Student Name: ______________________

Date: ______________________

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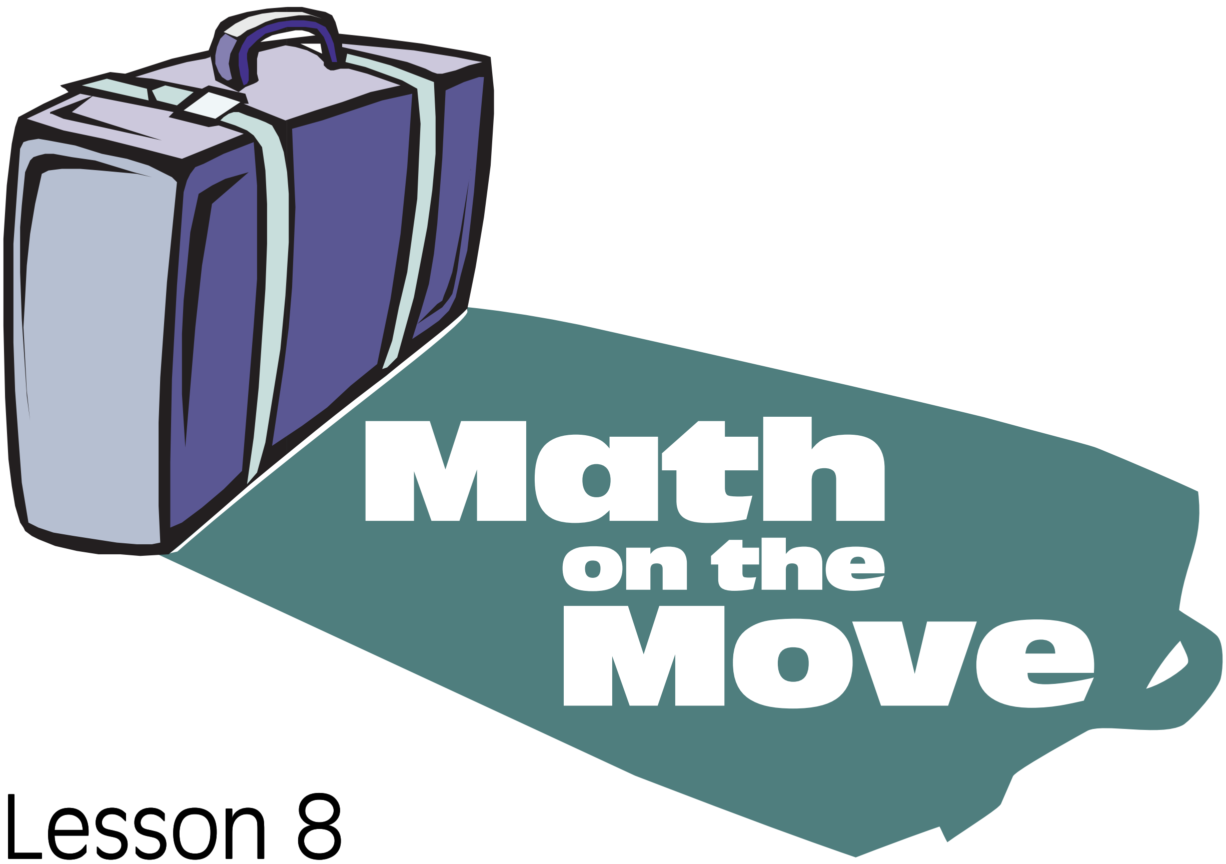

Lesson 8

Decimals

Objectives

- Understand conceptually what decimal notation means
- Be able to convert from decimals to mixed numbers and fractions
- Round decimals to a given place value

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After work, you go to the store to buy some produce for a fruit salad you want to make. You see a deal for bananas on a sign, "Ten bananas for one dollar."

