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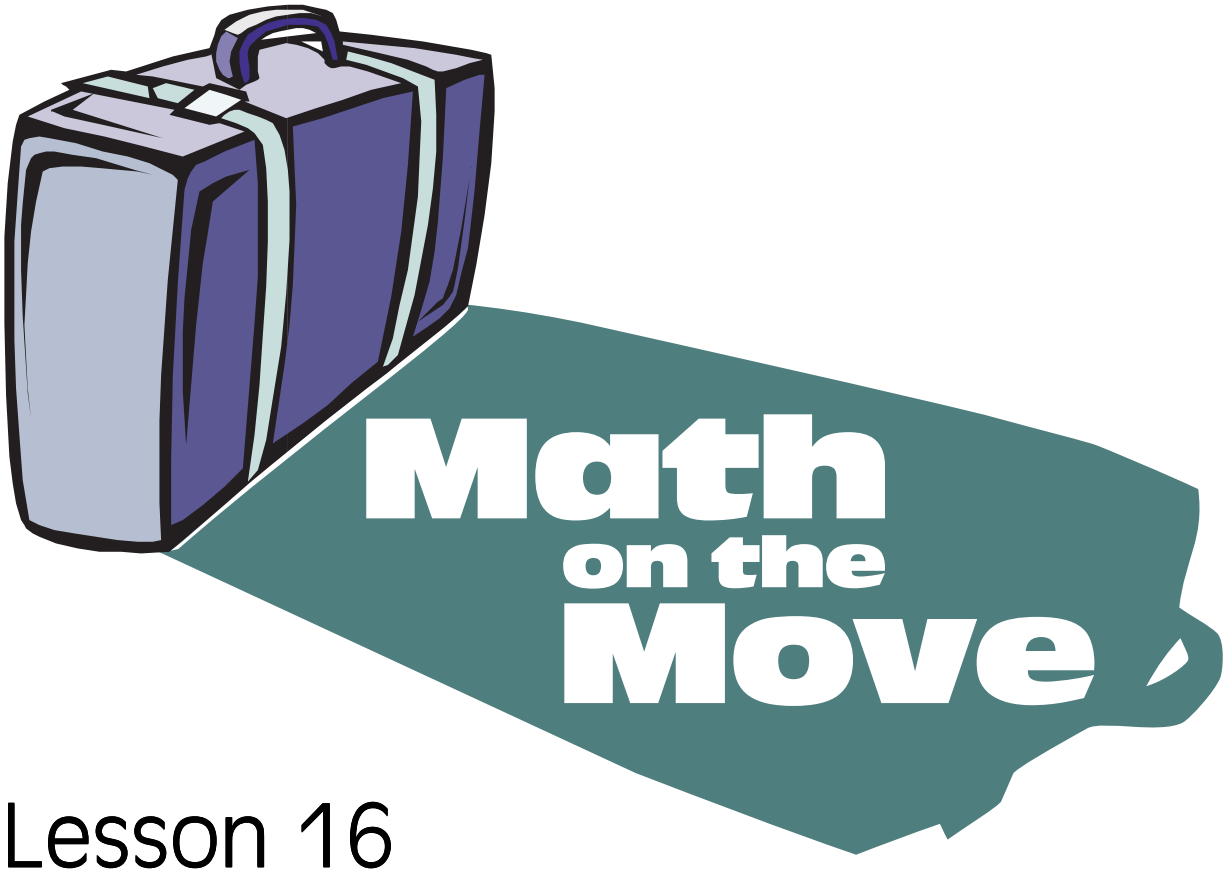

Lesson 16
Proportions

Objectives

- Understand what a proportion is
- Solve word problems using proportions

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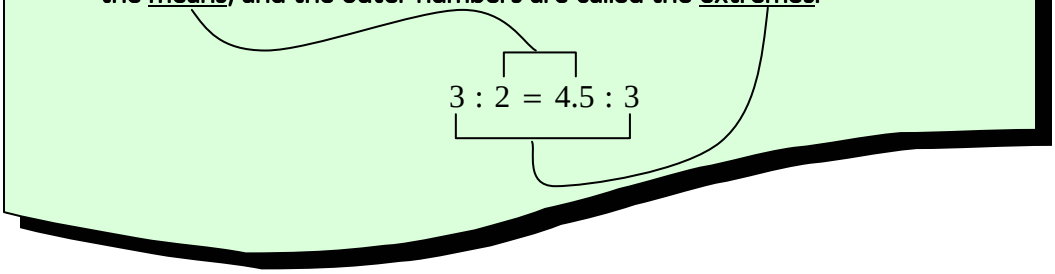
Raúl is having a surprise birthday party for his friend, Aimee. He goes to the store to buy some avocados to make guacamole dip. He sees a sign that says, "2 avocados for $3." If he wants to spend $15 dollars on avocados, how many can he buy?

