Student Name: _____ Date: _____	Contact Person Name: _____ Phone Number: _____

Texas Assessment of Knowledge and Skills

Exit Level Math Review

Lesson 28
Solving Problems

TAKS Objective 10 – Demonstrate an understanding of mathematical processes and tools used in problem solving

Lesson Objectives:

- Understand the 4-step problem-solving method
- Use the 4-step problem-solving method to solve problems

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The Texas Assessment of Knowledge and Skills (TAKS) exit level exam covers ten learning objectives. These lessons are designed to teach math concepts specific to each objective as well as strategies to consider when approaching typical TAKS questions. To successfully complete the TAKS exit level exam, the student should be able to:

1) **Describe functional relationships in a variety of ways.**
2) **Demonstrate an understanding of the properties and attributes of functions.**
3) **Demonstrate an understanding of linear functions.**
4) **Formulate and use linear equations and inequalities.**
5) **Demonstrate an understanding of quadratic equations and other nonlinear functions.**
6) **Demonstrate an understanding of geometric relationships and spatial reasoning.**
7) **Demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.**
8) **Demonstrate an understanding of concepts and uses of measurement and similarity.**
9) **Demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.**
10) **Demonstrate an understanding of the mathematical processes and tools used in problem solving.**

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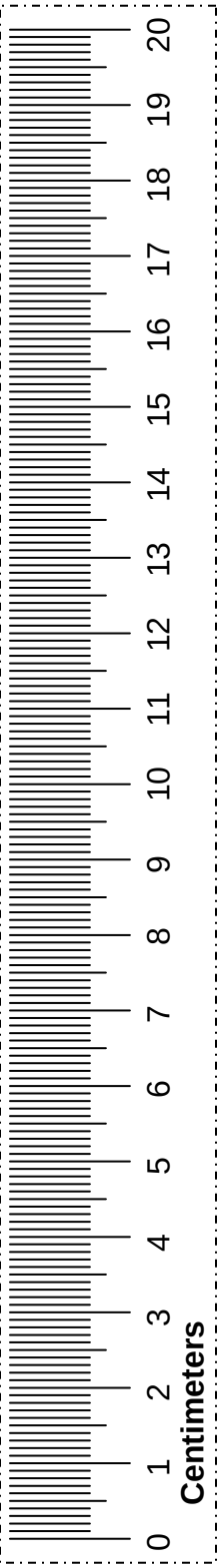
TAKS Mathematics Chart



Length

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

Capacity and Volume

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

Mass and Weight

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

Time

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

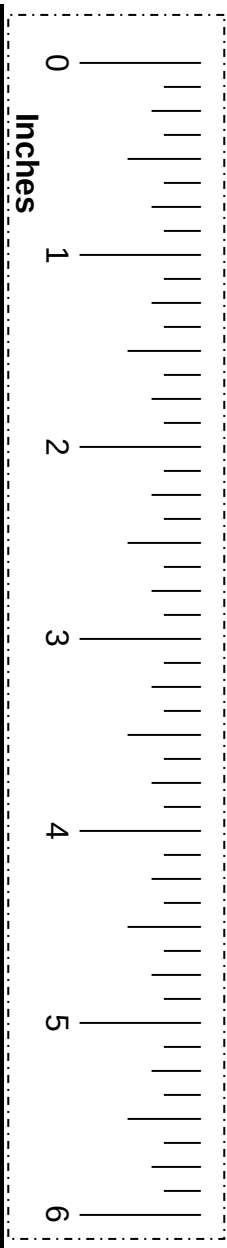
1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