From what we know about fractions, one banana costs

$$1 \div 10 = \frac{1}{10} \text{ of a dollar.}$$

From your knowledge of money, you also know that one tenth of one dollar is a dime, or $0.10

So,

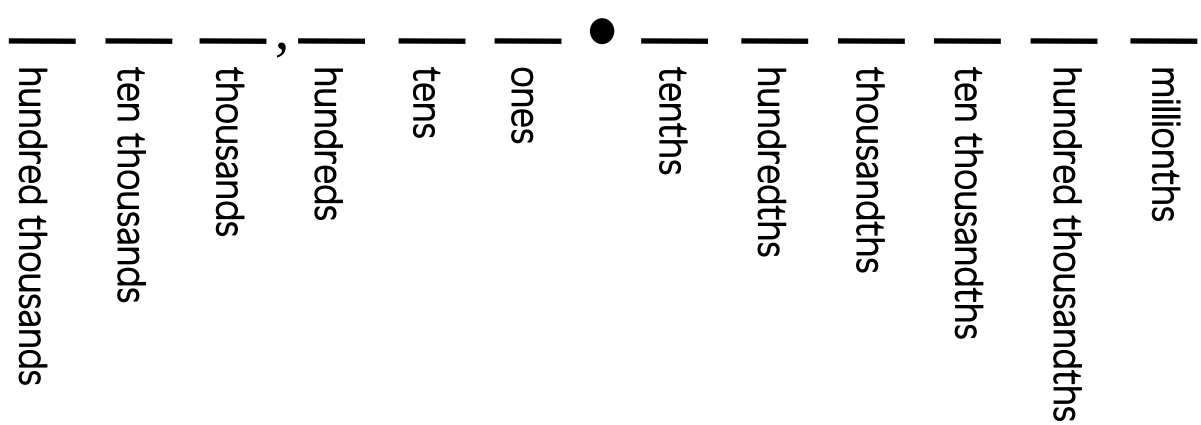
$$\frac{1}{10} = .10$$

Here is yet another way to represent fractions! 0.10 is an example of a **decimal**.

- A **decimal** is a number that can represent a whole part and a fractional part. A **decimal point**, written as a period (.), is written to separate the whole part from the fractional part.

  For example, 3.5 is a decimal, and so is 0.72.

A great way to begin an understanding of decimals is to think of them using money. With decimals, the first number after the decimal point is in the tenths place. This makes sense because one dime is $0.1, or one-tenth of a dollar. The second number to the right of a decimal point is in the hundredths place. With money, this number tells you the number of pennies you have. This makes sense since one penny is 1 one-hundredth ($\frac{1}{100}$) of a dollar. Here is a diagram to help you better understand whole number and decimal places.



Example

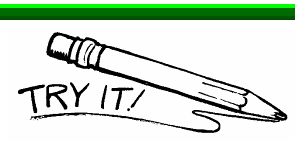
Write the place value of each digit in the number .123450

Solution

We see that 1 is in the tenths place, 2 is in the hundredths place, 3 is in the thousandths place, 4 is in the ten-thousandths place, 5 is in the hundred-thousandths place, and zero is in the millionths place.

We can also say that there is 1 tenth, 2 hundredths, 3 thousandths, 4 ten thousandths, 5 hundred thousandths, and zero millionths.

Now you give it a try.



1. Write each digit in the correct place value in the chart below.
Then write the place value of the digit that is farthest to the right.

a) 3.1 b) 2.03 c) 8.463 d) 7.1464 e) 13.00001

	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
a)				•					
b)				•					
c)				•					
d)				•					
e)				•					

So how do we write and say the entire decimal?

Example

Write out 3.413 using words.

Solution

This is a mixture of whole numbers and a fractional part. By counting the number of decimal places, we see this number goes to the thousandths place. We say it like this:

3.413

three and four hundred thirteen thousandths

Let's notice a few things here. When we say "and," we really mean to say, "there is a decimal point here." The next part we say is the fraction. We read it as if it's a new number, and then say the place value of the right-most digit. "Four hundred thirteen" is the number. "Thousandths" is the last place value.

This is also exactly how we say mixed numbers. Without much work, we can also write decimals as mixed numbers.

$$3.413 = 3\frac{413}{1000}$$

Once again, the number is read as three and four hundred thirteen thousandths

From mixed number form, we know how to change this into an improper fraction as well!

$$3\frac{413}{1000} = \frac{3000}{1000} + \frac{413}{1000} = \frac{3413}{1000}$$

$$\text{so } 3\frac{413}{1000} = \frac{3413}{1000}$$

Algorithm

To write a decimal with words:

1. Write the number to the left of the decimal as you would any whole number.
2. In place of the decimal point, use the word "and".
3. Write the number to the right of the decimal point, as you would any whole number.
4. At the end, write the final digit's place value. It should end in "ths." (tenths, hundredths, thousandths, ...)

