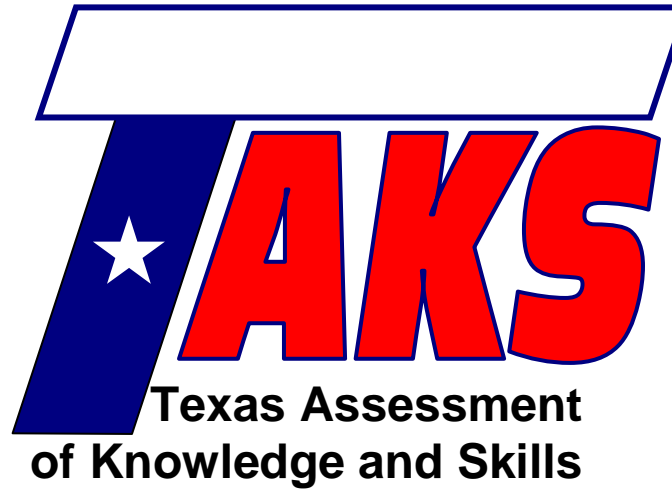


Student Name: _____

Date: _____

Contact Person Name: _____

Phone Number: _____



Exit Level Math Review

Lesson 25

Proportions with Percents and Rates

TAKS Objective 9 – Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems

Lesson Objectives:

- Use proportions for percent problems
- Find the percent of a number by converting percents to decimals
- Use proportions to solve rate problems

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

- 1) Describe functional relationships in a variety of ways.
- 2) Demonstrate an understanding of the properties and attributes of functions.
- 3) Demonstrate an understanding of linear functions.
- 4) Formulate and use linear equations and inequalities.
- 5) Demonstrate an understanding of quadratic equations and other nonlinear functions.
- 6) Demonstrate an understanding of geometric relationships and spatial reasoning.
- 7) Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.
- 8) Demonstrate an understanding of concepts and uses of measurement and similarity.
- 9) Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.
- 10) Demonstrate an understanding of the mathematical processes and tools used in problem solving.

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TAKS Mathematics Chart



Length

Metric

1 kilometer = 1000 meters

1 meter = 100 centimeters

1 centimeter = 10 millimeters

Customary

1 mile = 1760 yards

1 mile = 5280 feet

1 yard = 3 feet

1 foot = 12 inches

Capacity and Volume

Metric

1 liter = 1000 milliliters

Customary

1 gallon = 4 quarts

1 gallon = 128 fluid ounces

1 quart = 2 pints

1 pint = 2 cups

1 cup = 8 fluid ounces

Mass and Weight

Metric

1 kilogram = 1000 grams

1 gram = 1000 milligrams

Customary

1 ton = 2000 pounds

1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

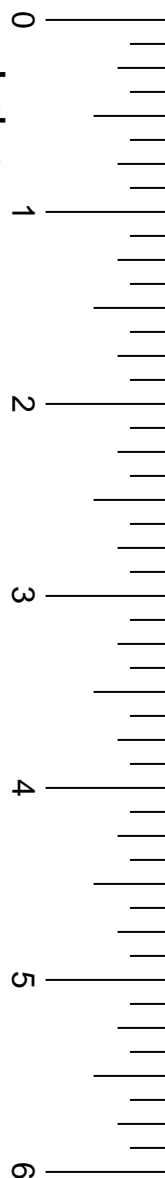
1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x, x\sqrt{3}, 2x$
	45°, 45°, 90°	$x, x, x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Inches



Think Back

Proportions are a set of equal ratios. We often see them written as equal fractions, and we solve a proportion problem using cross multiplication.

In previous lessons, we used proportions for geometry, but now we will apply them to algebra problems. Proportions can be very useful for solving percent problems.

Example

What percent of 40 is 15?

Solution

15 is some percent of 40. 15 is part of the whole, 40. Since a percent is a rate out of 100, we want to know what part this would be out of a whole 100. We find this using a proportion.

$$\frac{15}{40} = \frac{x}{100}$$

Solve the proportion for x.

$$\begin{array}{r} \frac{15}{40} = \frac{x}{100} \\ \swarrow \quad \searrow \\ 1,500 = \frac{40x}{40} \\ 37.5 = x \end{array}$$

Problem Solving Tip

Reducing fractions makes cross multiplication easier.

