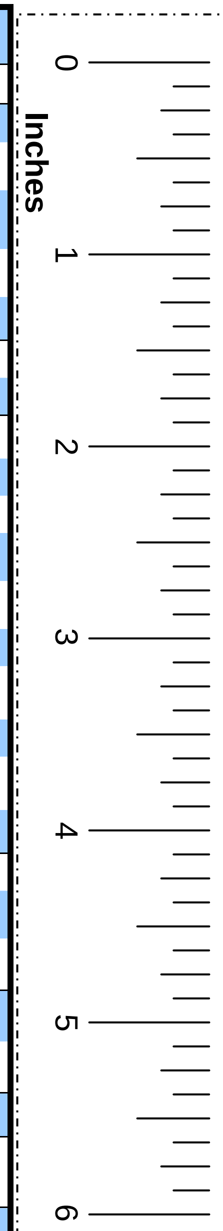
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
Triangles	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Often in mathematics, the result of a problem is surprising, and goes against your first instinct. Consider the following problem.

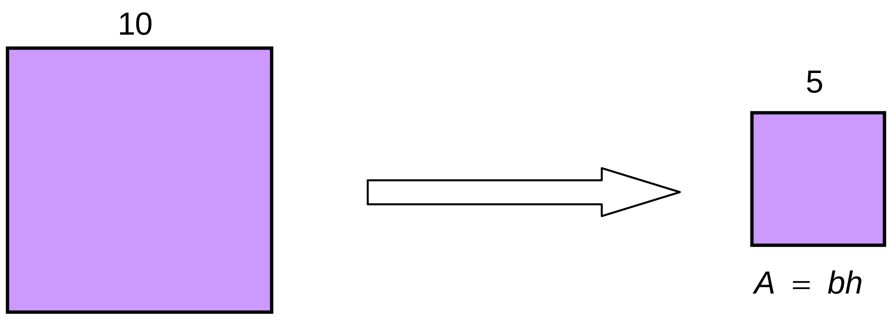
Example

The perimeter of a square is divided by two. What is the effect on that square's area?

Solution

Most people's initial reaction is to say that since the perimeter is cut in half, the area will also be halved. Let's see if this is actually the case.

Halving the perimeter of a square whose sides are each 10 units long will create a square whose sides are 5 units long.



$$A = bh$$
$$A = (10)(10)$$
$$A = 100 \textbf{ units}^2$$

$$A = bh$$
$$= (5)(5)$$
$$= 25 \textbf{ units}^2$$

$$\frac{25}{100} = \frac{1}{4}$$

Comparing their areas, we see that the new area is one-fourth as great.

FACT

To compare the size of two quantities, divide the new quantity by the original:

$$\frac{\textbf{new}}{\textbf{original}}$$

Example

Mr. Clinton's company manufactures a cylindrical can with a diameter of 8 inches and a volume of 245 cubic inches. If the diameter stays the same and the height is doubled, what will happen to the can's volume?

A It will remain the same.

B It will double.

C It will triple.

D It will quadruple.

Solution

Method 1: Work with the formula

The question asks what the effect will be on the volume of the cylindrical can. Refer to the volume section of the formula sheet.

Volume	prism or cylinder	$V = Bh$
	pyramid or cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$

The volume of a cylinder is $V = Bh$. We want to know what will happen when the height is doubled. That means we want to find what the volume will be when h is changed to $2h$.

$$V = Bh \rightarrow V = B(2h)$$

$$= B \bullet 2 \bullet h$$

$$= 2 \bullet B \bullet h = 2Bh$$

The new volume, $V = 2Bh$, is twice that of the original volume, $V = Bh$. Therefore, the answer is choice **B**.

Method 2: Create two concrete examples and compare the results.

The given diameter and volume is not needed to solve this problem. However, if numbers are easier for you to work with, find the volume of two cylinders where one has twice the height of the other. We will use the "friendliest" numbers possible.

Problem Solving Tip

Create a simpler problem that models a more difficult one.

Cylinder I	**Cylinder II**
Let $B = 1$ and $h = 1$	Let $B = 1$ and $h = 2$
$V = Bh$	$V = Bh$
$V = (1)(1)$	$V = (1)(2)$
$V = 1$	$V = 2$

Once again, we observe that doubling the height doubles the volume. Choice **B** is correct.

Method 3: Use the given information.

Maybe you did not realize that you did not need the given to solve this problem. Here is how to use the information given.

Step 1: Solve for h using the given information.

