Student Name: _____	Contact Person Name: _____
Date: _____	Phone Number: _____

Texas Assessment
of Knowledge and Skills

Exit Level Math Review

Lesson 26
Probability

TAKS Objective 9 – Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems

Lesson Objectives:

- Understand the difference between and be able to calculate theoretical and experimental probability
- Calculate the probability of dependent and independent events
- Use experimental probability and proportions to determine the likelihood of an event occurring

Authors:

Tim Wilson, B.A.
Jason March, B.A., M.S.Ed

Editor:

Linda Shanks

Graphics:

Tim Wilson
Jason March

The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

National PASS Center
Geneseo Migrant Center
3 Mt. Morris – Leicester Road
Leicester, NY 14481
(585) 658-7960
(585) 658-7969 (fax)
www.migrant.net/pass

Developed by the National PASS Center under the leadership of the National PASS Coordinating Committee with funding from the Region 20 Education Service Center, San Antonio, Texas, as part of the Mathematics Achievement = Success (MAS) Migrant Education Program Consortium Incentive project.

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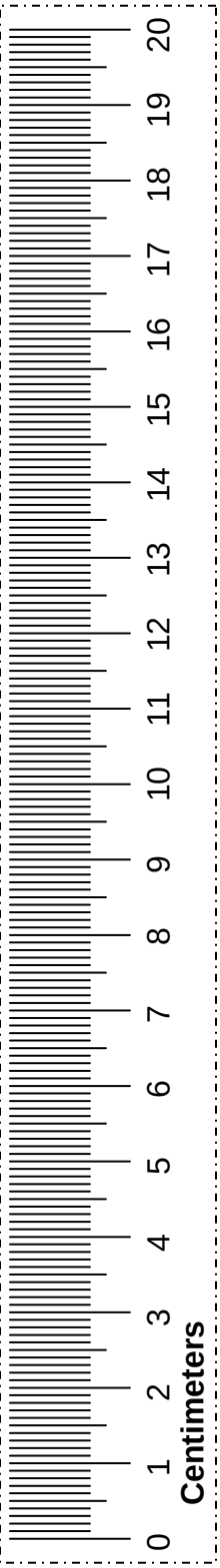
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

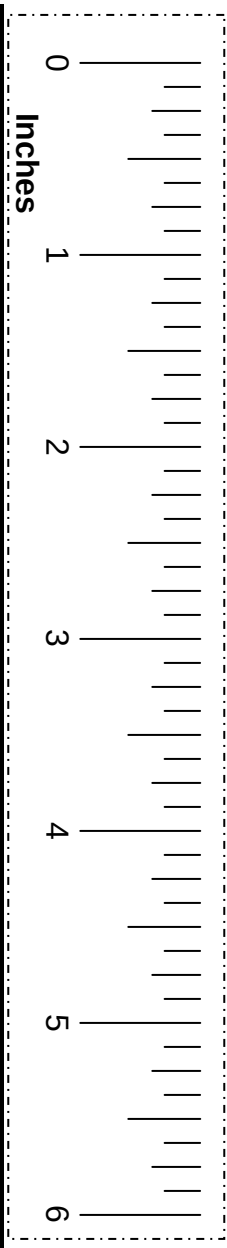
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