We can solve this problem by using **proportions**.

- A **proportion** is a set of ratios that are equal to each other. In a proportion, the product of the means equals the product of the extremes. The following are examples of proportions.

$$\frac{5}{2} = \frac{15}{6} \qquad 3 : 2 = 4.5 : 3 \qquad \frac{25}{40} = \frac{x}{50}$$

- When a proportion is written using colons, the inner numbers are called the means, and the outer numbers are called the extremes.



The second example shows that these ratios do not always have to use integers. In the third example, there is a variable used in the ratio on the right. Since the fractions are equal, we are able to figure out what x is. This will be very useful.

Example

Solve for x. $\frac{15}{30} = \frac{7}{x}$

Solution

We need to find the value of x that will make both of those fractions equal. In our lesson on equivalent fractions, we found equivalent forms of a fraction by either reducing the fraction or finding multiples of the numerator and denominator. In this case, we will do both.

First, notice that the fraction $\frac{15}{30}$ is not in lowest terms. So, we will reduce this fraction by dividing the top and bottom by the GCF.

$$\frac{15 \div 15}{30 \div 15} = \frac{1}{2}$$

Now that we have reduced our fraction, we can take equivalent forms of it to find an equivalent fraction with 7 in the numerator.

$$\frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$

So, $x = 14$.

$$\frac{15}{30} = \frac{7}{14}$$

This was not too difficult to solve, because we found equivalent forms of the fraction that worked. What if we were given a fraction where finding equivalent forms does not work?

Example

Solve for x. $\frac{25}{40} = \frac{x}{50}$

Solution

Let's try our first method where we reduce the fraction into lowest terms.

$$\frac{25 \div 5}{40 \div 5} = \frac{5}{8}$$

Now, multiply the numerator and denominator by the same number until we get 50 in the denominator. We run into the following problem.

$$\frac{5 \times 6}{8 \times 6} = \frac{30}{48}$$ **Too small!**

$$\frac{5 \times 7}{8 \times 7} = \frac{35}{56}$$ **Too large!**

With this method, we cannot find an equivalent form of $\frac{5}{8}$ with a 50 in the denominator.

Let's use what we know about equivalent fractions to make a new method to solve proportions – the cross product. We can use the cross product, because we know the two fractions are equal (by the definition of a proportion).

$$\frac{25}{40} = \frac{x}{50}$$

$$1250 = 40x$$

Think Back

The cross product method is used to prove two fractions are equivalent. If $\frac{1}{2} = \frac{2}{4}$, *then* $(1 \times 4) = (2 \times 2)$.

Now we can solve for x as we did with all other equations.

$$\frac{1250}{40} = \frac{40x}{40}$$

$$31.25 = x$$

So, $x = 31.25$. We can check our answer by plugging it back into the original proportion.

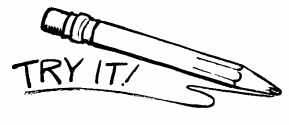
$$\frac{25}{40} = \frac{31.25}{50}$$

Remember that fractions are another way of writing division.

$$25 \div 40 = 31.25 \div 50$$

$$0.625 = 0.625 \quad \surd$$

Even though the solution for this proportion was a decimal, it is the only number that works. We could write our answer as a mixed number or even an improper fraction, but those would look confusing in a proportion.



Solve the following proportions for the given variable.

Then check your answer.

1. $\frac{8}{10} = \frac{x}{35}$

2. $\frac{27}{81} = \frac{3}{x}$

3. $\frac{a}{8} = \frac{15}{20}$

4. $\frac{7}{15} = \frac{z}{36}$

5. $\frac{10}{y} = \frac{25}{12}$

6. $\frac{x}{20} = \frac{2x + 6}{55}$

Did you check your answers? For the last problem in the Try It, make sure that you do not plug in the same number for both numerators.

