Student Name: ______________________________

Date: ______________________________

Contact Person Name: ______________________________

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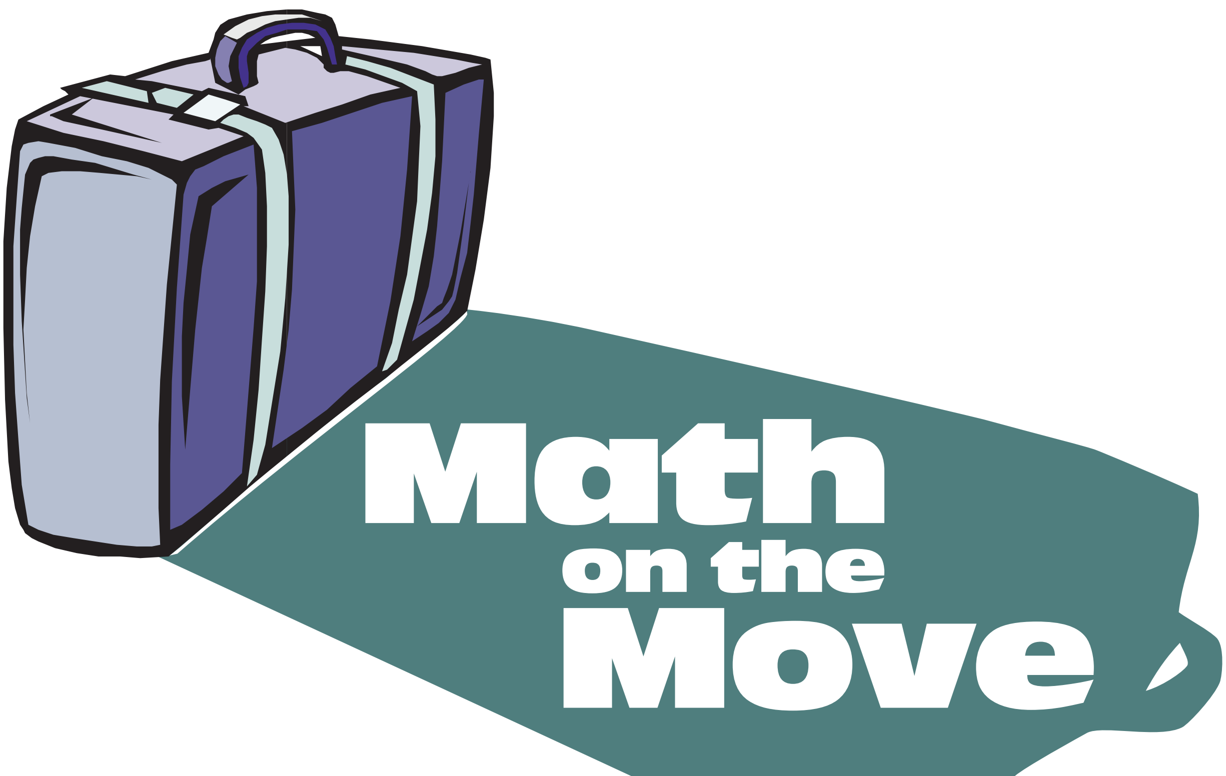

Lesson 6

Operations with Fractions

Objectives

- Add and subtract fractions with like and unlike denominators
- Multiply and divide fractions

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On your next shift at the pizza shop, a couple enters and orders a pizza cut into eight equal slices. You notice that the man eats three slices, and the woman only eats two. You think to yourself, "How much of that one pizza did they eat? How much pizza is left?"

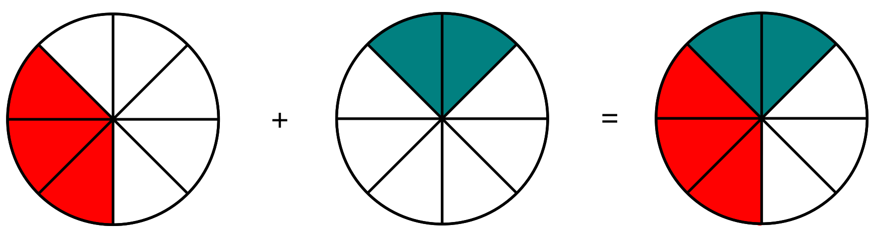
Let's think about this problem using fractions.