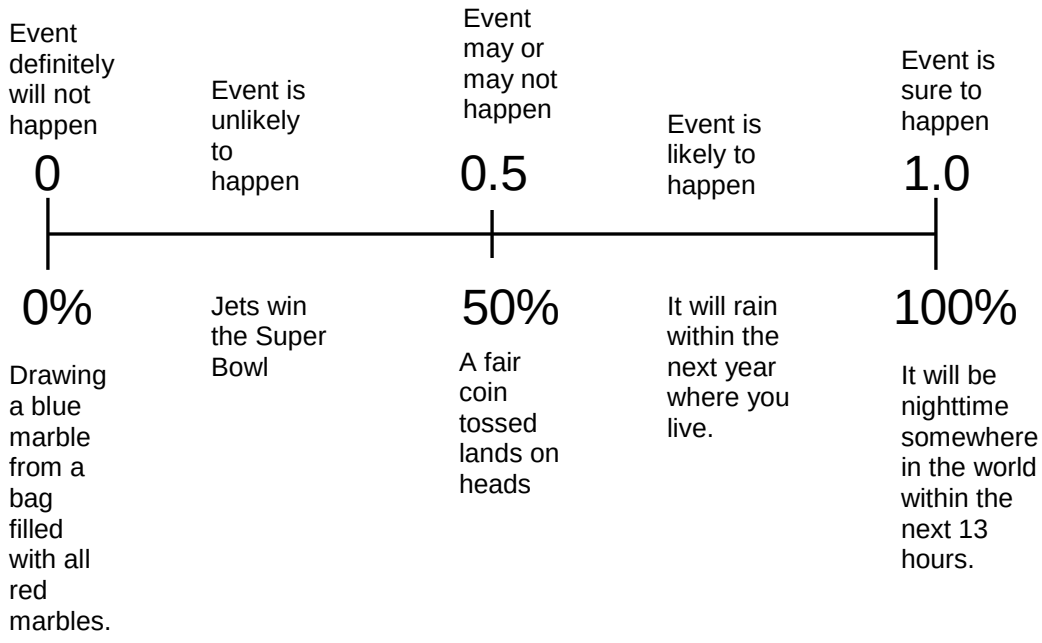
TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Probability is a prediction of the likelihood or the chances of an event occurring. Probability can be expressed as a fraction, decimal, or percent.

The chances of an event occurring are between and including 0 and 1.



The probability of an event occurring, expressed as a fraction is:

$$\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

By calculating probability in this way, we are finding what is called the **theoretical probability** of an event occurring. More on that later, though.

Example

Each letter of the name HOMER SIMPSON is written on a like-sized card and placed in a bag. Cards are drawn at random and are replaced in the basket. What is the probability of:

a) Drawing the letter "M"?

b) Drawing the letter "R"?

c) Drawing a vowel?

Solution

a) $$\frac{\text{number of Ms}}{\text{total number of letters}} = \frac{2}{12} = \frac{1}{6}$$

b) $$\frac{\text{number of Rs}}{\text{total number of letters}} = \frac{1}{12}$$

c) Recall, vowels are the letters A, E, I, O, and U.

$$\frac{\text{number of vowels}}{\text{total number of letters}} = \frac{4}{12} = \frac{1}{3}$$

Write the probability of each event occurring as a fraction in simplest form.

1) Landing on "tails" with a fair coin toss

2) Rolling an odd number with a fair, six-sided die

3) Picking a blue marble from a bag that contains 4 blue marbles, 8 red marbles, and 4 white marbles

You will be expected to find the probability of multiple events occurring. You must distinguish whether events are **independent** or **dependent**.

Independent Events

An event with more than one trial is called a **compound event**.

Compound events are **independent** if the outcome of one trial does not affect the outcome of the other(s).

Some examples of independent events:

- tossing a single coin three times
- spinning a spinner more than once
- drawing a marble from a bag, putting it back in the bag, and then drawing another marble

FACT

There is no such thing as luck in probability. If you toss a coin repeatedly, and it lands on heads for 99 tosses in a row, the probability of tossing heads on the 100th trial is still $\frac{1}{2}$.

To calculate the probability of a compound event, **multiply** the probabilities of the simpler events that make it up.

Example

A die is rolled twice. Find the probability of rolling a 4 on both rolls.

