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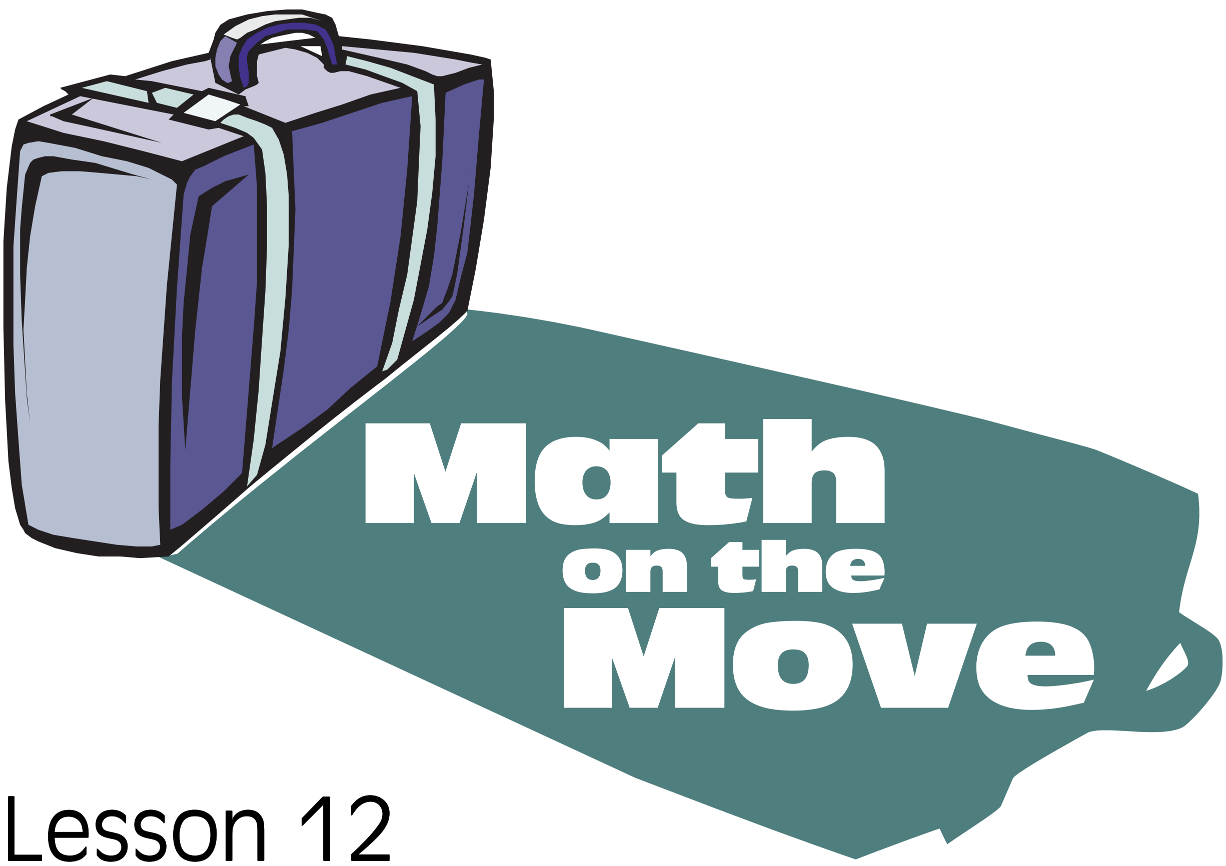

Lesson 12

Simplifying, Substituting, and Solving

Objectives

- Simplify algebraic expressions
- Substitute values for variables in algebraic expressions
- Solve and check two-step equations

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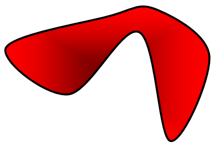
In the last lesson, we became familiar with the concept of a variable as something that is not known. Variables sometimes behave as whole numbers do.

Example

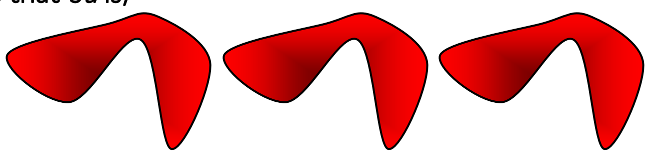
Simplify the expression $3a + 2a$.

