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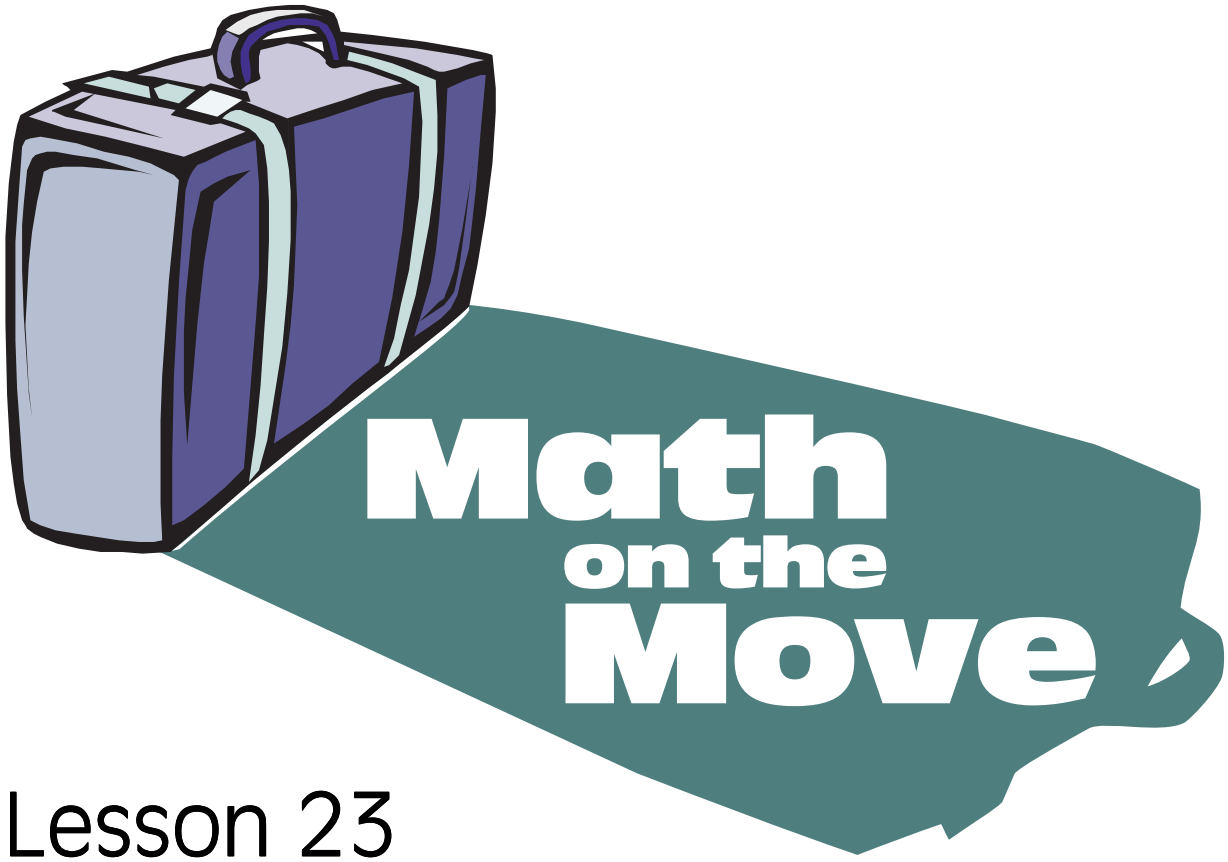

Lesson 23

Coordinate Geometry

Objectives

- Construct and label the x and y axes of a graph
- Plot a point, given its coordinates
- Identify coordinates of a plotted point
- Determine the slope and y-intercept of the graph of a line

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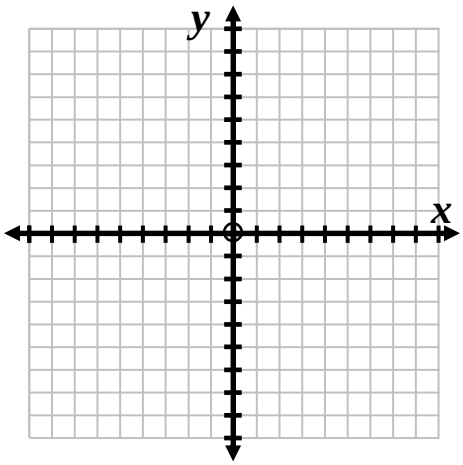
Developed by the National PASS Center under the leadership of the National PASS Coordinating Committee with funding from the Region 20 Education Service Center, San Antonio, Texas, as part of the Mathematics Achievement = Success (MAS) Migrant Education Program Consortium Incentive project. In addition, program support from the Opportunities for Success for Out-of-School Youth (OSY) Migrant Education Program Consortium Incentive project under the leadership of the Kansas Migrant Education Program.

