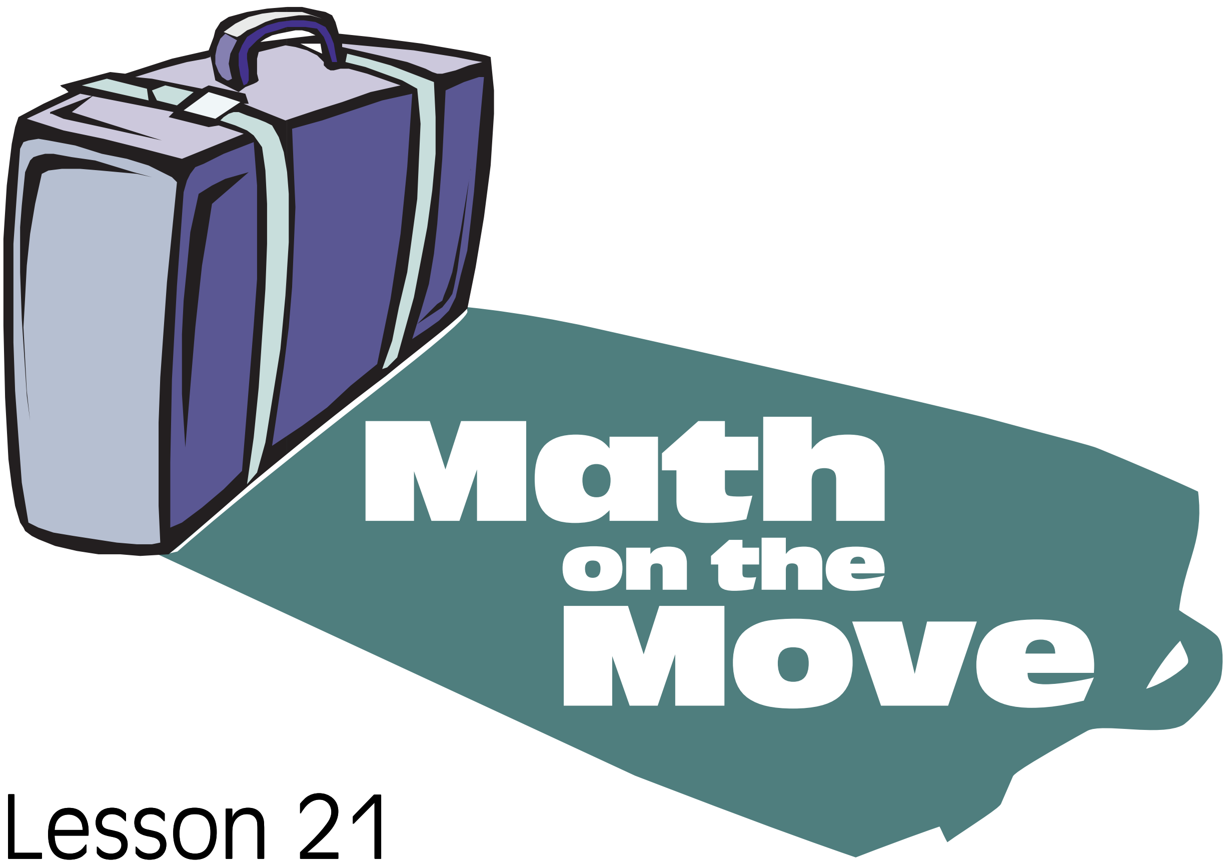
Student Name: ______________________

Date: ______________________

Contact Person Name: ______________________

Phone Number: ______________________



Lesson 21

Circles

Objectives

- Understand the concepts of radius and diameter
- Determine the circumference of a circle, given the diameter or radius
- Determine the area of a circle, given the diameter or radius

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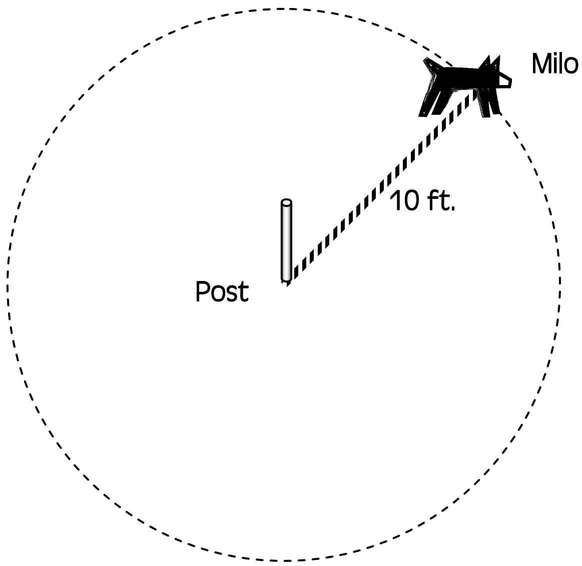
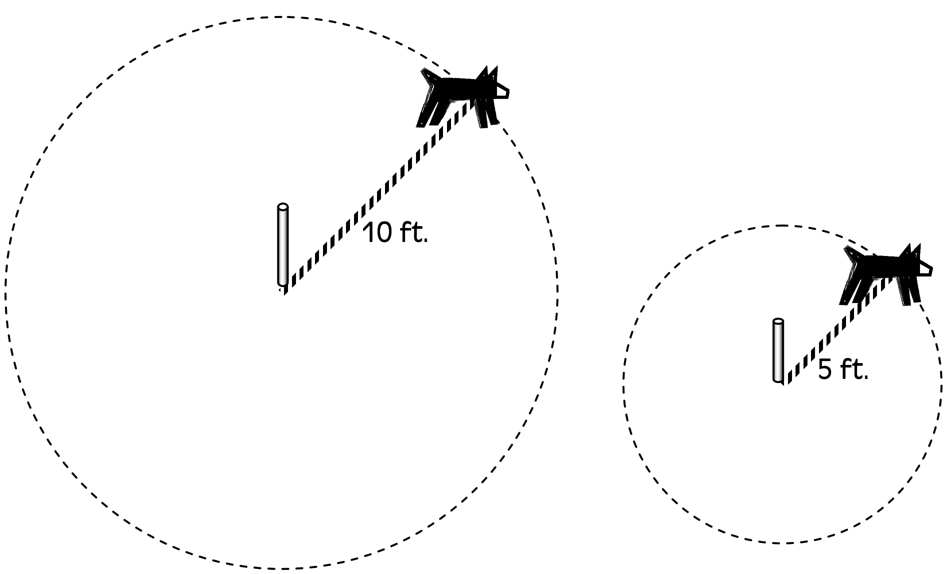
Imagine you have just gotten a new dog, Milo. You know that you cannot let him roam free in the yard because he would run away. You get a 10-ft. line of rope, and tie Milo to a post in the middle of the yard. After you tie him up, he stretches the rope out as far as it goes and walks around.