Recall that a cylinder has a circular base. Use the area formula of a circle in place of B.

$$V = Bh$$

$$V = \pi r^2 h$$

$$245 = \pi(4)^2 h$$

$$245 = 16\pi h$$

$$\frac{245}{16\pi} = \frac{\cancel{16\pi} h}{\cancel{16\pi}}$$

$$4.87 \approx h$$

Step 2: Multiply the value found for h by 2.

$$2\left(4.87\right) = 9.74$$

Step 3: Calculate the volume of a cylinder of height 9.74

$$V = Bh$$

$$V = \pi r^2 h$$

$$V = \pi(4)^2(9.74)$$

$$V = \pi(16)(9.74)$$

$$V \approx 489.59$$

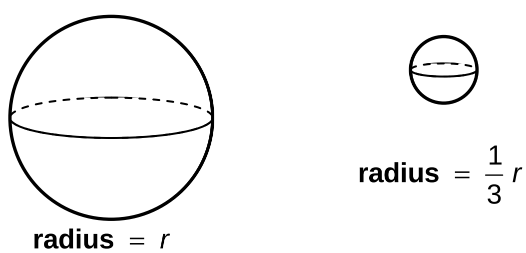
Step 4: Compare this new volume with the original volume given.

$$\frac{489.59}{245} \approx 2$$

Our calculations do not exactly produce 2, due to rounding. However, from the answer choices, we see that the best choice is **B**.

Example

The radius of the larger sphere shown below was multiplied by a factor of $\frac{1}{3}$ to produce the smaller sphere.



How does the surface area of the smaller sphere compare to the surface area of the larger sphere?

A The surface area of the smaller sphere is $\frac{1}{3}$ as large.

B The surface area of the smaller sphere is $\frac{1}{\pi}$ as large.

C The surface area of the smaller sphere is $\frac{1}{6}$ as large.

D The surface area of the smaller sphere is $\frac{1}{9}$ as large.

Solution

Use the formula for the surface area of a sphere.

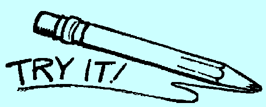
$S = 4\pi r^2$.

Substitute $\frac{1}{3}r$ in place of r.

$$S = 4\pi\left(\frac{1}{3}r\right)^2 = 4\pi\,\frac{1}{9}r^2$$

> ***Think Back***
>
> $(ab)^c = a^c b^c$

Notice that the only difference between the original formula and the new one is a factor of $\frac{1}{9}$. Therefore, the answer is choice **D**.



1) The sides of a cube are doubled. What is the effect on the surface area of the cube?

A The surface area is doubled.

B The surface area is tripled.

C The surface area is quadrupled.

D The surface area is multiplied by a scale factor of 2.

2) The height of a triangular pyramid is changed by a factor of 2.3. What is the effect on the volume of the pyramid?

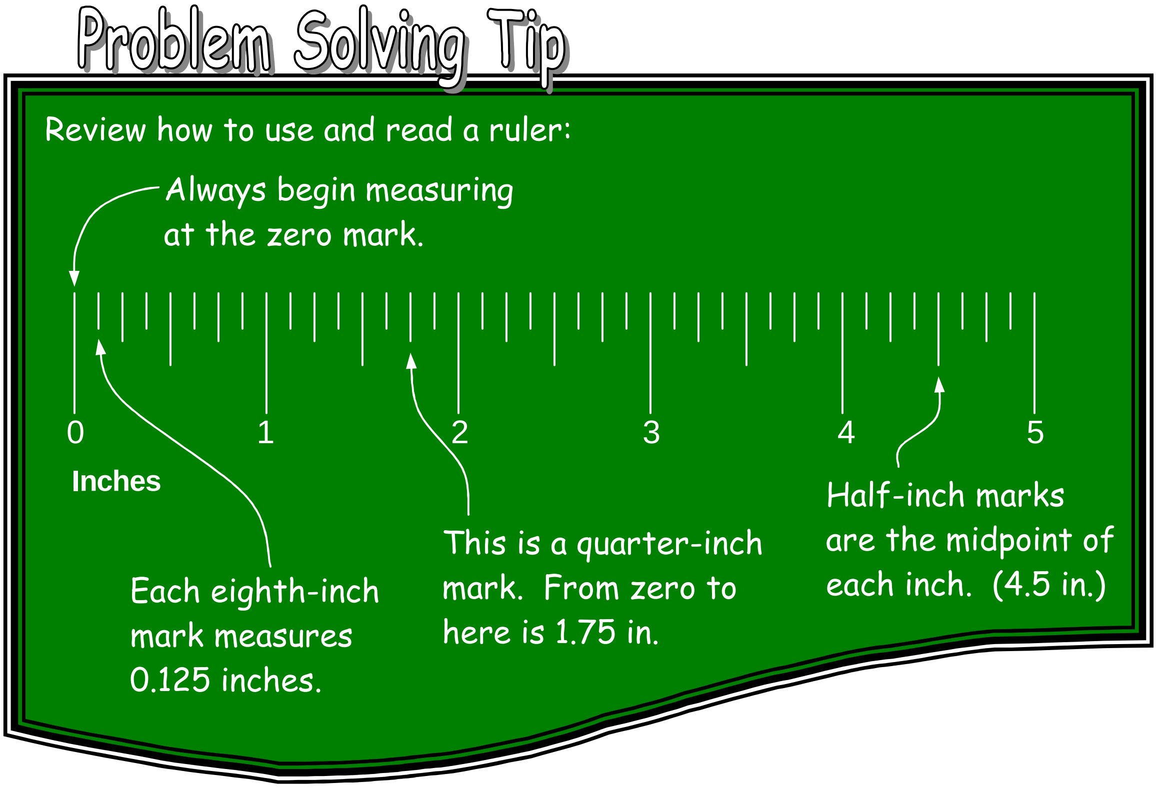
A The volume also changes by a factor of 2.3