The man ate three out of eight slices, or $\frac{3}{8}$. The woman ate two out of eight slices, or $\frac{2}{8}$.

If we add up how much each of them ate, we see that the total pizza eaten is

$$\frac{3}{8}+\frac{2}{8}.$$

We can show this visually as:



$$\frac{3}{8} \quad + \quad \frac{2}{8} \quad = \quad \frac{5}{8}$$

So, $\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$. The couple ate $\frac{5}{8}$ of their pizza!

Even though we added the two fractions, the denominators stayed the same.

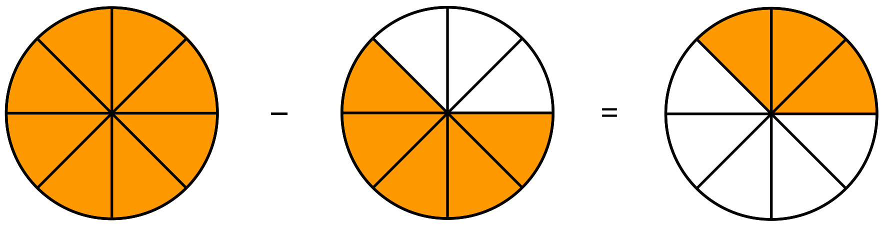
> ***Think Back***
>
> *The denominator tells you <u>what</u> you are talking about. In this example, we were adding eighths, so our answer will also be eighths.*

How much pizza was left?

The couple ordered a whole pizza made up of eight slices, or $\frac{8}{8}$. Then they ate $\frac{5}{8}$ of the pizza. In math, that is

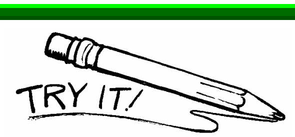
$\frac{8}{8}-\frac{5}{8}$.

We can use a picture to help us here as well.



$$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$$

We see that $\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$. Notice that we subtracted only the numerators! The denominators did not change. Remember, we are talking about pieces that are the same size. When two or more fractions have the same denominator, we can add or subtract them simply by adding or subtracting their numerators. <u>The denominators do not change</u>.



1. Add or subtract the fractions, then express the answers in simplest form.

a) $\frac{1}{3}+\frac{1}{3}$ b) $\frac{3}{5}-\frac{1}{5}$ c) $\frac{2}{6}+\frac{1}{6}$ d) $\frac{9}{4}-\frac{7}{4}$

Suppose the families of four and five want more pizza, but only three members from the family of four want another slice, and one member from the family of five wants another slice. In other words, you need to make $\frac{3}{4}$ of a pizza for the family of four, and $\frac{1}{5}$ of a pizza for the family of five.

Now you have a problem. How much pizza do you make? The slices of pizza were different sizes for the different families, and you don't want to be wasteful and make two whole pizzas just so you can cut each one a little differently. We can figure this out by using what we know about equivalent fractions, and what we just discovered about adding fractions that have the same denominator. So, the total amount of pizza you need to make is