Thus, 37.5% of 40 is 15.

**Algorithm****To solve a problem involving a percent:**

1. If it is there, underline the word "what." It will be the variable. Write x above it.
2. Translate the word problem into the following proportion:

$$\frac{\text{"is"}}{\text{"of"}} = \frac{\%}{100}$$

3. Solve the proportion.

Percent problems will frequently be presented as a word problem.

Example

Mr. Sanchez is grading papers from his math class. It takes him 30 minutes to grade 25% of his papers. How long will it take him to grade all the papers?

Solution

We have the ratio of the percent given.

$$25\% = \frac{25}{100}$$

We need a ratio representing the time to grade papers. Mr. Sanchez finishes part of the job in 30 minutes. Let x represent the number of minutes needed to finish grading all of the papers. We will set up the following proportion:

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

$$\frac{30}{x} = \frac{25}{100}$$

Now we can cross multiply and solve.

$$\begin{array}{r} \frac{30}{x} = \frac{25}{100} \\ \swarrow \quad \searrow \\ \frac{3000}{25} = \frac{25x}{25} \end{array}$$

$$120 = x$$

Therefore, it would take Mr. Sanchez 120 minutes to finish grading all the papers.

Problem Solving Tip

Percents can also be expressed as a decimal. To write a percent as a decimal, move the decimal two places to the left. For more help with converting percents visit: www.migrant.net/migrant/MOM/index.html

Example

Marco wants to buy two new speakers for his stereo. Audio Giant has the speakers he wants. They are advertised as follows: "Buy 1 speaker at full price, \$50, get the second for 20% off." How much will Marco spend for the speakers?

- A** \$100 **B** \$90
C \$50 **D** \$40

Solution

To find out how much he will spend, we need to determine the price of each speaker. The first speaker is \$50. To find the price of the second speaker, we need to find the discounted price.

$$\begin{aligned} \$50 - 20\% \text{ of } \$50 &= \\ 50 - (.20) 50 &= \\ 50 - 10 &= 40 \end{aligned}$$

The sale price of the second speaker is \$40. Marco will spend the sum of these prices. $\$50 + \$40 = \$90$. So, the answer is choice **B**.

Problem Solving Tip

A quicker method to find the price of the second speaker could be to multiply the original price by $1 - .20$ or $.80$. If we look at the equation for the sale price, $50 - (.20)50$, and factor out 50, we get $50(1 - .20)$ or $50(.80)$ which equals 40

Example

Maggie goes to a restaurant for dinner. Her bill comes to \$40 before tax. If the total bill is \$43.40, what percent was the sales tax on the meal?

- A** 8% **B** 8.5%
C .08% **D** .085%

Solution

There are two ways we can solve this problem. We could either (a) determine the percent increase or (b) solve it algebraically.

(a) The percent increase is the difference in the total price and the initial price.

$$\$43.40 - \$40 = \$3.40$$

If we let x represent the sales tax, we can find the percent using the following proportion:

$$\frac{3.40}{40} = \frac{x}{100}$$

Now we can cross multiply and solve.

$$\begin{array}{r} \frac{3.40}{40} = \frac{x}{100} \\ \frac{340}{40} = \frac{40x}{40} \end{array}$$

$$x = 8.5$$

The sales tax is 8.5%.

(b) Set up an algebraic equation with the initial price plus the sales tax equaling the total price. We will let x represent the sales tax (in decimal form).

$$\begin{array}{ccccc} & & 40 + 40x = 43.40 & & \\ & \nearrow & & \nwarrow & \\ \text{Initial price} & & \text{Percentage added from sales tax} & & \text{Total price} \end{array}$$

Now we can solve for x .

$$\begin{array}{r} 40 + 40x = 43.40 \\ \underline{-40} \qquad \qquad \underline{-40} \\ 40x = 3.40 \\ \underline{40} \qquad \qquad \underline{40} \\ x = .085 \end{array}$$

This answer is in decimal form. If we move the decimal point two places to the right, we find the sales tax is 8.5%. Thus, the answer is choice **B**.

Example

Bob's horse, Moon, weighed 1100 pounds. Over a two-month period, Bob increased the horse's food intake, and her weight increased by 10%. Over the following two months, Bob rode Moon more frequently causing her weight to decrease by 10%.