B The volume changes by a factor of $\frac{1}{2.3}$

C The volume changes by a factor of $(2.3)^2$

D The volume changes by a factor of 2.3π

Some test questions require you to use a ruler to find volume or surface area. Use the ruler on the formula sheet if you do not have one of your own (or if you are using a copy of this course from a computer printer or copier.)



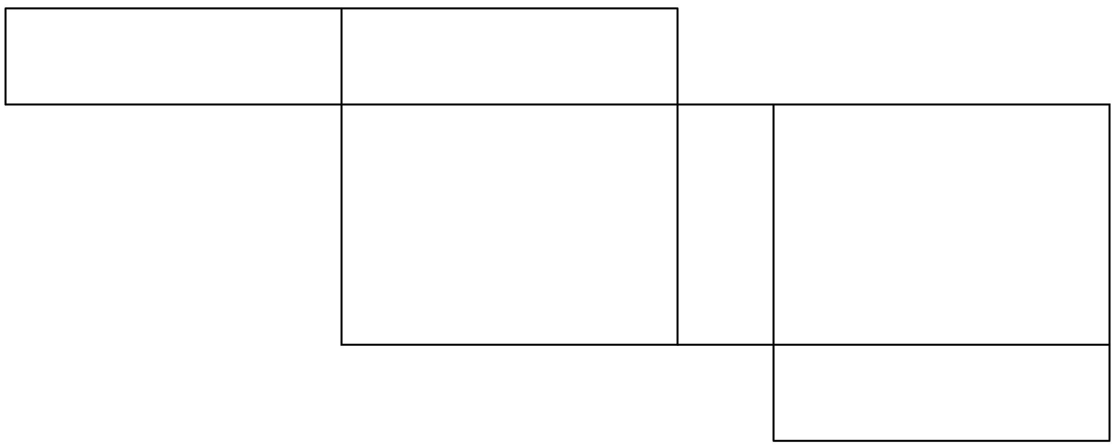
Think Back

<u>Volume</u> can be thought of as the amount of water it takes to fill a 3-D figure

<u>Surface Area</u> is a measure of how much paper it would take to perfectly wrap a 3-D figure.

Example

Ramona made a rectangular prism to hold her jewelry. The net of the rectangular prism is shown below. Use the ruler to measure the dimensions of the rectangular prism to the nearest $\frac{1}{4}$ inch.



Which is closest to the volume of this rectangular prism?