$$\frac{3}{4}+\frac{1}{5}.$$

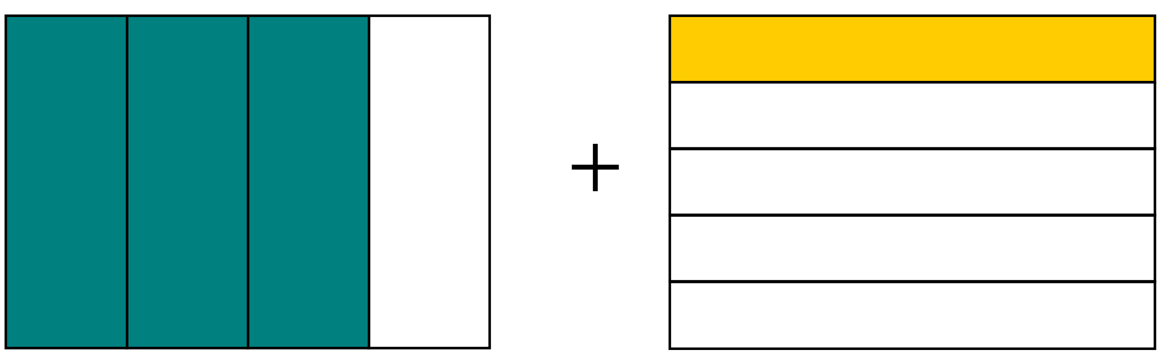
Maybe your first thought is to add the numerators and the denominators, and say

$$\frac{3}{4}+\frac{1}{5}=\frac{3+1}{4+5}=\frac{4}{9}.$$

Wait a minute here, something is not right. Using number sense, we see that we are adding something to $\frac{3}{4}$, and we know $\frac{3}{4}>\frac{1}{2}$, so our answer should be bigger than $\frac{1}{2}$. But the answer we got was $\frac{4}{9}$, and $\frac{4}{9}$ is definitely not bigger than $\frac{1}{2}$, so this way of thinking is wrong.

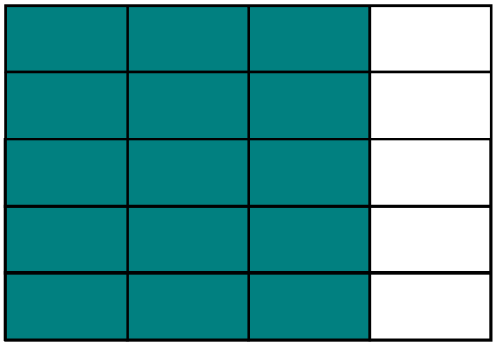
*It is **wrong** to add fractions by adding the numerators and denominators!*

Let's start over with a model. First, we show that $\frac{3}{4}+\frac{1}{5}$ can be drawn like this,



But we cannot add up the shaded pieces yet, because they are different sizes: fourths and fifths are not the same.

What if we redraw the picture like this?



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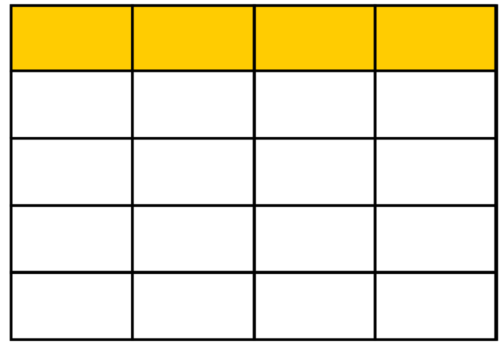
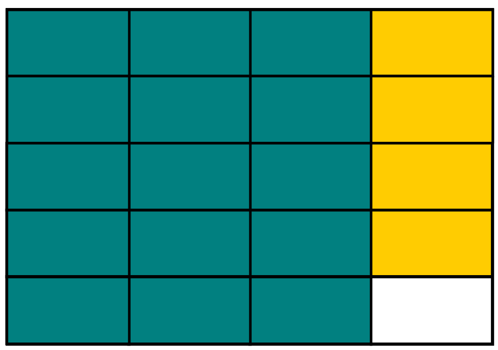


$\frac{15}{20}$ + $\frac{4}{20}$

Now we see that each piece is the same size, and there is still the same amount being shaded. Now we can combine the shaded areas and see that this is equal to



$$=\frac{19}{20}$$

We showed that both fractions could be divided into twenty boxes evenly, so we were able to add up the total number of shaded boxes. What we really did was found equivalent fractions with the <u>same denominator</u>. This allows us to add the fractions together as we just learned! The visual example gives us some good ideas for adding fractions with different denominators. Let's look at an example using numbers, so we can understand more.