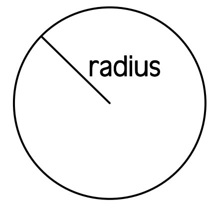
As you can see, Milo has walked in a circle. A circle is not a polygon, because it is not made of straight line segments. In a polygon, line segments are used to determine the dimensions. A circle has no sides. How do we find the dimensions of a circle?

What's interesting about circles is that they are all the same shape. (Therefore, they are all similar to one another!) They are all round. The only difference between circles is their size. Thinking of circles in this way, what would happen if you tied Milo up with a 5 ft. length of rope?



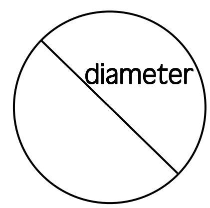
If we tie Milo up with a shorter piece of rope, the circle becomes smaller. So, the dimension that changes the size of the circle is the length of the rope. The length of the rope represents the **radius** of the circle.

- The **radius** of a circle is the distance from the center of the circle to the edge of the circle. The plural form of "radius" is *radii* (ray-dee-eye).



The radius of the larger circle is 10 ft., and the radius of the smaller circle is 5 ft. The **diameter** of a circle is directly related to its radius.

- The **diameter** is the distance across a circle that runs through the center of the circle.



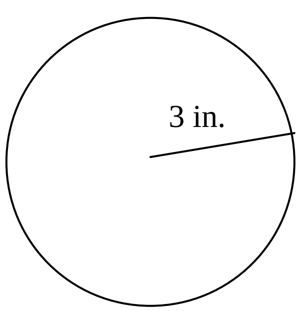
The diameter is twice the length of the radius.

If d = diameter and r = radius, then $d = 2r$.

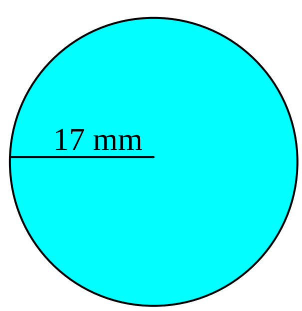
Example

Find the length of the diameter of the following circles.

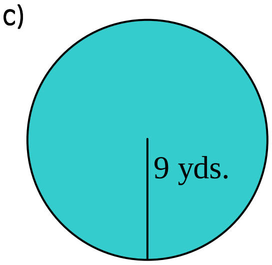
a)



b)



c)



Solution

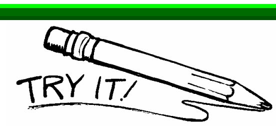
To solve each of these problems, remember that the diameter is <u>always</u> twice the radius.

$$d = 2r$$

a) r = 3 in., so $d = 2(3\text{ in.}) = 6\text{ in.}$

b) r = 17 mm, so $d = 2(17\text{ mm}) = 34\text{ mm}$.

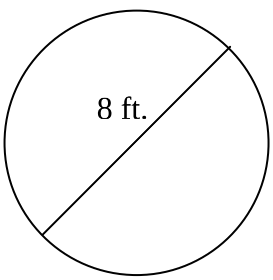
c) r = 9 yds., so $d = 2(9\text{ yds.}) = 18\text{ yds.}$



1. Find the diameter of a circle with <u>radius</u> 2.5 units.

Find the radius of the following circles.

2.