So far, we have been talking about all sorts of geometric figures and shapes, and finding out their properties. However, we have not yet considered their positions within space. For instance, a map-maker needs to place objects precisely on a map, according to their relative position to objects around them. An architect needs to make sure the size and placement of objects is accounted for in his or her blueprints of a building. For these reasons and countless others, we often graph figures on the **coordinate plane**.



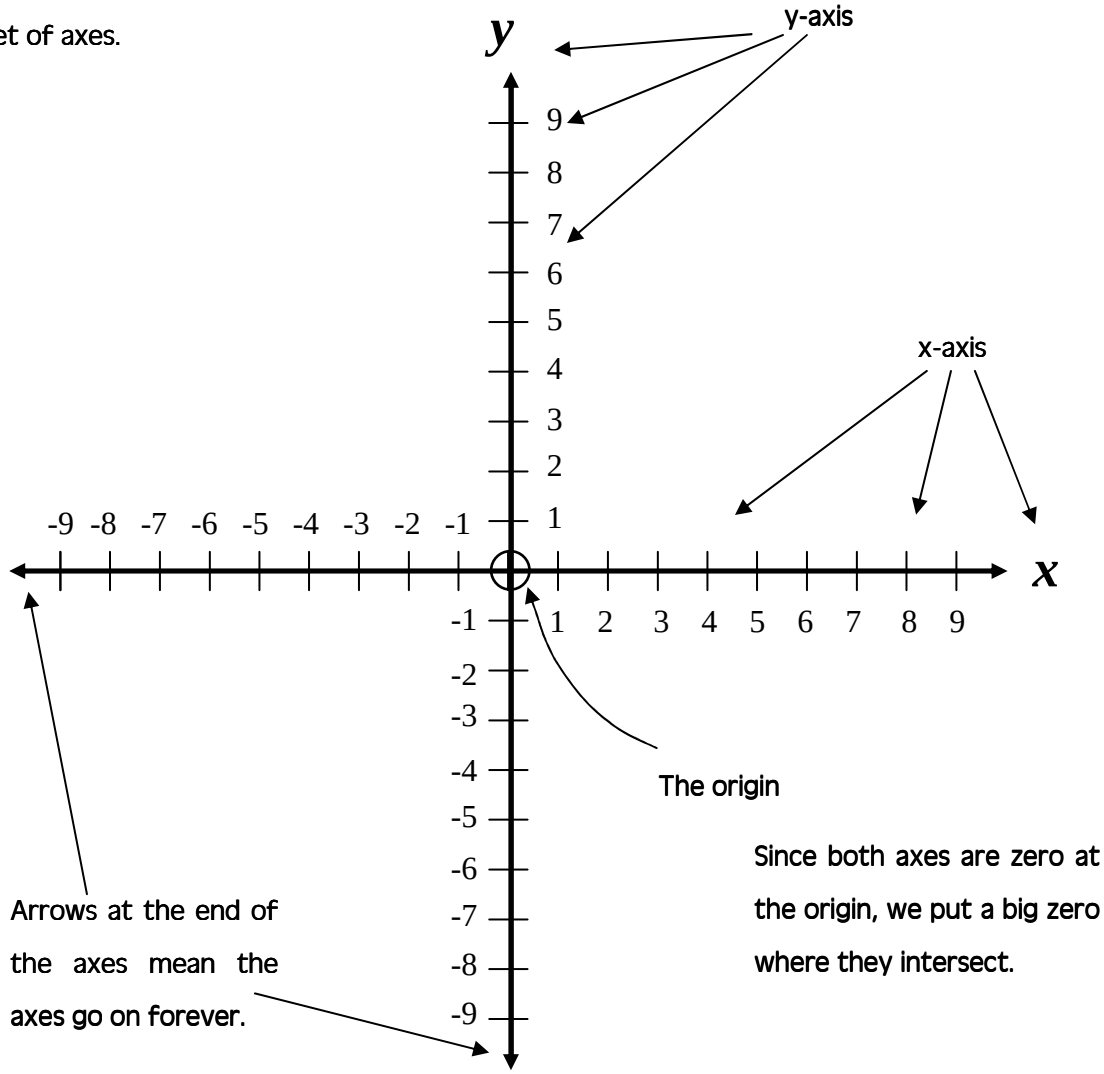
- The **coordinate plane** is used for graphing geometric figures, and finding their algebraic relationships.