Example

Find the sum. $\frac{1}{15}+\frac{7}{12}$

Solution

Think about the best way to solve this. We would not want to draw a model for fifteenths or twelfths, it would be too hard. We have to use numbers. Let's look at equivalent forms of each fraction to see if they have a common denominator.

Equivalent forms of $\frac{1}{15}$ are $\frac{2}{30}, \quad \frac{3}{45}, \quad \frac{4}{60}, \quad \frac{5}{75}, \quad \frac{6}{90}, \quad \frac{7}{105}, \ldots$

Equivalent forms of $\frac{7}{12}$ are $\frac{14}{24}, \quad \frac{21}{36}, \quad \frac{28}{48}, \quad \frac{35}{60}, \quad \frac{42}{72}, \quad \frac{49}{84}, \ldots$

Because $\frac{4}{60}$ and $\frac{35}{60}$ have the same denominator, we can add them just as we did before.

$$\frac{4}{60} + \frac{35}{60} = \frac{39}{60}$$

We are not finished yet! We need to check to see if 39 and 60 share a common factor. In fact, they do; it is 3. To put this fraction in simplest form, we divide the numerator and denominator by the GCF, which is 3.

$$\frac{39 \div 3}{60 \div 3} = \frac{13}{20}$$

When we are looking for a denominator that two fractions share, we can save some time by looking at multiples of each denominator.

Think Back

A multiple of a number is the product of that number and any number besides zero.

Example

Simplify $\frac{1}{6} + \frac{1}{2}$

Solution

Method 1

Multiples of 6: 6, 12, 18,…

Multiples of 2: 2, 4, 6, 8, 10,…

The first one they have in common is 6. For 6, $6 \times 1 = 6$ so we do not need to change $\frac{1}{6}$ to anything.

For 2, $2 \times 3 = 6$, so we must multiply both the numerator and the denominator by 3 to get

$$\frac{1}{2} = \frac{3}{6}$$

Think Back

In this method we found the least common multiple of the denominators.

With common denominators, we may add the numerators, and see

$$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}.$$

In lowest terms, $\frac{4}{6} = \frac{2}{3}$.

Method 2

If we multiply the denominators, we see that $2 \times 6 = 12$. We can convert to a denominator of 12, by multiplying $\frac{1}{6} \times \frac{2}{2}$ and $\frac{1}{2} \times \frac{6}{6}$ to get $\frac{2}{12} + \frac{6}{12} = \frac{8}{12}$

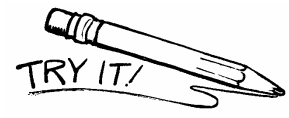
Expressed in lowest terms,

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

- Two fractions whose common denominator is their LCM are said to have the **lowest** or **least common denominator**, which is known as the **LCD**.

In Method 1, we found the least common multiple of the fractions' denominators and used it to write equivalent fractions with the lowest common denominator.

With Method 2, finding a common denominator is a little easier, but converting to simplest form requires a bit more work. Try both ways, then decide which method you feel is easier.



2. Solve using the least common denominator, and then again by multiplying the denominators. Write the answer in lowest terms.

a) $\frac{1}{4}+\frac{1}{12}$

b) $\frac{8}{9}-\frac{2}{3}$

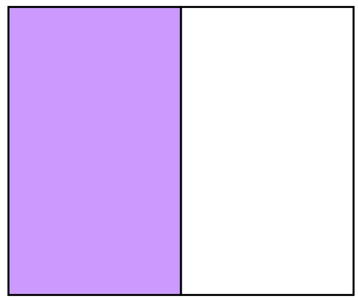
Now that we have conquered adding and subtracting fractions, we can move on to multiplying and dividing fractions.

You have decided that working at the pizza shop is too stressful and that you would rather work outdoors. You get a job working on a more peaceful grape farm. One day, you pick all the grapes from a vine and fill one $\frac{1}{2}$ gallon bucket $\frac{1}{3}$ of the way with grapes. As you move on to the next grapevine, you ask yourself, "How many gallons did I just pick?"

It was one-third of a half gallon. Mathematically, this is $\frac{1}{3}\times\frac{1}{2}$. Can we take one-third of a group of one-half? Let's make a model!