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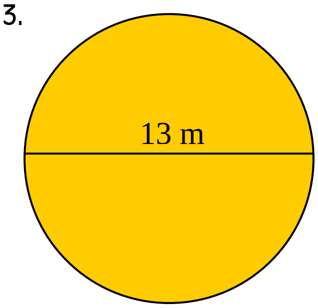

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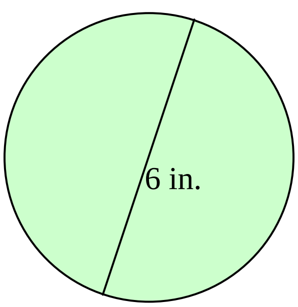
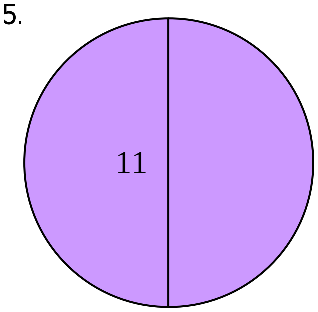
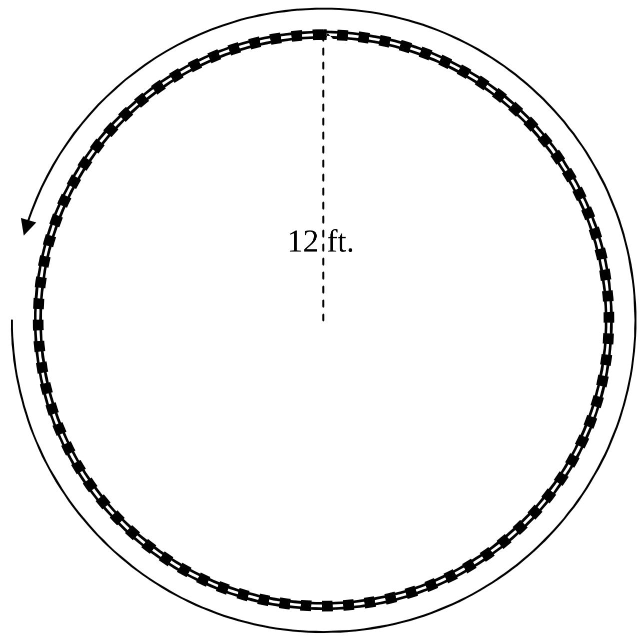


5.



After several days, you feel bad for tying your dog, Milo, up in the back yard. You decide to build a circular pen for him to play in. He was tied up with a 10-ft. length of rope. You want to build the pen slightly larger. You want the radius of the pen to be 12 ft. You need to figure out the length of fence you will need to build the pen.

The length of fence needed will be the distance all the way around the circular pen. The distance all the way around a circle is called the **circumference**.



- The **circumference** of a circle is the length around the outer edge of a circle. The circumference of a circle is similar to the perimeter of a polygon.

To find the circumference, we use a formula directly related to the radius and diameter. Remember: when Milo's rope was longer, the circle was bigger. When the radius of a circle increases, so does the circumference. Also, remember that the diameter is directly related to the radius. When the radius increases, so does the diameter. The formula we use for the circumference of a circle is:

$$C = \pi d$$

C = circumference; π = pi; d = diameter.

- The symbol π is actually the Greek letter, **pi** (rhymes with "sky"). In math, it denotes the number you get when a circle's circumference is divided by its diameter. Pi is not a variable, it is always the same number.
    - $\pi \approx 3.14 \approx \frac{22}{7}$

Pi is a number that never ends and never repeats. The first 25 decimal places of pi are

3.1415926535897932384626433…

For our purposes, we use the values 3.14 and $\frac{22}{7}$ *to estimate pi.*

You want to build a circle with a radius of 12 ft.

How can we find the circumference of a circle if we only have the radius?

Since the radius is 12 ft., the diameter is 24 ft. $(d = 2r = 2(12) = 24 \text{ ft.})$

With the given information, we can find the circumference of the pen.

$$C = \pi d = 24\pi \approx 24(3.14) \approx 75.36 \text{ ft.}$$

The circumference around the dog's pen will be about 75.36 ft. This means you will need 75.36 ft. of fencing.

Example

Madeline is building a circular swimming pool in her back yard. She wants the diameter of the pool to be 30 ft. If pool walling costs $12 per foot, how much will it cost Madeline to put up a pool?

Solution

First, we must find how many feet of pool wall Madeline needs to build her circular pool. Since we know the diameter of her pool, we can determine the circumference of the pool.

$$C = \pi d = 30\pi$$

$$30(3.14) = 94.2 \text{ ft.}$$

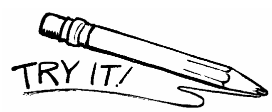
The circumference tells us how many feet go around the whole pool, which means we know how many feet of wall she needs. Madeline needs 94.2 ft. of wall to go around her pool.

Next, we need to find how much 94.2 ft. of wall will cost. If the wall costs $12 per foot, we need to multiply the price per foot by the total number of feet.

$$\$12 \times 94.2 = \$1{,}130.40$$

It will cost Madeline $1,130.40 to build her round pool.

Sometimes drawing a picture makes it easier to solve these problems. Try some circumference problems on your own.



6. Miguel ran around a circular track 5 times. If the radius of the circle is 60 meters, how far did he run? (Use $\pi \approx 3.14$)

7. Ana just bought a new bike. The tires on her bike have a diameter of 55 cm. Ana wants to know what the circumference of the tire is. (Use $\pi \approx \frac{22}{7}$) Round to the nearest <u>hundredth</u>.

8. Tomás just found that the circumference of the clock in town is 30 meters. Tomás wants to know what the diameter of the clock is. (Use $\pi \approx 3.14$) Round to the nearest <u>tenth</u>.

Let's go back to the story of Milo, the dog. After you finish building the circular pen for Milo, you wonder how many square feet of land are inside it. Now, you are looking for the <u>area</u> of the circle.