The coordinate plane is broken up by two **axes** (ax-ees).

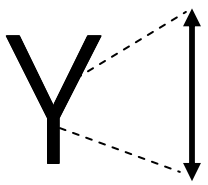
- The **x-axis** is a horizontal number line (left and right) on the coordinate plane.
- The **y-axis** is a vertical number line (up and down) on the coordinate plane.
    - The x and y axes always cross at zero. Where they cross is called the **origin**.

These definitions will make a lot more sense when we look at a coordinate graph more closely.

Here is a set of axes.



A good way to remember that the y-axis is the vertical axis is to notice that a capital "Y" has a vertical line in it.



So how do we use the axes to graph information? Let's start by learning how to graph the most basic geometric figure – a point.

Remember, the coordinate graph is made of a horizontal axis, going from left to right, and vertical axis, going up and down. It makes sense to have information that tells how far to the left or right, and how far up or down we need to travel. This information is given by an **ordered pair** of numbers.

- An **ordered pair** is a group of two numbers, shown inside parentheses "( )" and separated by a comma ",".
  (3,2) is an ordered pair, and so is (-5,0).

Ordered pairs have their name for two reasons. The most obvious reason is that there are <u>two</u> numbers in every ordered <u>pair</u>. The second, less obvious reason, is that the <u>order</u> in which the numbers are written is important. For example, (3,2) is a different ordered pair from (2,3). In other words,

$$(3, 2) \neq (2, 3).$$

Ordered pairs show up a lot in math, but when they are used to make graphs, they mean

$$(how\ far\ left\ or\ right,\ how\ far\ up\ or\ down)$$

$$(x, y)$$

Ordered pairs are the **coordinates** of a point.

- The **coordinates** of a point on a graph are the <u>directions</u> to get to that point. They tell you how to locate it.

Let's graph a point to show what we mean.