Calculator Tip

The basic calculator cannot show a repeating decimal. For example, if you plug in $1 \div 3$, *the calculator will display*

.333333

If you plug in $2 \div 3$, *the calculator will display*

.666667

These are both repeating decimals. The calculator uses an approximation, which is why the calculator puts a 7 at the end of the repeating sixes (it's rounding up).

Let's look at the problem from the beginning of the lesson.

Example

Raúl is having a surprise birthday party for his friend, Aimee. He goes to the store to buy some avocados to make guacamole dip. He sees a sign that says, "2 avocados for $3." If he wants to spend $15 dollars on avocados, how many can he buy?

Solution

There is a proportion hidden within this word problem. The third sentence says, "2 avocados for $3." Raúl can get 2 avocados for every $3 he spends. In other words, the ratio of avocados to money is 2 to 3. Since this ratio is proportional, we can say the following.

$$\frac{\text{avocados}}{\text{money}} = \frac{2}{3} = \frac{x}{15}$$

We know that he can buy 2 avocados with $3. Since we do not know how many he can buy with $15, we use x to represent the number of avocados.

$$\frac{2}{3} = \frac{x}{15}$$

$$\frac{30}{3} = \frac{3x}{3}$$

$$10 = x$$

So, Raúl can buy 10 avocados with $15.

Example

A metal rod 2 ft. long is 5 lbs. If the ratio of length : weight is proportional, how heavy is a 5 ft. metal rod?

Solution

This problem should not be too difficult because it tells us which ratio is proportional. We always write proportions with fractions.

$$\frac{\text{feet}}{\text{pounds}} = \frac{2}{5} = \frac{5}{x}$$

Cross multiply.

$$\frac{2}{5} = \frac{5}{x}$$

$$2x = 25$$

Solve.

$$\frac{2x}{2} = \frac{25}{2}$$

$$x = 12.5$$

A 5 ft. rod weighs 12.5 lbs.

Solve the following proportion word problems

7. Pablo read 40 pages of a book in 50 minutes. How many pages should he be able to read in 80 minutes?

8. Jeannie takes inventory of her closet and discovers that she has 8 shirts for every 5 pair of jeans. If she has 40 shirts, how many pairs of jeans does she have?

9. Diego found out that after working for 9 months, he had earned 6 days of vacation time. How many days will he have earned after working for two years?

Proportions can also be useful for rate problems.

Example

Joel and José are planning a road trip from San Diego, CA to the Grand Canyon. The trip is approximately 553 miles. If their average speed is 70 mph, how long will it take them to drive to the Grand Canyon?

Solution

We will use the formula for average speed to solve this problem.

$$70 = \frac{553}{x}$$

Since we do not know the time, we will use the variable x to represent time. This may not look like a proportion but we can change it to look like one.

$$\frac{70}{1} = \frac{553}{x}$$

Think Back

The formula for average speed is,

$$\text{speed} = \frac{\text{distance}}{\text{time}}.$$

FACT

Any integer can be turned into a fraction by using that number as the numerator and using 1 as the denominator.

Now that this looks like a proportion, we can solve for the variable by cross multiplying.

$$\frac{70}{1} = \frac{553}{x}$$

$$70x = 553$$

Solve.

$$\frac{70x}{70} = \frac{553}{70}$$

$$x = 7.9$$

We got 7.9 for *x*, but what does *x* represent? When we set up the problem, we used *x* to represent time because that was our unknown. The last thing we need to do is attach units to our answer. When we solved the problem, we took the distance (553 miles) and divided it by the speed (70 mph).

$$\text{miles} \div \frac{\text{miles}}{\text{hours}}$$

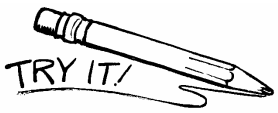
This is the same as

$$\cancel{\text{miles}} \times \frac{\text{hours}}{\cancel{\text{miles}}} = \text{hours}$$

So, our answer is 7.9 hours.

Think Back

When we divide by a fraction, it is the same as multiplying by its reciprocal. $a \div \frac{b}{c} = a \times \frac{c}{b}$



Solve the following proportion word problems.