Solution

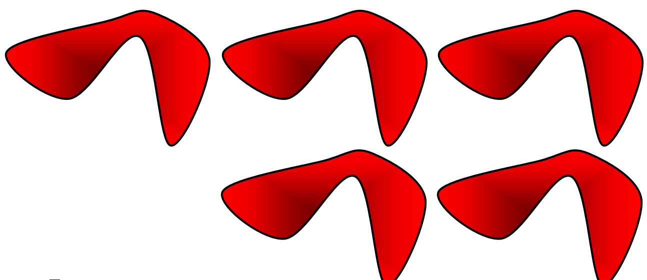
Perhaps a picture will help us. Let's say that *a* is some weird object, for instance,

 $= a$

This means that 3*a* is,



Now we'll add 3*a* + 2*a*



$$\begin{array}{r} 3a \\ +\ 2a \\ \hline 5a \end{array}$$

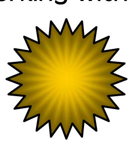
See, $3a + 2a = 5a$, just as 3 + 2 = 5

Example

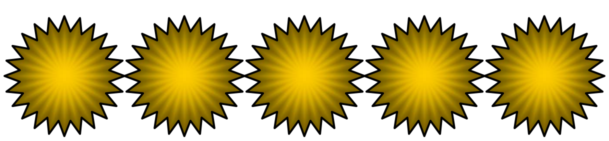
Simplify the expression $5x - 2x$

Solution

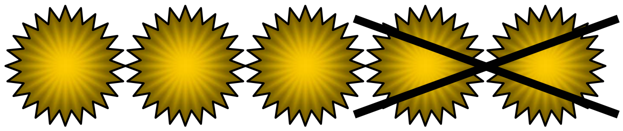
Now we're working with the variable *x*. Let's represent *x* with any object, say

 $= x$

This means,

 $= 5x$

Now, to show $5x - 2x$, we simply take two *x*'s away.

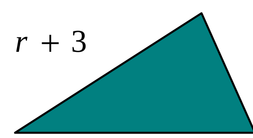


$$5x - 2x$$
$$= 3x$$

As we suspected, $5x - 2x = 3x$, just as 5 – 2 = 3. These two examples show that variables can be nice. So far, they are behaving just as whole numbers do, except they are a letter instead of a number. Let's look at the next example.

Example

Find the distance around the triangle in terms of *r*.



Solution

The distance around the triangle is the sum of the lengths of the sides. That is,

$$(r + 3) + (2r + 4) + (2r)$$
$$r + 3 + 2r + 4 + 2r$$

This expression has five **terms**.

- A **term** is anything being separated by addition or subtraction.

  For example,

$5x$	1 term	The term is $5x$.
$2a + 9$	2 terms	They are $2a$ and 9.
$4b - 12a + 8$	3 terms	They are $4b, 12a, 8$
$16ab + 2a - 3b + 7$	4 terms	$16ab,\ 2a,\ 3b,\ 7$

To simplify $r + 3 + 2r + 4 + 2r$, we need to combine *like terms*, or the terms that are similar to each other. In this example, there are terms with the variable *r* and terms with no variable.

This expression has five terms. We must combine the *r*-terms

$$r + 3 + 2r + 4 + 2r$$

If you do not see a number in front of a variable, assume there is a 1 there

$$= 5r + 3 + 4$$

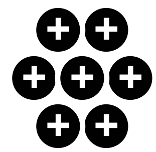
Now, we combine the number terms.

$$= 5r + 7$$

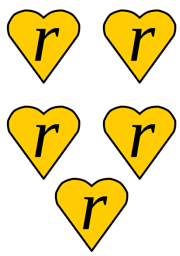
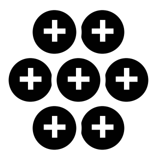
Now you may be wondering, "what next?" The answer is, we're done simplifying. Terms with letters do not combine with terms that have only numbers.

Let's say *r* is some random object.

From our work with integers, we know

 = 7

So how do you show

 +  ?

You can't! It just doesn't make any sense. That's why 5*r* + 7 is in simplest form, even though there are two terms.

In summary, combine <u>letters with letters</u> and <u>numbers with numbers</u>.

Example

Simplify $4n + 3 - n - 1$

Solution

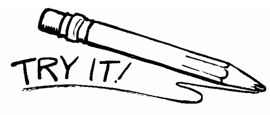
Remember, <u>letters with letters</u> and <u>numbers with numbers</u>.

First, we combine our like terms. Always include the sign to the left of each term.

$$4n + 3 - n - 1$$

$$= 3n + 3 - 1$$

$$= 3n + 2$$



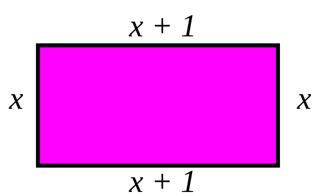
1. Simplify each expression.

a) $6m + m$ b) $4x - 2x$ c) $10a - 9a$

d) $4w - 3 + 7w - 1$ e) $10s + 4s - 6s$

f) $8z - 5 + 2z + 9$ g) $m + m + 1 + m + 2$

2. Find the distance around the rectangle.



In the previous lesson, each equation could be solved with one operation. For example, the equation

$$7 = 3 + x$$

was solved by subtracting 3 from both sides. Then, you were done. Most of the time in algebra, however, equations are a little more complicated and require more than one step to solve.