Solution

The probability of rolling a 4 on the first roll is

$$P(4) = \frac{1}{6}.$$

The probability of a die rolling 4 on both rolls is found by:

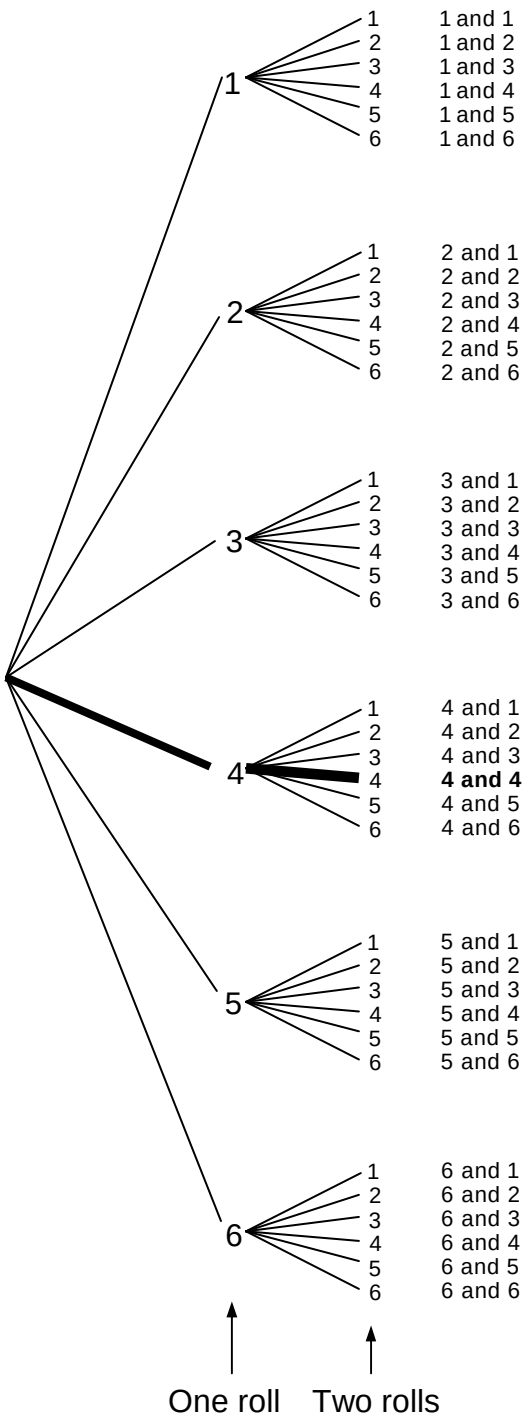
$$P(4 \text{ and } 4) = P(4) \cdot P(4) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

FACT

The probability of event A and event B happening is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

A good way to visualize the previous result is with a tree diagram. Observe below:



This represents all the rolls possible when rolling a single die two times. Highlighted, we see our desired outcome, 4 and 4. Notice that in the first roll, there is a $\frac{1}{6}$ chance of rolling 4, because there are 6 possible outcomes in the first roll. From the 4 in the first roll, there are six paths or possible outcomes for the second roll. This indicates a $\frac{1}{6}$ chance of rolling 4 in the second roll. However, we see there are 36 possible outcomes in total. The probability of getting one of those outcomes, 4 and 4, is $\frac{1}{36}$.

Problem Solving Tip

To review fractions, decimals, percents, and other basic concepts, there are English and Spanish lessons available at no cost. Visit:

www.migrant.net/migrant/MOM/index.html

Example

A bag contains 2 red marbles, 2 green marbles, and 1 yellow marble. You draw three marbles with replacement. What is the probability of drawing 1 green marble, 1 red marble, and then 1 yellow marble?

Solution

The key phrase in the question to focus on is "**with replacement**." It means that you put the marbles back in the bag after drawing them. Therefore, drawing marbles in this case is an independent event. Find the probability of an independent event by following these next steps.