TAKS Mathematics Chart

Perimeter	Rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$
Area	Rectangle	$A = lw$ or $A = bh$
	Triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1+b_2)h}{2}$
	Regular polygon	$A = \frac{1}{2}aP$
	Circle	$A = \pi r^2$
P represents the perimeter of the base of a three-dimensional figure.		
B represents the area of the base of a three-dimensional figure.		
Surface Area	Cube (total)	$S = 6s^2$
	Prism (lateral)	$S = Ph$
	Prism (total)	$S = Ph + 2B$
	Pyramid (lateral)	$S = \frac{1}{2}Pl$
	Pyramid (total)	$S = \frac{1}{2}Pl + B$
	Cylinder (lateral)	$S = 2\pi rh$
	Cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	Cone (lateral)	$S = \pi rl$
	Cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	Sphere	$S = 4\pi r^2$
Volume	Prism or Cylinder	$V = Bh$
	Pyramid or Cone	$V = \frac{1}{3}Bh$
	Sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$



Perhaps you have asked, "Why learn math? When am I ever going to use this?" The answer to this question, on the surface, is: you may never directly use the math techniques required to pass the TAKS exam. You may never again factor and solve a quadratic equation in your lifetime. This is not the whole answer, however. Hopefully, as a result of your math education, you have learned to **problem-solve**. Problem-solving skills are used in all walks of life, in every profession, at all age levels. In this sense, you can use the skills taught in math to solve any problem. The general, four-step problem-solving method is below.

Algorithm

To solve a problem:

1. Understand the problem.
2. Make a plan.
3. Carry out the plan.
4. Look back.

Do not be put off by how obvious this method seems. The following example will demonstrate its effectiveness.

Example

The spreadsheet below contains 20 cells. A cell in a spreadsheet can be identified first by the column letter and then by the row number. For example, the number 8 is found by cell C4.

	A	B	C	D	E
1	5	-6	5	1	6
2	10	-7	6	2	
3	15	-8	7	3	-36
4	20	-9	8	4	

If the number in cell A3 = B2 – 2(E2 + D3), which of the following must be the number in cell E2?

A –16 **B** –12

C –4 **D** –14

Solution

We will use the 4-step method.

Step 1: **Understand the problem**

- What does the problem ask me to find?
  - *The number in cell E2*
- What information is given to find this?
  - *A spreadsheet*
  - *A formula:* $A3 = B2 - 2(E2 + D3)$
- Can I restate the problem in my own words to better understand it?
  - "Use the formula $A3 = B2 - 2(E2 + D3)$ and the spreadsheet to find the number in cell E2."
  - Write your own wording of the problem below.

    _____

    _____

    _____

The real challenge of this problem is understanding what it is asking. Notice that none of the lessons in this review have taught the skill of reading a spreadsheet. You need to figure it out on your own. Once you do that, you are able to use the equation.

Step 2: **Make a plan**

- Substitute values from the spreadsheet into the equation given
- Solve the equation for E2.
  - Use a variable in place of E2.

Step 3: **Carry out the plan**

	A	B	C	D	E
1	5	-6	5	1	6
2	10	-7	6	2	
3	15	-8	7	3	-36
4	20	-9	8	4	

Problem Solving Tip

Do not go on until you understand how to fill in the equation from the spreadsheet.

$A3 = B2 - 2(E2 + D3)$

Read the spreadsheet and substitute values given for A3, B2, and D3. Use x as E2, since its value is unknown.

$15 = -7 - 2(E2 + 3)$

$15 = -7 - 2(x + 3)$

Solve the equation for x. Begin doing this by distributing -2.

$15 = -7 - 2x - 6$

Combine like-terms on the right-hand-side. $-7 + -6 = -13$.

$15 = -13 - 2x$

$+13 \quad +13$

Begin solving for the variable by adding 13 to both sides of the equal sign.

$28 = -2x$

$\frac{28}{-2} = \frac{-2x}{-2}$

Since the term $-2x = -2 \cdot x$, divide both sides by -2 to find x.

$-14 = x$

$-14 = E2$

Recall x stands for E2, which is what you wish to find.

The answer is choice **D**.

Step 4: **Look back**

- Have you answered the question?
  - *Yes, we have found the value in cell E2.*

Here is an expanded algorithm from the original one shown.