Example

Solve for *p*. $2p + 3 = 11$

Solution

We can use what we learned from the last lesson. In order to solve for a variable, we need to use inverse operations to get the variable by itself. Which operation should we use first? There are <u>two</u> operations to be undone,

$$2p + 3 = 11$$

multiplication and addition

We will undo the addition first by subtracting 3 from both sides.

$$\begin{array}{r|c|l} 2p + 3 & = & 11 \\ -3 & & \underline{-3} \\ 2p & = & 8 \end{array}$$

$$\begin{array}{r|c|l} 2p + 3 & = & 11 \\ -3 & & \underline{-3} \\ \dfrac{2p}{2} & = & \dfrac{8}{2} \\ p & = & 4 \end{array}$$

Since $2p = 2 \bullet p$, we must undo multiplication and *divide* by two.

Our final step is to check our answer.

Check: $p = 4$

$$2p + 3 = 11$$
$$2(\;\;) + 3 = 11$$
$$2(4) + 3 = 11$$
$$8 + 3 = 11$$
$$11 = 11 \checkmark$$

Algorithm

To solve for a variable:

1. Perform addition or subtraction.
2. Perform multiplication or division.

$$\begin{array}{r|c|l} 3 + 2y & = & 9 \\ -3 & & -3 \\ \dfrac{2y}{2} & = & \dfrac{6}{2} \\ y & = & 3 \end{array}$$

Example

Solve for *y*. $7 - 2y = 13$

Solution

Our variable is *y*. We need to get it all by itself.

$$\begin{array}{r|c|l} 7 - 2y & = & 13 \\ -7 & & -7 \end{array}$$

Subtract 7 from both sides.

$$\begin{array}{r|c|l} \frac{-2y}{-2} & = & \frac{6}{-2} \end{array}$$

Divide both sides by -2.

$$\begin{array}{r|c|l} y & = & -3 \end{array}$$

$\underline{Check}$: $y = -3$

$$7 - 2y = 13$$
$$7 - 2(\ \ ) = 13$$
$$7 - 2(-3) = 13$$
$$7 + 6 = 13$$
$$13 = 13$$

Think Back

***P**arentheses*

***E**xponents*

***M**ultiplication or **D**ivision*

***A**ddition or **S**ubtraction*

Example

Solve for *t*. $15 = 2t + 5$

Solution

$$\begin{array}{r|c|l} 15 & = & 2t + 5 \\ -5 & & \quad\ \ -5 \\ \frac{10}{2} & = & \frac{2t}{2} \\ 5 & = & t \end{array}$$

$\underline{Check}$: $5 = t$

$$15 = 2t + 5$$
$$15 = 2(\ \ ) + 5$$
$$15 = 2(5) + 5$$
$$15 = 10 + 5$$
$$15 = 15$$

Example

Solve for *s*. $94 = -18 - 2s$

Solution

$$
\begin{array}{r|c|l}
94 & = & -18 - 2s \\
+18 & & +18 \\
\frac{112}{-2} & = & \frac{-2s}{-2} \\
-56 & = & s
\end{array}
$$

> ***Think Back***
>
> *A negative number multiplied or divided by a negative number is a positive number!*

$\underline{Check}$: $-56 = s$

$$94 = -18 - 2s$$
$$94 = -18 - 2(\quad)$$
$$94 = -18 - 2(-56)$$
$$94 = -18 + 112$$
$$94 = 94 \checkmark$$

Example

Solve for *n*. $3n = 4n + 7$

Solution

Now we have terms with variables on both sides of the equals sign. We need to get the variable on one side of the equal sign.

$$
\begin{array}{r|c|l}
3n & = & 4n + 7 \\
-4n & & -4n \\
\frac{-n}{-1} & = & \frac{7}{-1} \\
n & = & -7
\end{array}
$$

Subtract 4*n* from both sides.

Since $-n$ is really $-1 \bullet n$, we will undo multiplication and *divide* by -1 on both sides.

$\underline{Check}$: $n = -7$

$$3n = 4n + 7$$
$$3(\quad) = 4(\quad) + 7$$
$$3(-7) = 4(-7) + 7$$
$$-21 = -28 + 7$$
$$-21 = -21 \checkmark$$

This check requires us to work on both sides of the equals sign at once. We know that our answer is correct, because the numbers at the end are the same.