10. Daniela is traveling from San Antonio to Houston – a distance of 200 miles. If she has an average speed of 75 mph, how long will it take for her to get to Houston.

11. Rafaela has a car that can drive 28 miles per gallon of gas. Her car can hold 12 gallons of gas. If she plans on driving from Chicago to Denver (1008 miles), how many times will she need to stop for gas?

12. Dion drives 125 miles in $2\frac{1}{2}$ hours. At the same rate, how far will he be able to travel in 6 hours?

☑ Review

1. Highlight the definition of proportion.

2. Highlight the "Fact" boxes.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 16

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 16, Set A and Set B.

Set A

Solve the proportions for the given variable.

1. $\frac{b}{2} = \frac{35}{7}$ 2. $\frac{90}{w} = \frac{10}{11}$ 3. $\frac{12}{15} = \frac{z}{40}$ 4. $\frac{a+5}{20} = \frac{a}{15}$

Set B

1. A piece of cable 8.5 cm long weighs 52 grams. What will a 10 cm length of the same cable weigh? (Round to the nearest hundredth)

2. The ratio of men to women at a class is 6 to 5. How many female students are there if there are 3600 men?

3. Hurricane Katrina dropped about 14 inches of rain over a two-day period. How much rain is this per hour? (Round your answer to the nearest tenth.)

   (*Hint*: How many hours is two days?)

4. A doctor sees each of her patients for 25 minutes during a typical appointment. How many patients can she see in a typical $7\frac{1}{2}$ hour day? (*Hint*: Make sure units of time are the same.)

1. $x = 28$ $\frac{8}{10} = .8$ $\frac{28}{35} = .8$

2. $x = 9$ $\frac{27}{81} = .\overline{3}$ $\frac{3}{9} = .\overline{3}$

3. $a = 6$ $\frac{6}{8} = .75$ $\frac{15}{20} = .75$

4. $z = 16.8$ $\quad \frac{7}{15} = .4\overline{6}$

$$\frac{16.8}{36} = .4\overline{6}$$

5. $y = 4.8$ $\quad \frac{10}{4.8} = 2.08\overline{3}$

$$\frac{25}{12} = 2.08\overline{3}$$

6. $\frac{x}{20} \times \frac{2x+6}{55}$ $\qquad \frac{8}{20} = .4$

$$55x = 20(2x+6) \qquad \frac{22}{55} = .4$$

$$55x = 40x + 120$$

$$-40x \quad -40x$$

$$\frac{15x}{15} = \frac{120}{15}$$

$$x = 8$$

$$2x + 6 = 22$$

7. $\frac{40 \text{ pgs.}}{50 \text{ min.}} \times \frac{x}{80 \text{ min.}}$

$$\frac{3200}{50} = \frac{50x}{50}$$

$$64 \text{ pages} = x$$

8. $\frac{8 \text{ shirts}}{5 \text{ jeans}} = \frac{40 \text{ shirts}}{x}$

$$\frac{8x}{8} = \frac{200}{8}$$

$$x = 25 \text{ pairs of jeans}$$

9. 2 years = 24 months

$$\frac{9 \text{ months}}{6 \text{ days}} = \frac{24 \text{ months}}{x}$$

$$\frac{9x}{9} = \frac{144}{9}$$

$$x = 16 \text{ days}$$

10. $\frac{75 \text{ miles}}{1 \text{ hr.}} = \frac{200 \text{ miles}}{x}$

$$\frac{75x}{75} = \frac{200}{75}$$

This is the same as 2 hr. 40 min.

$$x = 2.\overline{6} \text{ hr.}$$

11. $\frac{28 \text{ miles}}{1 \text{ gallon}} = \frac{1008 \text{ miles}}{x}$

$$\frac{28x}{28} = \frac{1008}{28}$$

$$x = 36 \text{ gallons}$$

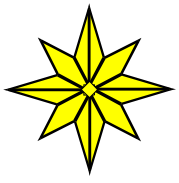
$$\frac{36 \text{ gallons}}{12} = 3 \text{ stops for gas}$$

12. $\frac{125 \text{ miles}}{2.5 \text{ hr.}} = \frac{x}{6 \text{ hr.}}$

$$\frac{750}{2.5} = \frac{2.5x}{2.5}$$

$$300 \text{ miles} = x$$

NOTES



End of Lesson 16