Let's show a group of halves below.

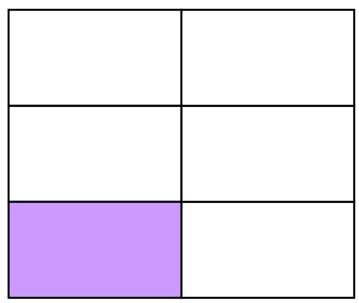
$$\frac{1}{2}$$



The shaded half shows the amount that a bucket can hold out of one whole gallon.

But we are talking about "one-third of one-half." So we must now divide our halves into thirds.

$$\frac{1}{3}\times\frac{1}{2}$$



We can see that one-third of one-half of one gallon is really one sixth of a gallon!

We see now that we get one out of six equal pieces, so

$$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$$

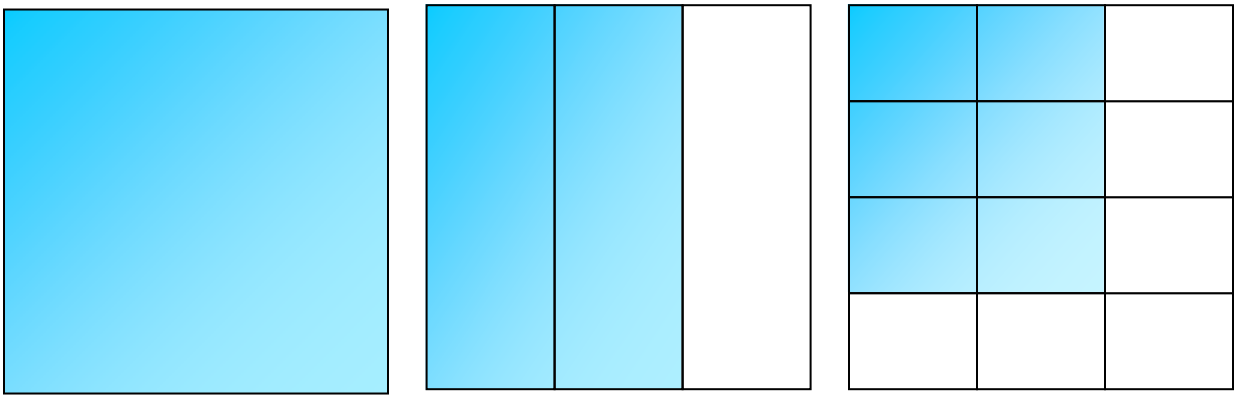
Here is another example.

Example

Maria was cutting glass to fit a window. She cut the glass $\frac{2}{3}$ of a meter wide by $\frac{3}{4}$ of a meter tall. What is the *area* of the glass Maria cuts?

Solution

We need to find how much of a square meter of glass Maria cuts. The area of the glass she cuts is how much of the whole piece she takes. This is given by $\frac{2}{3}\times\frac{3}{4}$. Let's use a model again.



First we take a square meter, cut it $\frac{2}{3}$ meters wide, then cut that $\frac{3}{4}$ meters tall.

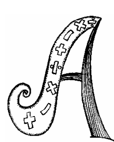
So,

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} \text{ of a square meter, or } \frac{1}{2} \text{ square meters.}$$

Think Back

Product is the answer to a multiplication problem.

So far, we have seen that $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$, and that $\frac{2}{3}\times\frac{3}{4}=\frac{6}{12}=\frac{1}{2}$. Notice that each time, the answer is the <u>product</u> of the numerator over the <u>product</u> of the denominator.