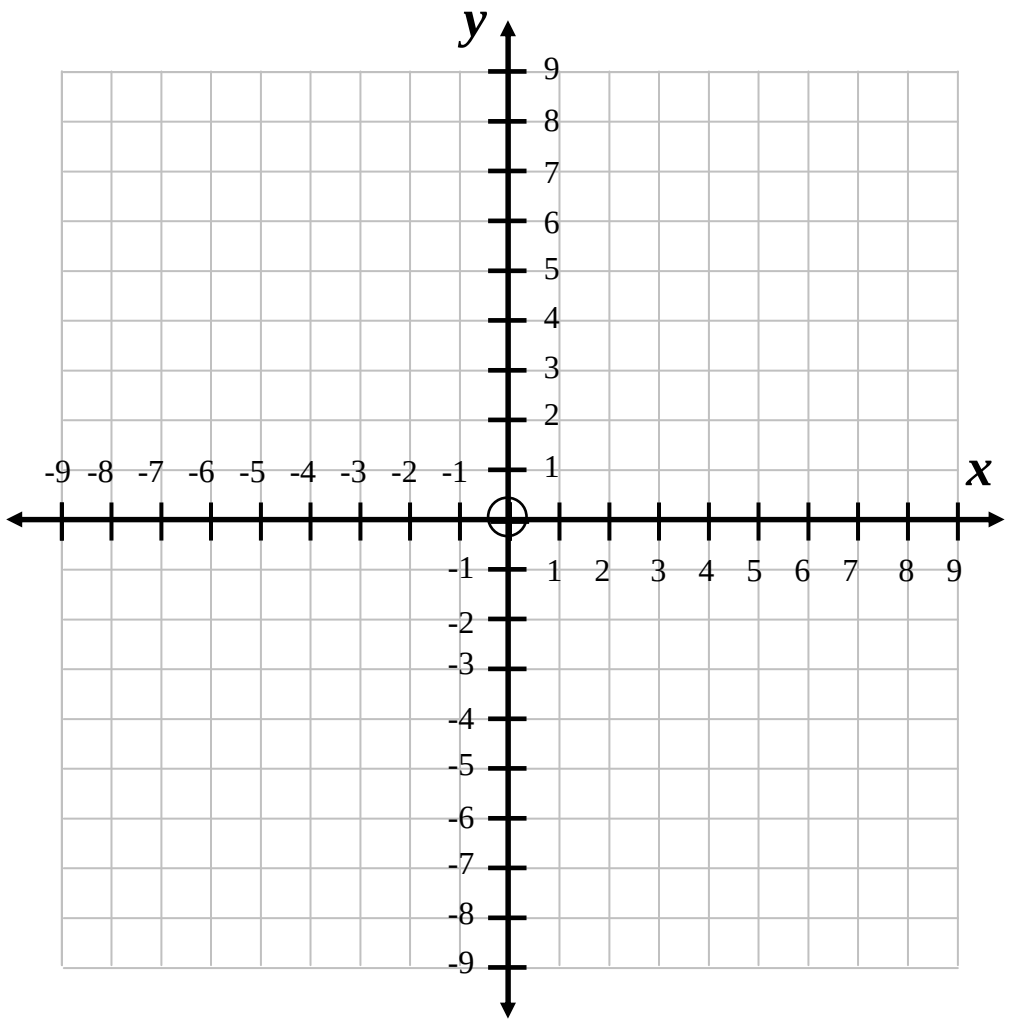
Example

Graph a point with the coordinates, (4, 8)

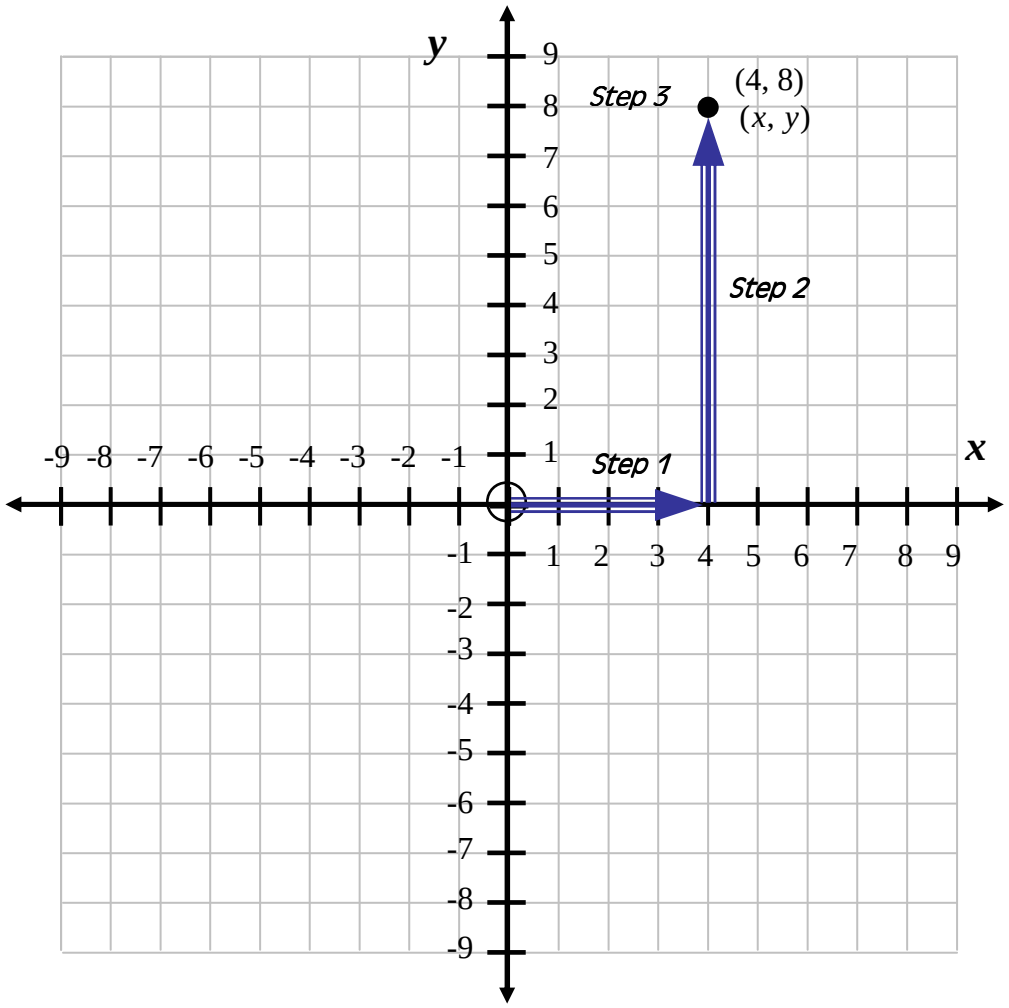
Solution

In order to graph anything, we must have some axes to graph on, so we will start with that.

We will also use grid lines to make it easier to graph correctly.