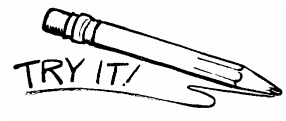
Algorithm

To write a decimal as a mixed number:

1. Rewrite all the digits to the left of the decimal point; this is the whole number part.
2. Write all the digits to the right of the decimal point as the numerator of the fraction.
3. For the denominator, write the place value of the right-most digit. (10, 100, 1000, 10000, 100000, ...)

For example, $17.927 = 17\frac{927}{1000}$

Now you give it a try.



2. Write each decimal using words, then as a mixed number, then as a fraction in lowest terms.

a) 2.6

Words:____

Mixed Number:

Fraction:

b) .43

Words:____

Mixed Number:

Fraction:

c) 1.6524

Words:____

Mixed Number:

Fraction:

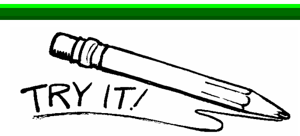
Consider two whole numbers, 340 and 00340. Believe it or not, 340 = 00340. The number 00340 looks a little strange, doesn't it? We don't usually see numbers written this way, since the first zeros before the 3 are meaningless. However, we do need the zero after the 4. If we drop the zero at the end of 340, it changes its meaning.

Similar things can be done with decimals. The following decimals are all equal.

$$\begin{aligned} &0.43 \\ &= 0.430 \\ &= 0.4300 \\ &= 0.43000 \\ &= 0.430000 \\ &= 0.43000000000000000000 \end{aligned}$$

They're the same because the place value of the 4 and 3 never change.

Any number of zeros may be added at the end of a decimal without changing the value of the decimal.



3. True/False

a) 07 = 7

b) 4 = 40

c) 00030 = 00300

d) 3.4 = 03.4

e) 8.42300 = 8.423

f) 900.163200 = 0900.1632

Knowing this helps us put decimals in order.

Example

Which is larger, .2, or .19?

Solution

You might be tempted to say that .19 is larger than .2, since $19 > 2$. But let's think about this first.

We know that .2 = .20

Thinking in terms of money, we also know that $0.20 is more money than $0.19.

Therefore, $.2 > .19$

What about the next two decimals?

Example

Which is larger, 0.2 or .199999999999999999 ?

Solution

Let's line up these two numbers according to their place values.

$.2$

$.199999999999999999$

Notice that the top number has 2 tenths, and the bottom only has 1 tenth plus something that is less than one tenth, so $.2 \; > \; .199999999999999999$.

We see that decimals are nice to work with when looking at the size of two numbers. This is why we use decimals instead of fractions for money. What we have observed here will help us use the next method of comparing the size of two decimals.

Algorithm

To compare the size of decimals:

1. Line up the two decimals according to place value. An easy way to do this is to make sure the decimal points are on top of each other.
2. Compare place values until a difference is found.

   First check the whole number parts. If those are the same, check the tenths place of each. If they are the same, check the hundredths, then the thousandths, etc. until you find a place value where the digits aren't the same.
3. Determine which is larger.

   In the place value where you find the difference, the larger digit tells you which number is larger.

Example

Compare 1.1324549 and 1.1324639

Solution

Step 1: Line up the two numbers by their decimal points.

$$1.1324549$$
$$1.1324639$$

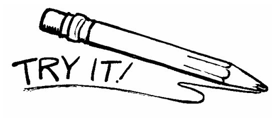
Step 2: Compare place values until a difference is found.

We see that the first difference comes in the hundred thousandths place.

Step 3: Determine which is larger.

We circled the digits 6 and 5.

$6 > 5$, so $1.1324639 > 1.1324549$.