Step 1: Count the total number of marbles.

$$2 + 2 + 1 = 5 \text{ marbles}$$

Step 2: Find the probability of drawing red.

$$P(\text{red}) = \frac{2}{5}$$

Step 3: Find the probability of drawing green.

$$P(\text{green}) = \frac{2}{5}$$

Step 4: Find the probability of drawing yellow.

$$P(\text{yellow}) = \frac{1}{5}$$

Step 5: Multiply the probabilities found in steps 2, 3, and 4.

$$P(\text{red and green and yellow}) = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} = \frac{4}{125}$$

To get a better idea of what these chances are, convert the fraction to a percent.

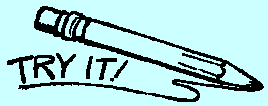
$$\frac{4}{125} = 4 \div 125 = 0.032 = 3.2\%$$

FACT

*The phrase "**with replacement**" indicates an independent event.*

Think Back

$$\frac{a}{x} \cdot \frac{b}{y} = \frac{ab}{xy}$$



Write the probability of each event occurring, rounded to the nearest percent.

4) Spinning 3 then 5 on a spinner with five equal-sized sections labeled one through five

5) Rolling a single die twice, and it landing on an even number, then 6

Dependent Events

A compound event is **dependent**, if the outcome of an earlier trial affects the probability of a future trial.

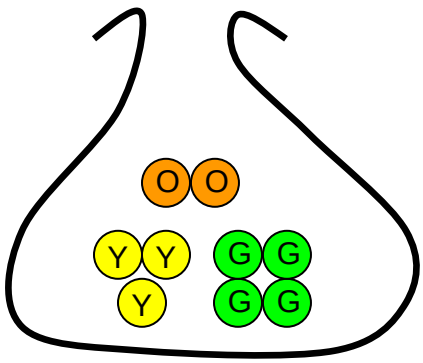
FACT

*A question with the phrase "**without replacement**" describes a dependent event.*

To understand this definition, we will use an example.

Example

You are eating colored candies from a bag, without looking. The bag contains 3 yellow candies, 4 green candies, and 2 orange candies. What is the probability that you will eat an orange candy, then a green candy, then a yellow candy?

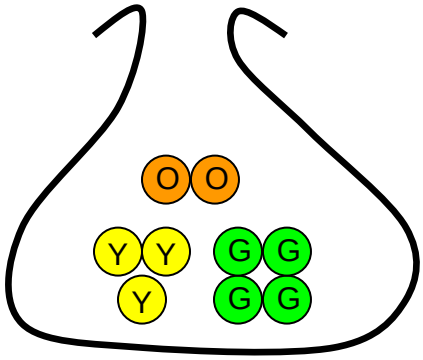


Solution

We will walk through this example step-by-step.

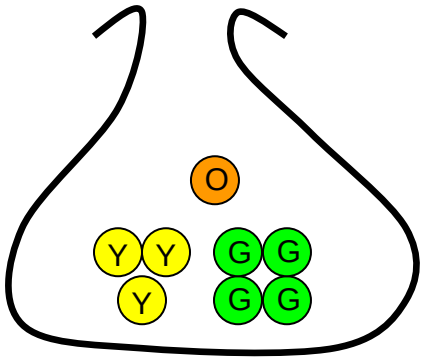
First, find the probability of orange.

$$P(\text{orange}) = \frac{2 \text{ orange}}{9 \text{ total}} = \frac{2}{9}$$



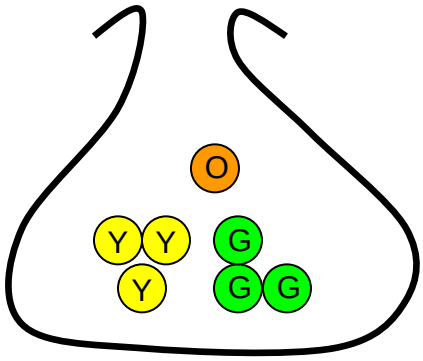
Now, imagine that you select orange and eat it. It is now gone from the bag, leaving 8 total candies. Now that you have removed orange, calculate the probability of green.

$$P(\text{green}) = \frac{4 \text{ green}}{8 \text{ total}} = \frac{4}{8} = \frac{1}{2}$$