What did Moon weigh at the end of this four-month period?

- A 1089 pounds
- B 979 pounds
- C 1100 pounds
- D 1210 pounds

Solution

First, we need to determine Moon's weight after the first two months. To do that, we need to find 10% of 1100 and add it to 1100.

$$1100 + (.10) 1100 = 1210$$

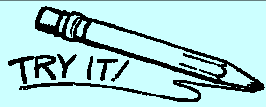
Problem Solving Tip

A quicker method to find the weight of the horse could be to multiply the original weight by 1.10. Increasing by 10% means the horse weighs 110% of its original weight.

Next, we need to find the weight of the horse after the last two months. To do that, we need to find 10% of 1210 and subtract it from 1210.

$$1210 - (.10) 1210 = 1089$$

So, Moon weighs 1089 pounds at the end of the four-month period. Choice **A** is the answer.



- 1) Sefkan goes to the store to buy a leather jacket. The jacket is on sale for 25% off. Its original price is \$160. If the sales tax on the jacket is 8.5%, what is the total price Sefkan will have to pay?
- A \$43.40
 - B \$130.20
 - C \$222
 - D \$120
- 2) Naomi buys a dress that is on sale for \$71.60. If the original price of the dress is \$89.50, what is the percent she saves?
- A 80%
 - B .8%
 - C 20%
 - D .2%
- 3) Mrs. O'Neill has a bin filled with 56 markers. She has 20 students in her class, and she gives each student two markers. What percent of markers does she have remaining?
- A 36%
 - B 71%
 - C 64%
 - D 29%

Proportions can also be very useful for rates.

Example

If y is directly proportional to x , and $y = 10$ when $x = 16$, what is the value of x when $y = 4$?

A $x = 5\frac{2}{5}$

B $x = 6\frac{4}{5}$

C $x = 6\frac{2}{5}$

D $x = 7\frac{4}{5}$

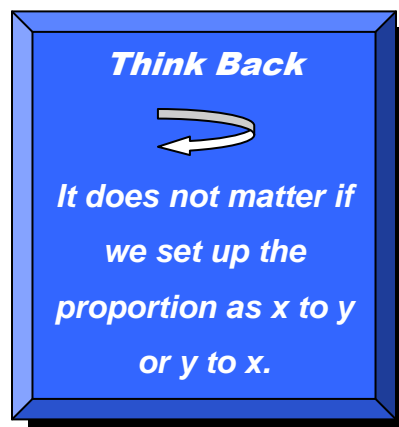
Solution

If x and y are directly proportional, we can set up a proportion with the ratio of x to y .

$$\frac{16}{10} = \frac{x}{4}$$

$$\frac{64}{10} = \frac{10x}{10}$$

$$6.4 = x$$



We get $x = 6.4$, but that does not match any of our answers. We need to turn this into a mixed number. The number 4 is in the tenths place, which means

$6.4 = 6\frac{4}{10}$. This can then be reduced to $6\frac{2}{5}$, so choice **C** is the answer.

Example

Wanda's car gets 20 miles per gallon of gas. If she takes a 360-mile road trip, how many gallons of gas will Wanda's car use?

A 18 gallons

B 180 gallons

C 1.1 gallons

D 11 gallons

Solution

A rate is a ratio using two different units. More specifically, the rate "miles per gallon" is a unit rate, and we can turn it into a ratio by putting it over 1.

$$20 \text{ miles per gallon} = \frac{20 \text{ miles}}{1 \text{ gallon}}$$

TAKS Review

Let x represent the unknown number of gallons needed to complete the trip.

Set up a proportion with miles over gallons and solve.

$$\begin{array}{r} \frac{20}{1} \quad \frac{360}{x} \\ \swarrow \quad \searrow \\ \frac{20x}{20} = \frac{360}{20} \\ x = 18 \end{array}$$

Wanda needs 18 gallons of gas to complete the trip. So, choice **A** is the answer.

Sometimes you may have a rate problem with changing units.

Example

In how many fewer minutes will a car complete a 20-mile drive at an average speed of 60 miles per hour than at an average speed of 50 miles per hour?