A 2.19 $\textbf{in}^3$ **B** 1.09 $\textbf{in}^3$

C 7.00 $\textbf{in}^3$ **D** 0.88 $\textbf{in}^3$

Solution

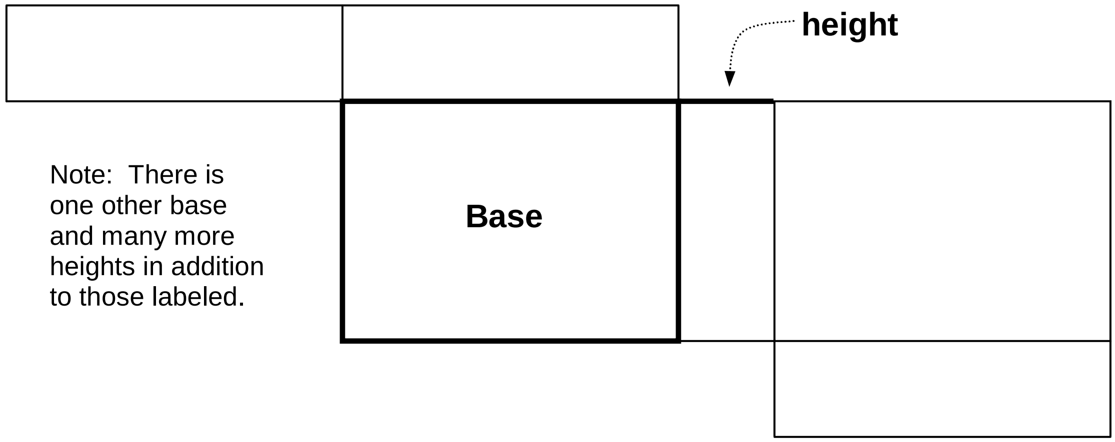
First, identify what the question asks you to find: the volume of the rectangular prism.

Next, identify what information is needed to calculate the volume.

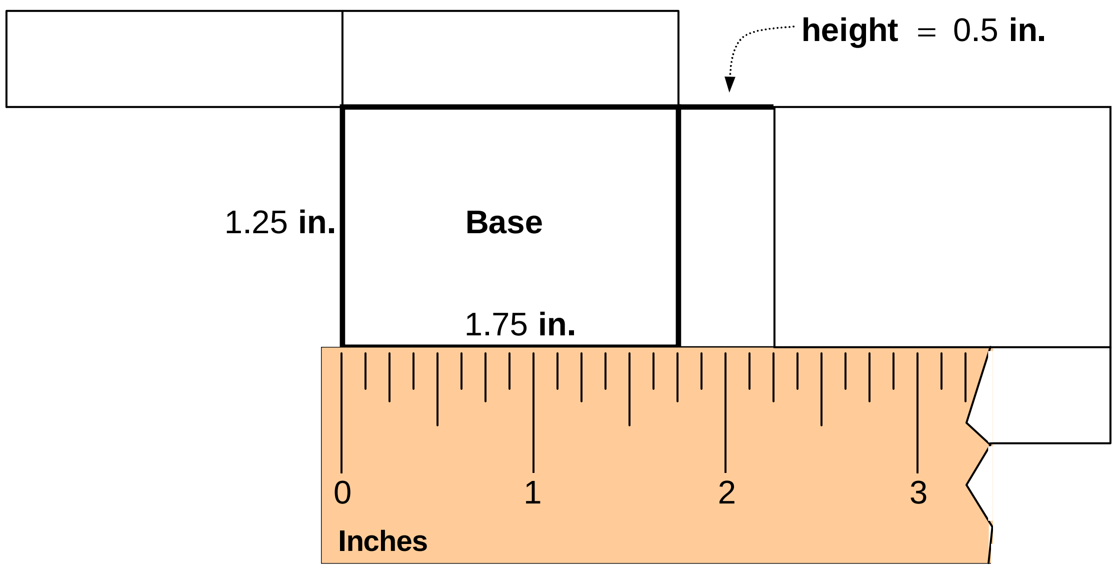
$$V = Bh$$

$$V = (\textbf{area of the base})(\textbf{height})$$

Third, using the net provided, identify the base and the height.



Finally, measure the length and width of the base, as well as the height of the prism. Use the formula to calculate the volume.

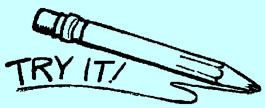


$$V = Bh$$

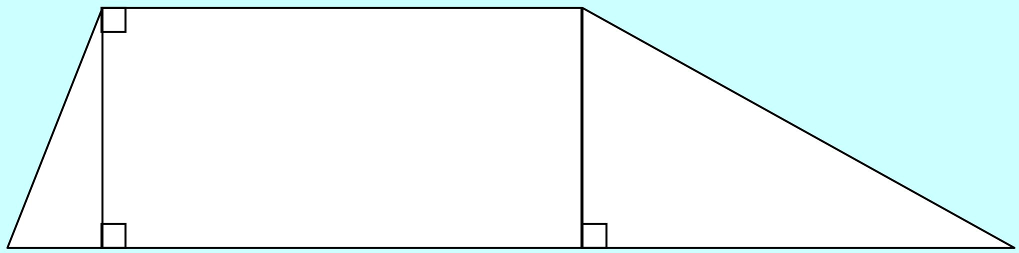
$$V = (1.25)(1.75)(0.5)$$

$$V \approx 1.09 \text{ in.}^3$$

The answer is choice **B**



3) Use the ruler to measure the dimensions of the composite figure to the nearest eighth of an inch.