Algorithm

To solve a problem:

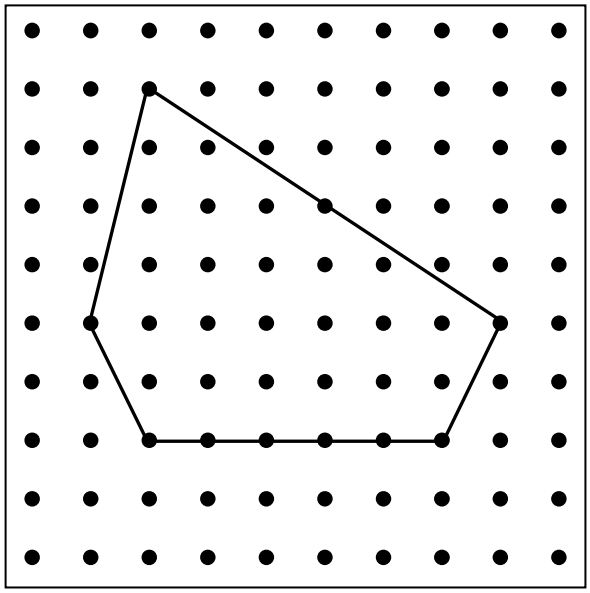
1. **Understand the problem.**
   - Do I understand all the words?
   - Can I restate the problem in my own words?
   - What is given?
   - What is the goal of the problem?
   - Is there enough information?
   - Is there unneeded information?
   - Is this problem similar to other problems I have solved?
2. **Make a plan.**

▪ Guess and test	▪ Draw a diagram	▪ Make a list
▪ Draw a picture	▪ Use number sense	▪ Solve a simpler problem
▪ Use a variable	▪ Consider every case	
▪ Look for a pattern	▪ Solve an equation	▪ Look for a formula

3. **Carry out the plan.**
   - Use your selected plan until the problem is solved, or until it is clear a new plan is needed.
   - Give yourself enough time (at least two minutes) to solve a problem on the test.
   - **Do not be afraid to start over.**
   - **Do not be afraid to start a problem, put it aside, and come back to it later.**
4. **Look back.**
   - Is your answer reasonable?
   - Have you answered all parts of the question?

Example

If the horizontal or vertical distance between adjacent pegs on the geoboard below is 1 unit, which is closest to the area of the polygon modeled on the geoboard?



A 34 units^2

B 23 units^2

C 26 units^2

D 18 units^2

Solution

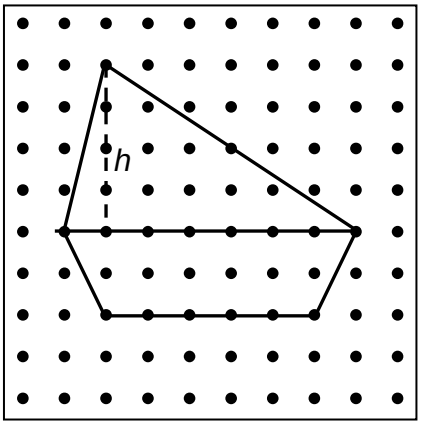
Step 1: **Understand the problem**

- What does the problem ask me to find?
  - *the area of the polygon*
- Do I understand all the words in the problem?
  - Using the words before and after those you do not understand, take your best guess at the meaning of words you do not know.
    - adjacent: next to
    - geoboard: a grid of equally spaced dots

Step 2: **Make a plan**

- Draw lines to break the polygon into familiar shapes and add their areas.

Step 3: **Carry out the plan**



Break the polygon into a triangle and a trapezoid.