- A** 20 minutes
- B** 24 minutes
- C** 12 minutes
- D** 4 minutes

Solution

We need to find the time it takes for both trips and subtract them from each other. First, convert 60 mph to a ratio.

$$60 \text{ miles per hour} = \frac{60 \text{ miles}}{1 \text{ hour}}$$

Next, we will let x represent the unknown number of hours needed to complete the faster trip. Set up the proportion and solve.

$$\begin{array}{r} \frac{60}{1} \quad \frac{20}{x} \\ \swarrow \quad \searrow \\ \frac{60x}{60} = \frac{20}{60} \\ x = \frac{1}{3} \end{array}$$

The faster trip takes one-third of an hour. The answers are all in minutes, so we need to convert this answer into minutes.

$$\frac{1}{3} \times 60 = 20 \text{ minutes}$$

Then, let y represent the number of hours to complete the slower trip. Set up the proportion and solve.

$$\begin{array}{r} \frac{50}{1} \times \frac{20}{y} \\ \frac{50y}{50} = \frac{20}{50} \\ y = \frac{2}{5} \end{array}$$

The slower trip takes two-fifths of an hour. We need to convert this answer into minutes.

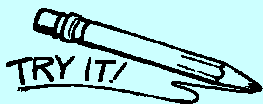
$$\frac{2}{5} \times 60 = 24 \text{ minutes}$$

The difference in times of the two trips is 4 minutes, so the answer is choice **D**

FACT

The formula for average speed is

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$



- 4) Joel and José are planning a road trip from San Diego, California to the Grand Canyon. The trip is approximately 553 miles. If their average speed is 70 mph, how long will it take them to drive to the Grand Canyon?
- A 7.9 hours
 - B 6.5 hours
 - C 7.9 minutes
 - D 6.5 minutes

- 5) Pablo read 40 pages of a book in 50 minutes. How many pages should he be able to read in 2 hours?
- A 2.5 pages
 - B 1.6 pages
 - C 96 pages
 - D 150 pages
- 6) Diego found out that after working for 9 months, he had earned 6 days of vacation time. How many days will he have earned after working for two years?
- A 16 days
 - B 16 years
 - C 3 days
 - D 3 years

Review

Know these concepts:

1. To solve a percent problem using a proportion, use one of the following proportions:

$$\frac{\text{"is"}}{\text{"of"}} = \frac{\%}{100} \qquad \frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

2. To find a percentage of a number, multiply that number by the decimal form of that percent.
 - a. To convert a percent to a decimal move the decimal point two places to the left. $8\% = .08$
 - b. To convert a decimal to a percent move the decimal point two places to the right. $.15 = 15\%$
3. A unit rate can be turned into a ratio by putting the rate over 1.



Practice Problems

Lesson 25

Directions: Write your answers in your math journal. Label this exercise
TAKS Review – Lesson 25.

- 1) Greg is downloading software for his computer. If it takes 45 minutes to download 60% of the software, how long does it take to download all the software?
A 75 hours **B** 75 minutes
C 27 hours **D** 27 minutes

- 2) Gabby goes to a restaurant for dinner. Her bill came \$23.95, and she paid \$29. Approximately what percent did Gabby leave as a tip?
A 15% **B** 18%
C 20% **D** 21%

- 3) Daniela is traveling from San Antonio to Houston – a distance of 200 miles. If her average speed is 75 mph, how long will it take for her to get to Houston?
A 2 hours and 20 minutes **B** 1 hour and 40 minutes
C 3 hours **D** 2 hours and 40 minutes

- 4) A doctor sees each of her patients for 30 minutes during a typical appointment. How many patients can she see in a typical $7\frac{1}{2}$ hour day?
A 14 patients **B** 15 patients
C 16 patients **D** 17 patients

TAKS Review

5) Speedy Wash can clean one car in five minutes. Laser Wash can clean three cars in twenty minutes. How many more cars can Speedy Wash clean than Laser Wash in an 8 hour day?

A 3 cars

B 24 cars

C 6 cars

D 18 cars



1) B

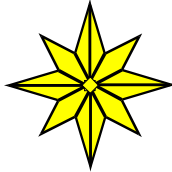
2) C

3) D

4) A

5) C

6) A



End of Lesson 25