4. Compare the following decimals using < , > , or =.

a) .12 and .13

b) .102 and .13

c) 1.35 and .999

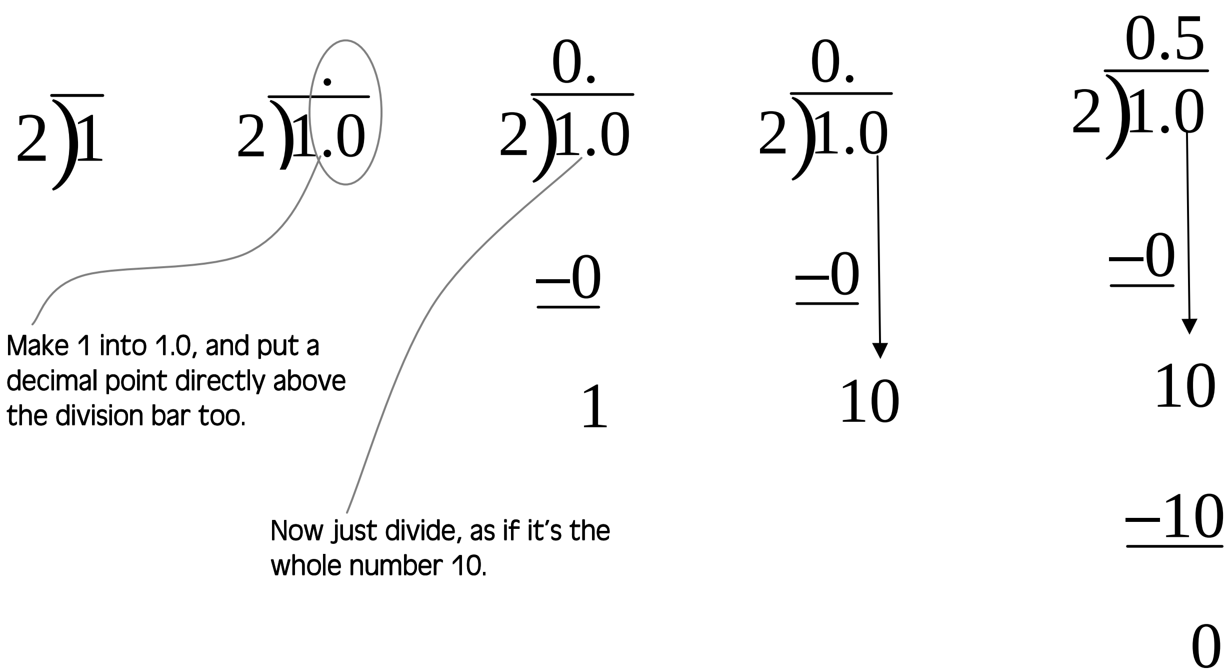
d) 16.82736 and 16.82747

You continue walking through the grocery store and see another sign that reads, "Tuna, half off of $1." You understand that half of $1.00 is $0.50. This means that,

$$\frac{1}{2} = 0.5$$

You remember that $\frac{1}{2}$ also means $1 \div 2$. Therefore, $1 \div 2 = 0.5$. But wait, shouldn't $1 \div 2 = 0R1$?

The answer is, that before, we were only dividing with whole numbers. Now, we may use decimals instead of remainders. Here's how:



Example

Write $\frac{5}{8}$ as a decimal.

Solution

Remember, $\frac{5}{8} = 5 \div 8$

$8\overline{)5}$

$$\begin{array}{r} 0. \\ 8\overline{)5.0} \\ -\underline{0} \\ 5 \end{array} \qquad \begin{array}{r} 0. \\ \overline{)5.0} \\ -\underline{0} \\ 50 \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)5.0} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 2 \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)5.00} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 20 \end{array}$$

If there is a remainder, create a decimal and keep adding zeros to the dividend until there is no remainder.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 20 \\ -\underline{16} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

Walking through the aisles at the grocery store, you see a sign that reads, "Markers, 3 for $1 or 1 for $.35," and you wonder which one is the better deal.

In the first deal, 3 markers for one dollar, the price of one marker is represented as $\frac{1}{3}$ of a dollar. How much is this? Let's remember that fractions mean division, so

$$\frac{1}{3} = 1 \div 3$$

$$\frac{1}{3} = 1 \div 3 \text{, and}$$

$$1 \div 3$$

$$= 3\overline{)1.0000\cdots}\quad 0.3333\ldots$$

$$\begin{array}{r} -\underline{.9} \\ .10 \\ -\underline{.09} \\ .010 \\ -\underline{.009} \\ .0001 \\ \vdots \end{array}$$

No matter how long we keep dividing, this decimal will never end, and we will keep adding on another 3 forever! To show that a decimal will never end and goes on in the same pattern, we write a bar over the part that repeats. So, 0.3333333333333333333... is written as $.\overline{3}$. Because we are dealing with money, we will round this decimal to the hundredths place (Rounding will be explained later in this lesson). Thus, one marker from the first deal will cost $.33. That is better than the $.35 of the other deal.