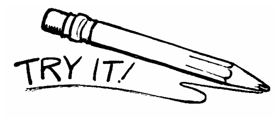
Algorithm

To multiply fractions:

1. Multiply their numerators.
2. Multiply their denominators.
3. Write your answer as $\frac{Step1}{Step2}$
4. Put your answer in simplest form.

$$\frac{1}{3}\times\frac{3}{5} = \frac{1\times 3}{3\times 5} = \frac{3}{15} = \frac{1}{5}$$

or $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$



3. Find the product of the following fractions, and simplify:

a) $\frac{2}{3} \times \frac{7}{8}$ b) $\frac{2}{5} \times \frac{4}{11}$ c) $\frac{1}{8} \times \frac{2}{3}$

Recall that we use fractions to divide one whole into parts. In the grape farm problem, we divided one whole gallon into half gallons. We then divided one half gallon into thirds to represent $\frac{1}{3} \times \frac{1}{2}$.

Since a fraction is really a division problem, we can rewrite this as

$$\begin{aligned} \frac{1}{3} \times \frac{1}{2} &= \frac{1}{3} \times 1 \div 2 \\ &= \frac{1}{3} \div 2 \\ &= \frac{1}{3} \div \frac{2}{1} \end{aligned}$$

Think Back

The identity property of multiplication says that anything multiplied by one equals itself!

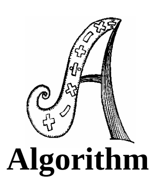
Look at that!

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{3} \div \frac{2}{1}$$

- Given a fraction $\frac{a}{b}$, we call the "flipped" form of it, $\frac{b}{a}$, the **reciprocal**. For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
    - The product of a number and its reciprocal is 1. $\frac{2}{3}\times\frac{3}{2}=\frac{6}{6}=1$

From the observation above, it seems that we can show the multiplication of fractions as one fraction being divided by the other fraction's reciprocal.

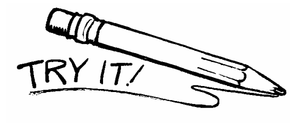
If this is true, we can represent the quotient of two fractions as one fraction being multiplied by the reciprocal of the other.



To divide fractions:

1. Change the division sign to a multiplication sign, and take the reciprocal of the divisor (the fraction that is on the right).
2. Multiply the numerators and denominators.
3. Put your answer in simplest form.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$



4. Find the quotient (the answer to the division problem).

a) $5 \div \frac{1}{8}$

b) $\frac{1}{2} \div \frac{3}{7}$

c) $\frac{5}{12} \div \frac{16}{17}$

Review

1. Highlight the following definitions:
    a. Least Common Denominator (LCD)
    b. reciprocal

2. Highlight the “Algorithm” boxes.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 6

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 6, Set A and Set B.

Set A

1. Add or subtract the following fractions. Write your answers in simplest form.

a) $\frac{2}{7}+\frac{3}{7}$ b) $\frac{4}{9}-\frac{3}{9}$ c) $\frac{1}{4}+\frac{3}{5}$ d) $\frac{11}{12}-\frac{4}{5}$

2. Find the product or quotient. Write your answers in lowest terms.

a) $\frac{1}{2}\times\frac{7}{8}$ b) $\frac{9}{10}\times\frac{5}{6}$ c) $\frac{1}{2}\div 9$ d) $\frac{2}{5}\div\frac{10}{13}$

Set B

1. Melissa wrote on a test that $\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$. Is her answer right? Explain why or why not using words and/or pictures.

2. If you can pick $\frac{1}{6}$ of a gallon of grapes from each grapevine, how many grapevines would you have to pick to fill a half-gallon bucket? What about 3 half-gallon buckets?

3. Nine acres of land is being divided into $\frac{3}{4}$-acre plots to build some new houses. How many lots will there be? What happens to the number of lots if the number of acres of land is doubled?

ANSWERS TO TRY IT

1. a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{2}$ d) $\frac{1}{2}$

2. a) $\frac{4}{12} = \frac{1}{3}$ or $\frac{16}{48} = \frac{1}{3}$ b) $\frac{2}{9}$ or $\frac{6}{27} = \frac{2}{9}$

3. a) $\frac{14}{24} = \frac{7}{12}$ b) $\frac{8}{55}$ c) $\frac{2}{24} = \frac{1}{12}$

4. a) 40 b) $\frac{7}{6}$ c) $\frac{85}{192}$

End of Lesson 6