As with circumference, area is dependent on the radius of the circle. If the radius of the circle increases, so does the area.

The area formula for circles is

$$\text{Area} = \text{pi} \times (\text{radius})^2 \qquad \text{or} \qquad A = \pi r^2$$

So, to find the area of the circular pen built for Milo, you need the radius. You made the pen with a radius of 12 ft., so plug that into the formula.

$$A = \pi r^2 = \pi(12)^2 = 144\pi$$

$$144(3.14) = 452.16 \text{ sq. ft.}$$

The area of Milo's pen is 452.16 sq. ft.

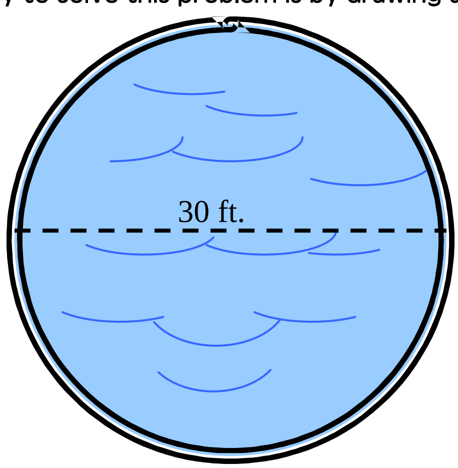
> ***Think Back***
>
> *When we wrote numbers next to variables, we wrote the number first. This rule applies for a number next to pi, as well.*

Example

Madeline built her swimming pool with a diameter of 30 ft. She wants to buy a pool cover, so she needs to know how many square feet the pool's surface is.

Solution

The easiest way to solve this problem is by drawing a picture.



She must find the <u>area</u> of this circle in order to get the right sized pool cover.

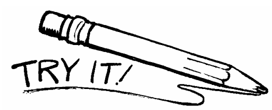
First, she needs to find the radius of the pool. Since the diameter of the pool is 30 feet, then the radius is half of that, or 15 ft.

Then, she plugs the radius into the area formula for circles.

$$A = \pi r^2 = \pi(15)^2 = 225\pi$$

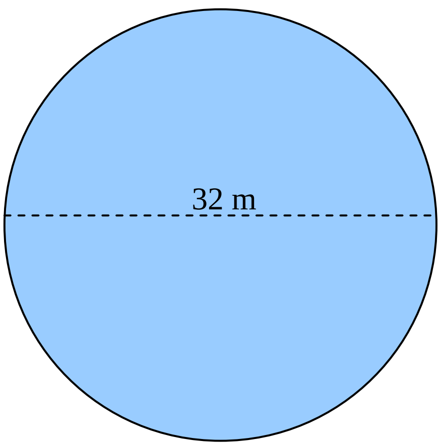
$$225(3.14) = 706.5 \text{ sq. ft.}$$

So, Madeline's pool is 706.5 sq. ft. Now she knows what size pool cover to get.

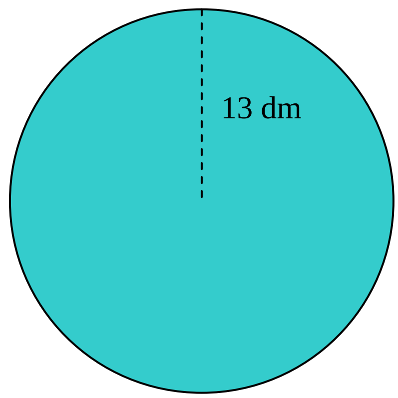


Find the area of the following circles. Use $\pi \approx \frac{22}{7}$, and round to the nearest digit.

9.



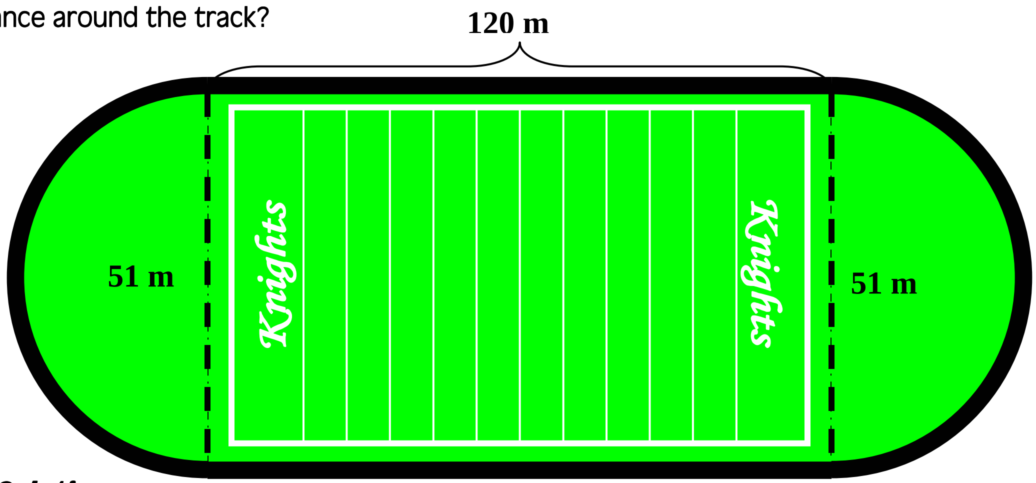
10.



Sometimes, math problems involve a combination of shapes and circles.