Another example of a decimal that never ends is 0.643712121212121212121212... and this is written as $0.6437\overline{12}$. Notice that we write the bar only over the numbers that repeat, which makes the decimal easier to read.

These are examples of **repeating decimals**.

- A **repeating decimal** is a decimal that has an infinite number of digits, and the digits continue in a set pattern.

  For example, $.3333333\ldots = .\overline{3}$, and $.473473473473473\ldots = .\overline{473}$ are repeating decimals.
- A decimal that ends is called a **terminating decimal**.

  For instance, $.173$ and 33.2 are terminating decimals.

Any fraction can be made into either a terminating or a repeating decimal!

Example

Write $\frac{4}{5}$ as a decimal.

Solution

We will use long division

$$\begin{array}{r} 0.8 \\ 5\overline{)\ 4.0} \\ -\underline{4.0} \\ 0 \end{array}$$

so $\frac{4}{5} = 0.8$, a terminating decimal.

Example

A sign in the store reads "Paper towels, 11 rolls for $3.00". How much will 1 roll of paper towels cost?

Solution

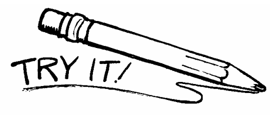
We must divide 3.00 into 11 equal groups, so we must solve

$$3 \div 11$$

$$\begin{array}{l} \quad 0.27272... \\ 11\overline{)3.00000...} \\ \ \ -\ \underline{2.2} \\ \qquad 80 \\ \quad\ -\ \underline{77} \\ \qquad\ \ 30 \\ \quad\ \ -\ \underline{22} \\ \qquad\quad 80 \\ \qquad\ -\underline{77} \\ \qquad\qquad \ddots \end{array}$$

As soon as you see the same remainder twice, you can tell that the decimal is a repeating decimal, and you may stop dividing!

Our answer is $3 \div 11 = .\overline{27}$



Convert the following fractions to decimals. The decimals may either terminate or repeat.

5) $\frac{9}{11}$

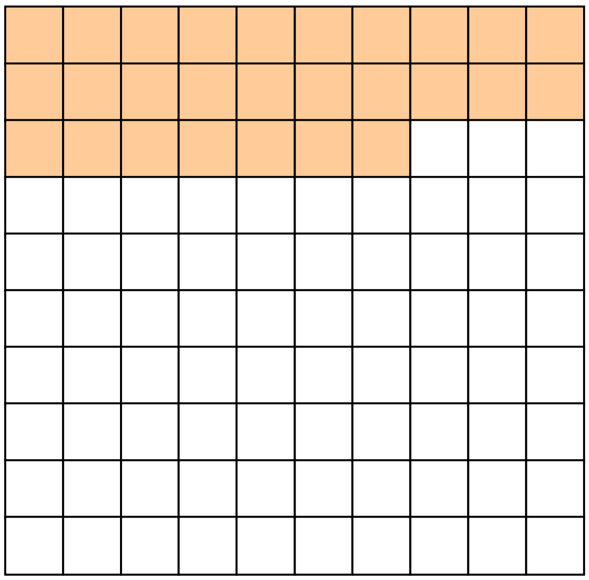
6) $\frac{11}{8}$

7) $\frac{5}{6}$

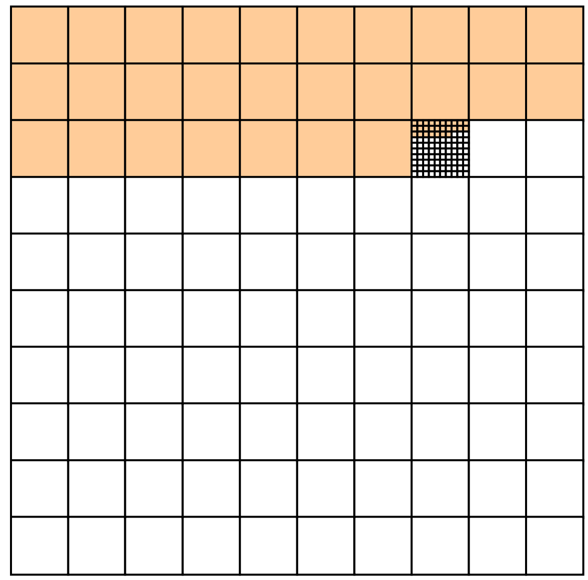
In the previous example, we found that one roll of paper towels costs $\$0.\overline{27}$.