Which represents the approximate area of this composite figure?

A 5.2 **in.**^{2}

B 3.9 **in.**^{2}

C 4.8 **in.**^{2}

D 16.4 **in.**^{2}

FACT

To change a measurement into a decimal, write it as a fraction first, and then divide the top by the bottom using your calculator. For instance, $\frac{3}{8}$ **in.** is

$3 \div 8 = 0.375$ **in.**

Problem Solving Tip

Only use the ruler when a question specifically tells you to use it.

Review

Know these concepts:

1. Changing one dimension of a two- or three-dimensional figure affects it in ways that are not usually predictable.

- **a.** Use the formula sheet to answer these questions, either by:
  - **i.** Changing the variable in question
  - **ii.** Creating two numeric examples that fit the question

2. Use a ruler to find area or volume

- **a.** Only use a ruler if you are told to do so in the question.

Practice Problems

Lesson 24

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 24.

1) The radius of a sphere is reduced to half its length. Which best describes the effect on the sphere's volume?

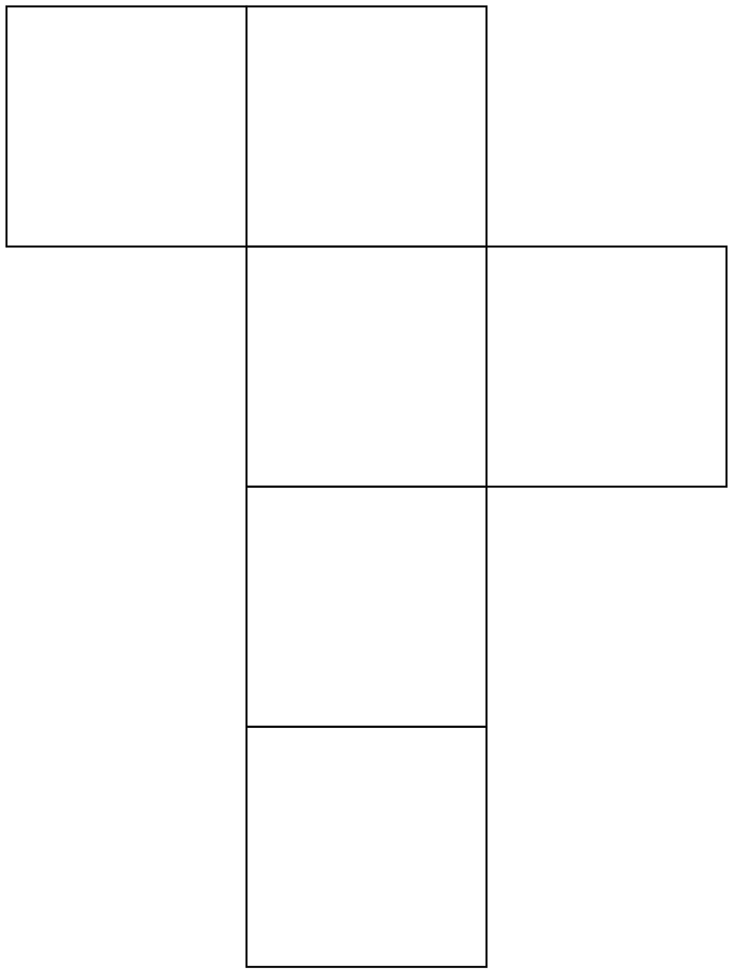
A The volume of the new sphere will be $\frac{4}{3}$ its original volume.

B The volume of the new sphere will be $\frac{1}{6}$ its original volume.

C The volume of the new sphere will be $\frac{1}{8}$ its original volume.

D Not here

2) The net of a cube is shown below. Use a ruler to measure the dimensions of the cube to the nearest $\frac{1}{4}$ inch.



Which is closest to the surface area of the cube?

A 1.25 $in.^2$

B 2.0 $in.^2$

C 8.5 $in.^2$

D 9.4 $in.^2$

1) **C** **2)** **A** **3)** **C**

End of Lesson 24