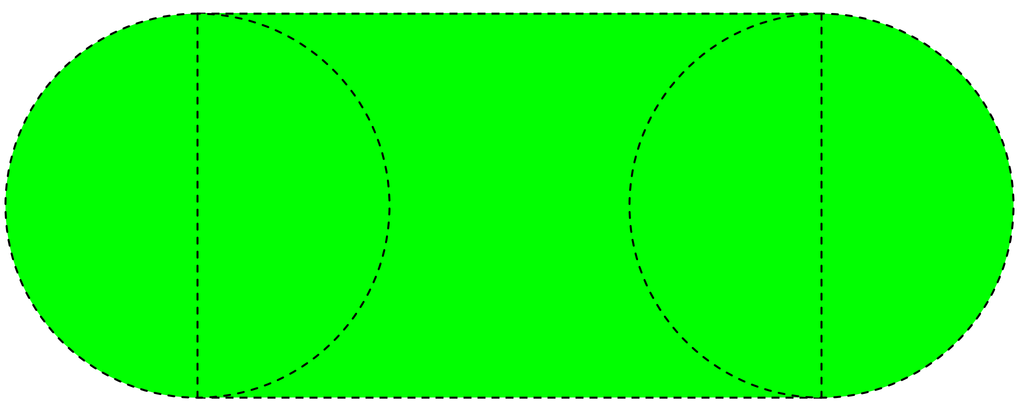
Example

The high school track goes around the football field as shown below. Given the dimensions, what is the distance around the track?



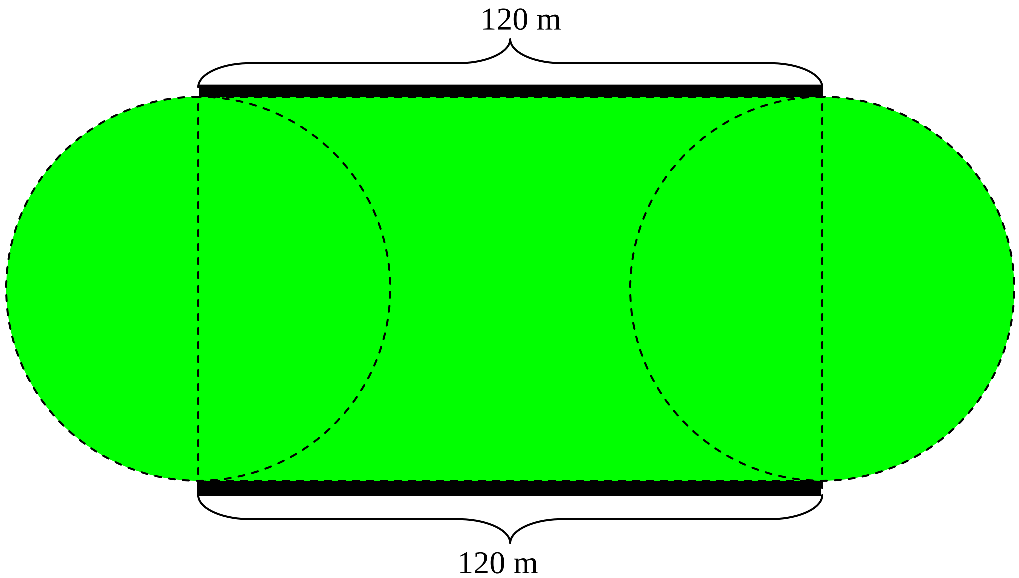
Solution

To solve this problem, we must figure out what shape we are looking at. If we break apart the diagram into familiar shapes, we get this:



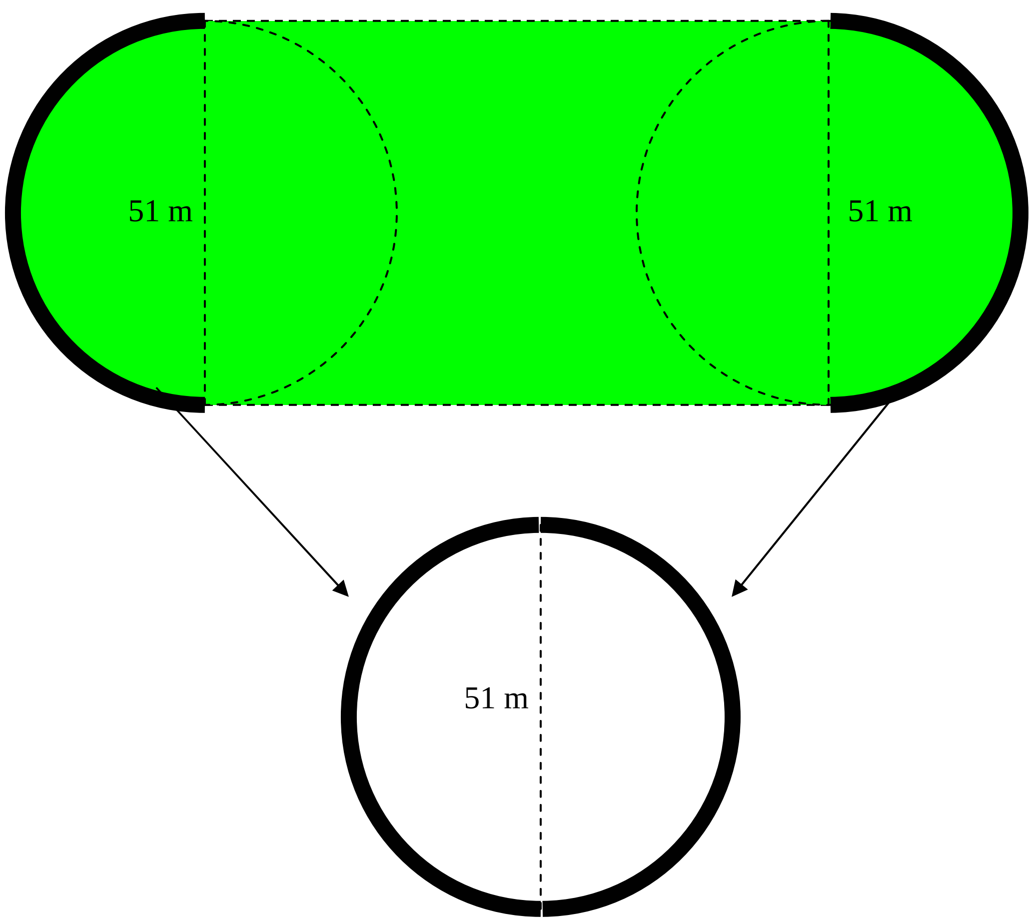
As we can see, the track is made of a rectangle and two semicircles.