Example

Solve for *j*. $\dfrac{4}{j} = 2$

Solution

Remember that we must multiply by the denominator of a fraction to undo it. This means that we will start by multiplying both sides by *j*.

$$j \cdot \frac{4}{j} = 2 \cdot j$$

$$4 = 2j$$

$$\frac{4}{2} = \frac{2j}{2}$$

$$2 = j$$

<u>*Check*</u>: $2 = j$

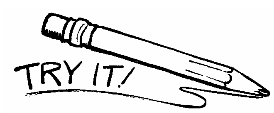
$$\frac{4}{j} = 2$$

$$\frac{4}{(\ )} = 2$$

$$\frac{4}{(2)} = 2$$

$$2 = 2$$

✓



3. Solve for each variable and check.

a) $4n + 2 = 34$

b) $9 + 2a = 25$

c) $-n - 6 = 50$

d) $2 = 6x - 10$

e) $\dfrac{c}{2} + 3 = 7$

f) $4 = -2v - 10$

g) $25g - 17 = 183$

h) $3n + 2 = 2n - 1$

i) $4 = 3x - 8$

j) $\frac{15}{x} = 3$

Sometimes you will be given equations with more than one variable, and you will be asked to substitute a number for one of the variables.

Example

In the equation $y = -8x + 3$, find the value of *y* when *x* = 1.

Solution

To answer this question, we need to substitute the value 1 in for *x*.

$y = -8x + 3$	Rewrite the equation.
$y = -8(\ ) + 3$	Substitute 1 for *x*.
$y = -8(1) + 3$	Use PEMDAS to simplify.
$y = -8 + 3$	
$y = -5$	

Example

If *m* = 4 and *a* = 5, find the value of *y* in the equation $y = 4m + a^2$.

Solution

We must substitute 4 for *m* and 5 for *a*.

$y = 4m + a^2$

$y = 4(\ \ ) + a^2$ If it helps you, only substitute one variable at a time.

$y = 4(4) + a^2$

$y = 4(4) + (\ \ )^2$

$y = 4(4) + (5)^2$

$y = 4(4) + 25$

$y = 16 + 25$

$y = 41$

Example

For the equation $d = rt$, find *d* when $r = 25$ and $t = 6$

Solution

Remember, when letters are together with no signs in between them, they are being multiplied.

$d = rt$ really means

$d = r \bullet t$

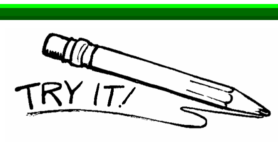
Now we substitute 25 for *r*, and 6 for *t*.

$d = r \bullet t$

$d = (\ \ )\bullet(\ \ )$

$d = (25)\bullet(6)$

$d = 150$



4. When $z = 2$, find the value of $4z - 3$.

5. When $h = 5$ and $a = 2$, find $3h + ha$.

6. When $x = 3$, what is the value of $\frac{6}{x^2}$?

7. If $a = -12$, simplify $a^2 + 5a - 24$.

8. Is the equation $a^2 + b^2 = c^2$ true when $a = 1$, $b = 2$, and $c = 3$?

☑ Review

1. Highlight each step of the "To solve for a variable" Algorithm.

2. Highlight the Objectives.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems
Math On the Move Lesson 12

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 12, Set A and Set B.

Set A

1. Simplify each algebraic expression

a) $2r + 4r - r$ b) $7a + 1 - 2a + 2$

c) $4x + y - 3x + 2y$ d) $h - 4h + 2 - 3$

2. Solve for each variable and check.

a) $6x + 2x = 32$ b) $12y + 2 = 26$

c) $2 = \frac{10}{t}$ d) $8p = 7p - 1$ e) $-2k - 4 = -30$

3. When $p = 12$, what is the value of $2p + 11$?

4. If $a = 7$ and $d = -6$, how much is $2a + 4d$?

5. In the equation $d = b^2 - 4ac$, find *d* when $b = 9$, $a = 2$ and $c = 3$

Set B

1. Here's an equation to solve that will require more than two steps!

Solve for *x*. $3x - 4 = x + 2$

2. Simplify the following expression. (*Hint:* Simplify the top first.)

$$\frac{3a + 2 - 2a - 2}{a}$$

ANSWERS TO TRY IT

1. a) $7m$ b) $2x$ c) a d) $11w - 4$

   e) $8s$ f) $10z + 4$ g) $3m + 3$

2. $4x + 2$

3. a) $n = 8$ b) $a = 8$ c) $n = -56$

   d) $x = 2$ e) $c = 8$ f) $v = -7$

   g) $g = 8$ h) $n = -3$ i) $x = 4$

   j) $x = 5$

4. $4z - 3 = 4(2) - 3 = 5$

5. $3h + ha = 3(5) + (5)(2) = 25$

6. $\frac{6}{x^2} = \frac{6}{3^2} = \frac{6}{9} = \frac{2}{3}$

7.

$$a^2 + 5a - 24$$
$$= (-12)^2 + 5(-12) - 24$$
$$= 144 - 60 - 24$$
$$= 60$$

8. No

$$a^2 + b^2 = c^2$$
$$1^2 + 2^2 = 3^2$$
$$1 + 4 = 9$$
$$5 \neq 9$$

NOTES

End of Lesson 12