1) Find the area of the triangle.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(7)(4)$$

$$A = \frac{1}{2}(28)$$

$$A = 14$$

2) Find the area of the trapezoid.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(7 + 5)(2)$$

$$A = \frac{1}{2}(12)(2)$$

$$A = 12$$

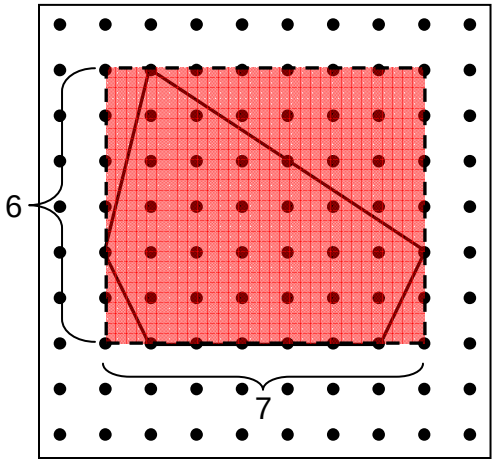
3) Add the area of the triangle and the trapezoid.

$$14 + 12 = 26 \text{ units}^2$$

The answer is choice **C**.

Step 4: **Look back**

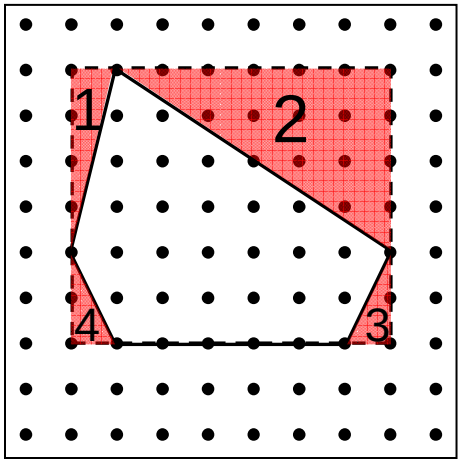
- Can you see an easier solution?
    - Suppose you didn't remember how to find the area of a trapezoid. Think of another way to solve the question.
        - *Draw a rectangle around the polygon and subtract the extra areas.*
            - *We will demonstrate this method on the next page.*



1) Find the area of the large rectangle.

$$A = bh$$

$$A = 7 \cdot 6 = 42$$



2) Find the areas of each triangle outside the polygon (shaded), and add them.

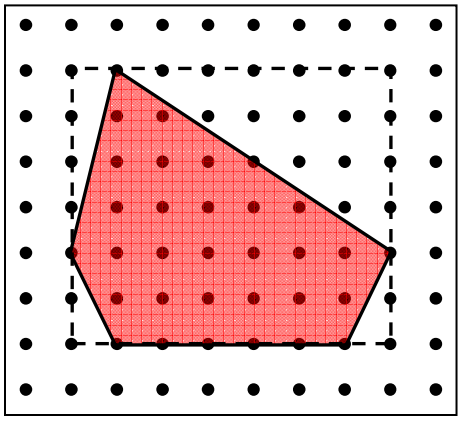
$$A_1 = \frac{1}{2} \cdot 4 \cdot 1 = 2$$

$$A_2 = \frac{1}{2} \cdot 6 \cdot 4 = 12$$

$$A_3 = \frac{1}{2} \cdot 2 \cdot 1 = 1$$

$$A_4 = \frac{1}{2} \cdot 2 \cdot 1 = 1$$

$$2 + 12 + 1 + 1 = 16$$



3) Subtract the area of triangles from the area of the rectangle.

$$42 - 16 = 26 \text{ units}^2$$

Once more, the answer is choice **C**.

Use the 4-step problem-solving method to solve each of the following problems.

1) ΔABC has a right angle at point A. Point D is between points B and C. $\overline{DA}$ is perpendicular to $\overline{BC}$. According to this information, which of the following is true?

A ΔCAD is similar to ΔABC

B ΔACD is similar to ΔABC

C ΔBDA is similar to ΔABC

D ΔDAC is similar to ΔABC

2) Casey has a circular piece of cardboard with a 12-inch diameter. She wants to cut a 12-inch-by-2-inch rectangle from the circle. She also wants to cut 12 square pieces that are 1 inch on each side. Which information makes this impossible to do?

A There will be no cardboard left once the squares are cut.