First, we will find the distance around the rectangular portion of the track.



The distance along the rectangular portion of the track is 120 + 120 = 240 meters.

Next, we will find the distance around the two semicircles. Because the two semicircles have the same diameter, we can put them together to make a full circle.



The circumference of the circle, then, is

$$C = \pi d = 51\pi$$

$$51(3.14) = 160.14 \text{ m}$$

If the distance around the circular part of the track is 160.14 m, and the distance around the rectangular part of the track is 240 m, then the total distance is

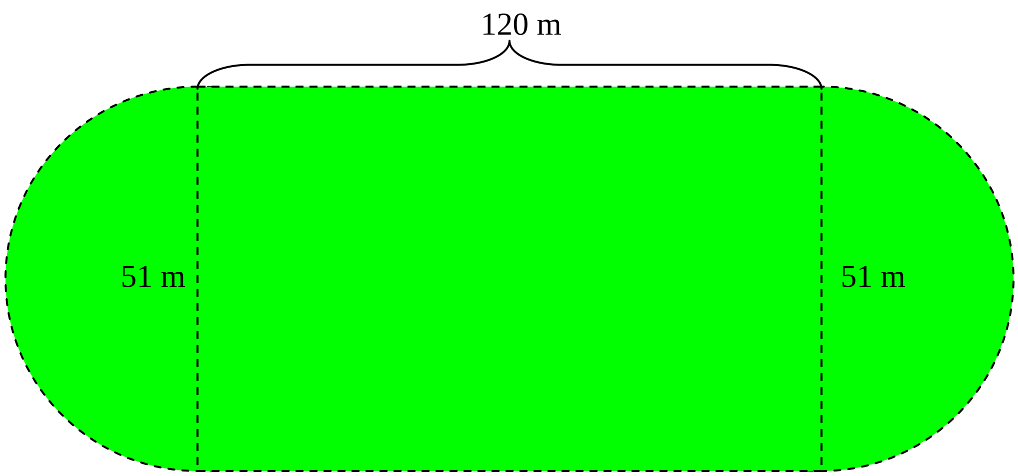
$$240 + 160.14 = 400.14 \text{ m}.$$

Example

Suppose your friend Jorge was asked to cut the grass inside the track. How many square meters of grass does he have to cut?

Solution

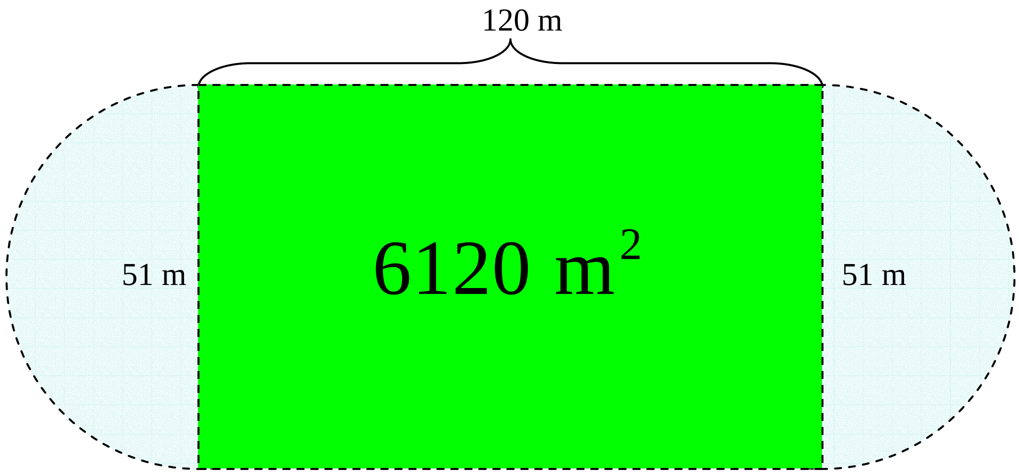
To solve this problem, we have to break down the area into familiar shapes again.