This seems strange. Have you ever seen the price of something shown as a repeating decimal? Stores don't say that something costs $.272727272727...

Let's think about why this is. Look at a picture of .27, and of .2727



.27



.2727

In order to count the extra .0027 on the right, we split one cent into 100 equal pieces, and we took 27 of them. One cent is worth so little. If we break it into 100 pieces, the value of one is one ten-thousandth of a cent. This value is so small that it is not needed when dealing with money. We don't care about it! Because of this, stores <u>round</u> to the nearest cent.

Therefore, our answer to the previous example is that one roll of paper towels costs $0.27

Let's learn how to round!

Example

Round 173.9378429329 to the nearest <u>tenth</u>.

Solution

Step 1: Look at the number to the right of the tenths place.

173.9<u>3</u>76429329

Step 2: Compare the number to 5. Notice that $3 < 5$, so we must round <u>down</u> and leave the tenths place the same.   Our answer is 173.9

Here is an algorithm to help you round

Algorithm

To round a number to a given place value:

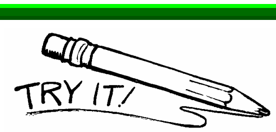
1. Look at the number to the right of the place value you are asked to round to.
2. Compare that number to 5.
   a. If that number is less than 5, round down and leave the given place value the same
   b. If that number is greater than or equal to 5, round up and increase the given place value by 1
      i. If the number in the given place value is a 9, make it a 0 and increase the value of the number to the left of our given place value

Round 1.895 to the nearest hundredth

1.89(5)

5 = 5
Round up

1.895 rounds to 1.90



Convert the following fractions to decimals. The decimals may either terminate or repeat. Then, round each decimal to the nearest <u>hundredth</u>.

8. $\frac{3}{8}$

9. $\frac{2}{3}$

10. $\frac{5}{11}$

Review

1. Highlight the following definitions:
    a. decimal
    b. decimal point
    c. repeating decimal
    d. terminating decimal

2. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 8

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 8, Set A and Set B.

Set A

1. Write the decimal with words, then as a mixed number, then as an improper fraction in simplest form.

4756.10974

2. Use an inequality sign (> or <) to compare each pair of decimals.

   a) 3.425 and 6.425 b) 1.089 and 1.1

   c) 0.001 and 0.01 d) 142.284756 and 142.284755

3. Round each decimal to the nearest <u>hundredth</u>.

   a) 7.43232 b) 14.267239 c) 9.473

   d) 1.1111111111 e) 0.9877654 f) $13.\overline{8}$

Set B

1. Write the following decimals so that their place values are lined up

24971894781.34 and 32.823743239

2. Write the amount as a decimal part of a dollar. (Hint: think of how many <u>cents</u> each equals.)

   a) 1 quarter b) 4 nickels c) 89 pennies d) 14 dimes

3. A number is between 0 and 1. The place value farthest to the right is the thousandths place. The number contains the digits 0, 2, and 5. Using each digit, what is the <u>smallest</u> number that these digits can make? What is the <u>greatest</u>? *Explain* your thinking.

ANSWERS TO TRY IT

1) a) Tenths b) Hundredths c) Thousandths d) Ten Thousandths

e) Hundred Thousandths

2) a) 2.6 is two and six tenths, $2\frac{6}{10} = 2\frac{3}{5} = \frac{13}{5}$

b) .43 is forty three hundredths, $\frac{43}{100}$

c) 1.6524 is one and six thousand, five hundred twenty-four ten-thousandths,

$$1\frac{6524}{10000} = 1\frac{1631}{2500} = \frac{4131}{2500}$$

3) a) True b) False c) False d) True e) True f) True

4) a) $.12 < .13$ b) $.102 < .13$ c) $1.35 > .999$ d) $16.82736 < 16.82747$

5) $0.\overline{81}$

6) 1.375

7) $0.8\overline{3}$

8) $0.375 \approx 0.38$

9) $0.\overline{66} \approx 0.67$

10) $0.\overline{45} \approx 0.45$

NOTES

End of Lesson 8