B A 12-inch long rectangle cannot be cut from the circular cardboard.

C Squares cannot be cut from the circle.

D There will not be enough cardboard to cut all the 1-inch square pieces indicated.

Use the 4-step problem-solving method to solve each of the following problems.

3) Given the function $y = 7.62x - 44.32$, which statement best describes the effect of increasing the *y*-intercept by 33.14?

A The new line is parallel to the original.

B The new line has a greater rate of change.

C The *x*-intercept increases.

D The *y*-intercept decreases.

4) A 25-foot ladder is leaning against a house so that its top touches the top of the wall. The bottom of the ladder is 10 feet away from the wall. Which of these can be used to find the height of the wall?

A In a right triangle with a 25-foot leg and a 10-foot leg, find the length of the hypotenuse.

B In a right triangle with a 25-foot leg and a 10-foot leg, find the altitude to the hypotenuse.

C In a right triangle with a 25-foot hypotenuse and a 10-foot leg, find the length of the other leg.

D In a right triangle with a 25-foot hypotenuse and a 10-foot leg, find the altitude to the hypotenuse.

Review

Know these concepts:

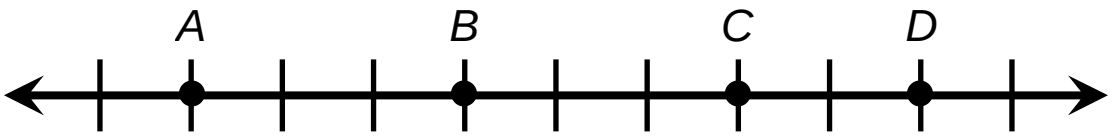
1. The 4-step problem-solving method

- **a.** Understand the problem
- **b.** Make a plan
- **c.** Carry out the plan
- **d.** Look back

Practice Problems
Lesson 28

Directions: Write your answers in your math journal. Label this exercise TAKS Review – Lesson 28.

1) Which of the following relationships could not be used to determine the length of $\overline{AD}$, as shown below?



A $AB + BC + CD = AD$

B $AC + CD = AD$

C $AC + BD = AD$

D $AB + BD = AD$

2) If $a + b = c$ and $x = y$, which of the following must be true?

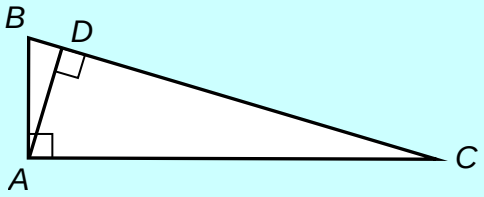
A $a + b - x = y - c$

B $a + b - c = x + y$

C $a + b + c = x - y$

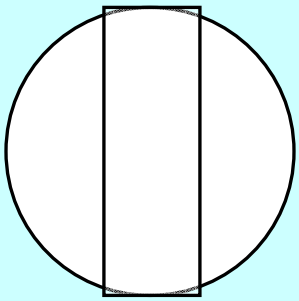
D $a + b + x = c + y$

1) A good strategy to use is to draw a picture.

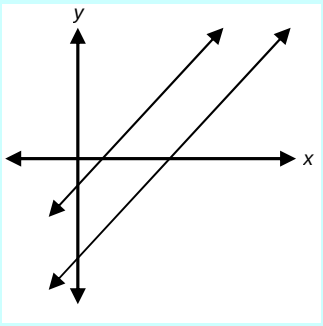


The answer is choice C.

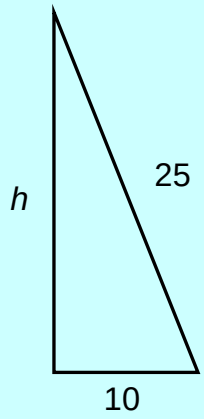
2) Draw a picture. The answer is choice **B**.



3) Sketch the two graphs. The answer is choice **A**



4) Draw a picture and use a variable. The answer is choice **C**.



End of Lesson 28