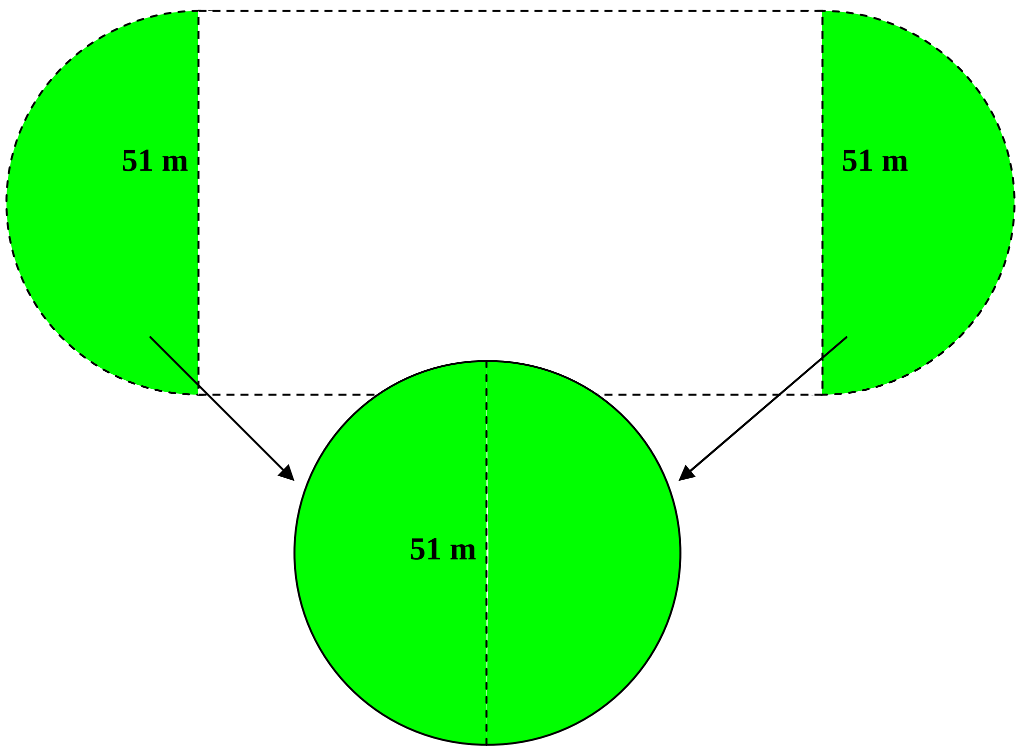
We must find the area of the shaded region, which is made of a rectangle and two semicircles.

First, we find the area of the rectangular portion.

The rectangle has a base of 120 meters and a height of 51 meters, so the area of the rectangle is 6120 m^2. $51 \times 120 = 6120 \text{ m}^2$



Next, we find the area of the two semicircles. Again, the two semicircles have the same diameter. We can put them together to make a full circle.



To find the area of this circle, we need the radius first. If the diameter is 51 meters, the radius is half of that. The radius is 25.5 meters. We plug the radius into the area formula for circles.

$$A = \pi r^2 = \pi(25.5)^2 = 650.25\pi$$

$$650.25(3.14) = 2041.785 \text{ m}^2$$

Finally, we add the area of the rectangle to the area of the semicircles. The total area inside the track is,

$$6120 + 2041.785 = 8161.785 \text{ m}^2$$

As before, when we find the area of a figure, our answer is in square units. The distance around an object (circumference or perimeter) is just in units.

The circumference and area formulas can be very hard to remember because they both have the number pi in them. To make them easier to remember, use the saying:

"Cherry pie is delicious. Apple pies are too."

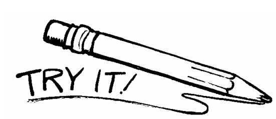
$$C = \pi d \qquad A = \pi r^2$$

When you say the two phrases together, it flows well. Say the sentences and formulas out loud a couple of times.

"Cherry pie is delicious, apple pies are too."

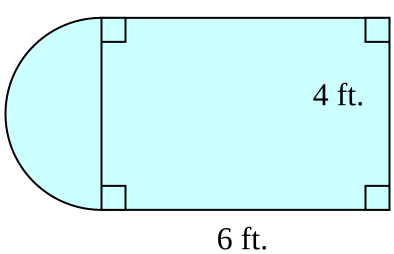
$$C = \pi d \qquad A = \pi r^2$$

Try these problems. Add the areas of familiar objects.

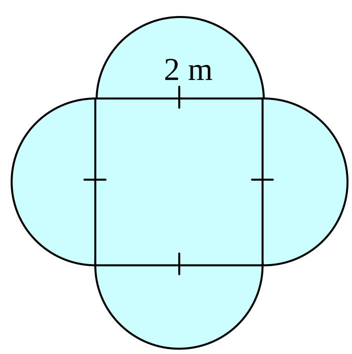


Find the area of the following objects. Use $\pi = 3.14$

11.



12.



Review

1. Find the definitions of the following terms, and highlight them in the lesson.
    a. radius
    b. diameter
    c. circumference
    d. pi (π)
2. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

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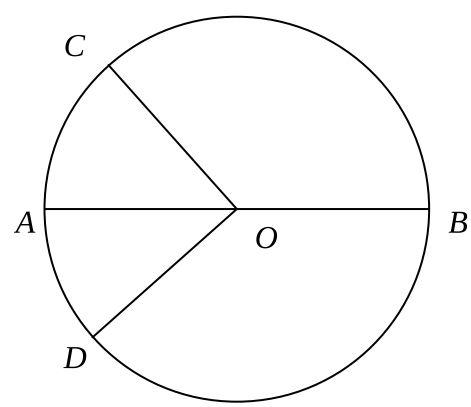
Practice Problems

Math On the Move Lesson 21

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 21, Set A and Set B.