Once more, imagine that you have reached in the bag and picked a green candy. This leaves 7 candies total. Next, find the probability of yellow.

$$P(\text{yellow}) = \frac{3 \text{ yellow}}{7 \text{ total}} = \frac{3}{7}$$



Finally, since this is a compound event, we must multiply the probabilities.

$$\frac{2}{9} \cdot \frac{1}{2} \cdot \frac{3}{7} = \frac{6}{126}$$

If you like, convert this to a decimal, then to a percent.

$$\frac{6}{126} = 6 \div 126 = 0.047... \approx 5\%$$

6) Octavio has 11 hardcover books. Three of the books are mysteries, three are science fiction, and the rest are romances. If Octavio were to randomly select two books from this set without replacing them, what is the probability that the first book selected will be science fiction and the second book selected will be a romance?

A $\frac{15}{121}$

B $\frac{3}{22}$

C $\frac{9}{110}$

D $\frac{8}{11}$

Theoretical vs. Experimental Probability

As mentioned at the beginning of the lesson, we have been calculating **theoretical probability**.

Theoretical probability is $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

If we tossed a coin 20 times, theoretical probability predicts it will land on heads half of the tosses, or 10 times. However, this is not a guarantee. As you can imagine, the coin could land on heads any number of times: 9 times, 11 times, zero times, all 20 times, and so on.

Experimental probability is

$$\frac{\text{number of times the desired outcome occurred during an experiment}}{\text{number of trials in the experiment}}$$

This means that if a coin lands on heads 7 times in 20 tosses, its experimental probability is $\frac{7}{20} = 0.35 = 35\%$.

Experimental probabilities are useful in the real world for predicting the outcome of certain events, such as a political race.

Example

The student council at Avon High School is planning a school wide trip. A local roller-skating rink will provide a discounted rate, if at least 250 of the school's 2,340 students sign up for the trip. The student council surveyed a random sample of students and asked which of the following activities the students would prefer as a school trip.

Survey Results

Activity	Students
Skating	26
Bowling	33
Watching a movie	127
Swimming	19

Based on the data in the survey, how many students are likely to choose skating?

A 297 **B** 377

C 217 **D** 1,450

Solution

Step 1: Find the experimental probability of students who want to go skating.

$$\frac{\text{surveyed students who chose skating}}{\text{total students surveyed}} = \frac{26}{205}$$

Step 2: Set up a proportion.

$$\frac{26}{205} = \frac{\text{students in the school who want to skate}}{\text{total number of students in the school}}$$

$$\frac{26}{205} = \frac{x}{2340}$$

Step 3: Solve the proportion for x.

$$\frac{26}{205} \times \frac{x}{2{,}340}$$

$$60{,}840 = 205x$$

$$\frac{60{,}840}{205} = \frac{205x}{205}$$

$$296.7... = x$$

$$x \approx 297 \text{ students}$$

Think Back

To solve a proportion, cross multiply, then divide.

The answer is choice **A**.

Example

The Irondequoit Eagles have won 5 games and lost 7 games this season. Based on these results, which is the best prediction of the number of games the Eagles must play in order to win 9 games?

A 6

B 8

C 13

D 22

Problem Solving Tip

Try to solve this now. If you make a mistake, you can learn from it before you actually take the test.

Solution

From the experimental results, the Eagles have won 5 games out of 12 games total. Set up a proportion.

$$\frac{\text{games won}}{\text{total games}} = \frac{9 \text{ wins}}{\text{total games}}$$

$$\frac{5}{12} = \frac{9}{x}$$

$$5x = 108$$

$$x = 21.6$$

$$x \approx 22$$

The answer is choice **D**.

7) Tim, Kate, and Jason are running for president of their school's student council. A random survey of 60 students was taken to determine whom they planned to vote for in the election. The results are shown in the table below.