Set A

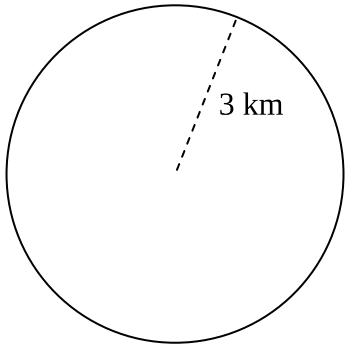


1. Using the diagram of circle *O*,
    a) name a diameter
    b) name all radii

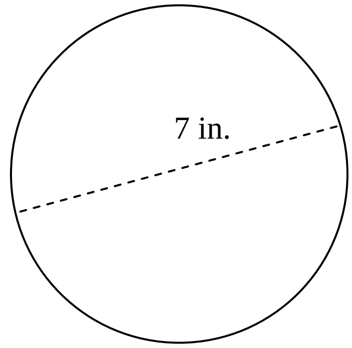
2. If the circle *O* has a radius of 6 mm. Find the length of:
    a) $\overline{AB}$
    b) $\overline{CO}$

3. Find the area and circumference of the following circles. (Round to the nearest tenth)

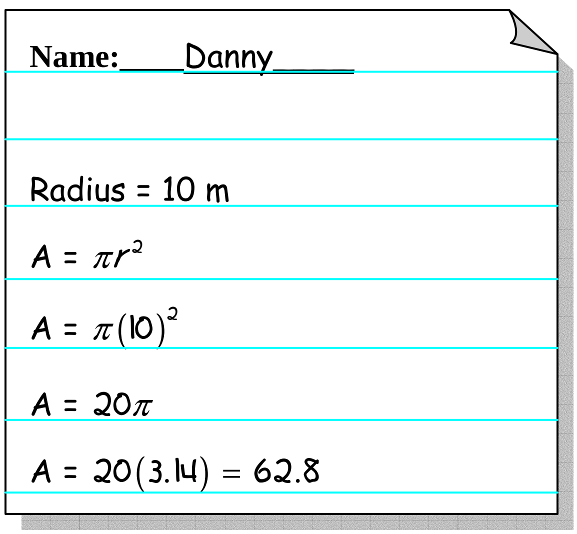
a)

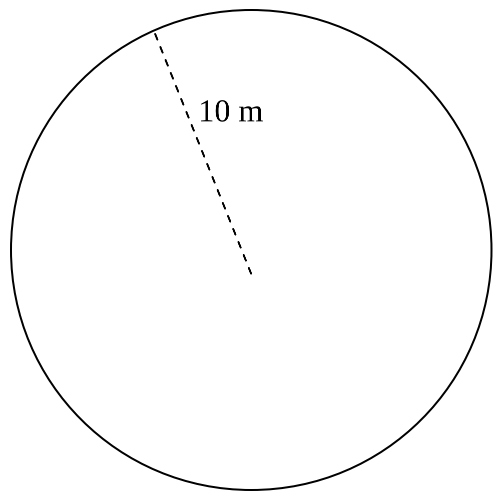


b)



4. Danny found the area of the circle, but feels his answer is not right. What did Danny do wrong? What is the actual area of the circle?

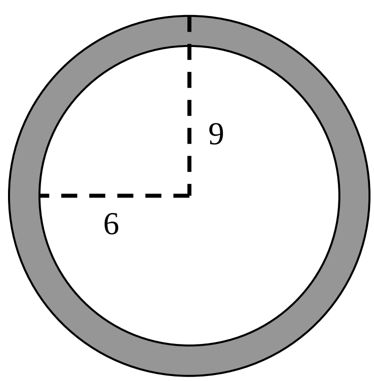




Set B

1. You own a pizza shop. A customer comes in and asks what the area of a single slice of pizza is. You know that a whole pizza has a diameter of 12 inches, and that each pizza is cut into 8 equal slices. Find the area of one slice. (*Hint*: Draw a picture to help)

2. What is the area of the shaded ring?



ANSWERS TO TRY IT

1. 5 units
2. 4 ft.
3. 6.5 m
4. 3 in.
5. 5.5 units
6. In the circle, the diameter is 120, so $\begin{array}{l} C = \pi d = 120\pi \\ 120(3.14) = 376.8 \text{ m} \end{array}$. The circumference of the track is 376.8 m. If Miguel went around 5 times, he ran $376.8 \times 5 = 1884$ m.
7. The diameter is 55, so $\begin{array}{l} C = \pi d = 55\pi \\ 55\left(\frac{22}{7}\right) = \frac{1210}{7} \approx 172.86 \text{ cm} \end{array}$
8. The circumference is given, and we are looking for the diameter. We know the formula is $C = \pi d$, and since we know the circumference, we need to divide the circumference by pi to get the diameter. $30 \div 3.14 \approx 9.6$ m
9. 805 m^2
10. 531 dm^2
11. 30.28 $ft.^2$
12. 10.28 m^2

End of Lesson 21