Survey Results

Candidate	Number of Students
Tim	12
Kate	28
Jason	20

Based on the data in the table, which of the following is the best prediction of the number of students who will vote for Kate if 2500 students vote?

A 1208 **B** 292

C 1167 **D** 833

Review

Know these concepts:

1. Probability is the likelihood of an event occurring.
    a. Theoretical probability
    b. Experimental probability
2. Independent events
    a. "with replacement"
3. Dependent events
    a. "without replacement"
4. Experimental results can be used to predict the outcome of an event

Practice Problems

Lesson 26

Directions: Write your answers in your math journal. Label this exercise

TAKS Review – Lesson 26.

Write the probability of each of the following events occurring as a fraction in simplest form.

1) Rolling a 6 on a standard six-sided die

2) Drawing a magenta marble from a bag containing 3 magenta marbles and 4 green marbles

3) Flipping a coin three times, and it landing on heads three times

4) Rolling an even number with a standard twelve-sided die

5) Spinning a 7, then an odd number on a spinner with seven equal-sized sections labeled one through seven.

6) Each letter of the word MISSISSIPPI is written on a like-sized card and placed in a bag. Cards are drawn at random, without replacement. What is the probability of drawing S, then P?

7) In a bag of 20 marbles (10 blue, 8 green, and 2 yellow) you draw 3 marbles at random without replacement. Find the probability of drawing two blues then a green marble.

8) You are on a bomb squad. To successfully disarm a bomb and save the lives of millions, you must cut two colored wires in the right order. There are three wires: red, blue, and green. What is the probability that the correct combination of wire cutting is green, then red?

9) A total of 1755 customers at an electronics store were asked to identify which item they planned to purchase in the next month. The table below shows the results of the survey.

Results of Customer Survey

Item	Number of Customers
HD Camcorder	598
Blu-Ray Player	264
1080p HDTV	325
Laptop Computer	312
Netbook Computer	256

According to the information above, which of the following statements is true?

A About $\frac{3}{20}$ of the customers planned to purchase a Blu-Ray player.

B About $\frac{1}{19}$ of the customers planned to purchase a 1080p HDTV.

C About $\frac{2}{3}$ of the customers planned to purchase a netbook computer.

D More than $\frac{1}{2}$ of the customers planned to purchase either a computer or a Blu-Ray player.

10) Paco is playing a videogame. He has managed to destroy 4 alien fighter ships out of a total of 6 alien fighter ships that flew by on the screen. Based on these results, which is the best prediction of the number of alien fighter ships that will fly by on the screen in order for Paco to destroy 6 of them?

A 8 **B** 9

C 12 **D** 15

11) Casey conducted an experiment by rolling a fair cube whose faces were labeled with odd numbers. The table below shows the results of her experiment.

Fair-Cube Tosses

Number Landing Face-Up	Frequency
1	11
3	12
5	8
7	10
9	10
11	9

What is the difference between the experimental results and the theoretical probability of a 3 or 7 landing face-up?

A $\frac{19}{60}$ **B** $\frac{11}{30}$

C $\frac{1}{30}$ **D** $\frac{11}{60}$

1) $\frac{1}{2}$ **2)** $\frac{1}{2}$ **3)** $\frac{4 \text{ blue}}{16 \text{ total}} = \frac{1}{4}$

4) $\frac{1}{5} \bullet \frac{1}{5} = \frac{1}{25} = 0.04 = 4\%$

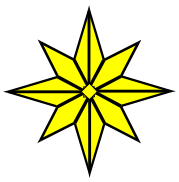
5) $\frac{1}{2} \bullet \frac{1}{6} = \frac{1}{12} = 0.08\overline{3} \approx 8\%$

6) There are 5 romances.

$\frac{3}{11} \bullet \frac{5}{10} = \frac{3}{11} \bullet \frac{1}{2} = \frac{3}{22} \rightarrow$ **B**

7) $\frac{28}{60} = \frac{x}{2500} \rightarrow$ **C**

NOTES



End of Lesson 26