Now, we must plot the coordinates, (4, 8). From our definition, we know that we must start at zero and move 4 units in the positive direction (to the right) on the x-axis (Step 1). Then, from there, we will move 8 units in the positive direction (up) along the y-axis (Step 2). Lastly, place a point where we end up (Step 3). Observe:



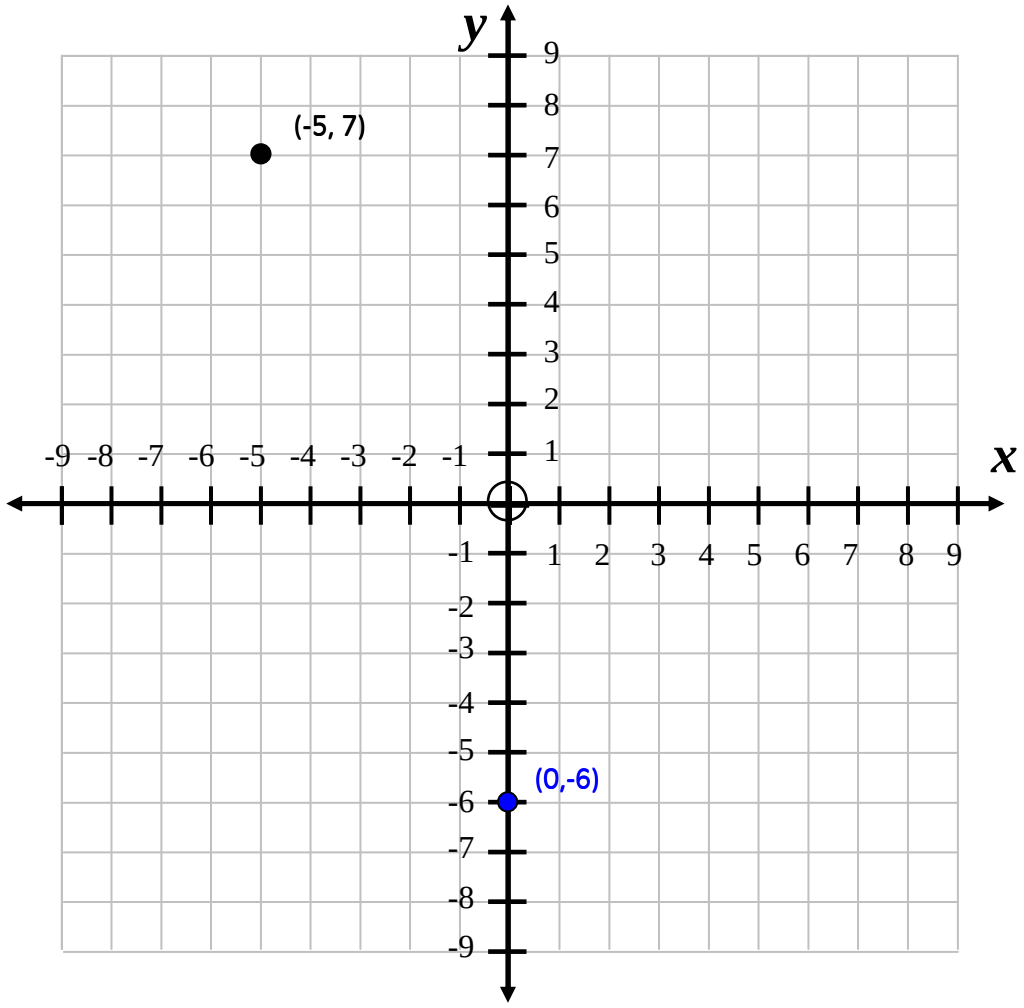
- Because the first number in the coordinate pair tells us which way to move on the x-axis, the first number is called the **x-coordinate**. For the same reason, since the second number in the coordinate pair tells us which way to move on the y-axis, the second number is called the **y-coordinate**.

Example

Graph a point at the coordinate (-5, 7).

Solution

First, we must move –5 units on the x-axis. Count 5 units in the negative direction (to the left) from zero. Then, we move 7 units in the positive direction (up) on the y-axis.

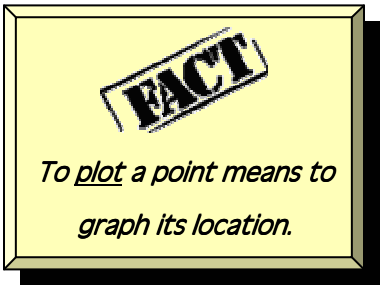


Example

Using the grid above, plot the point (0,-6)

Solution

The *x*-coordinate is 0. We move neither left nor right on the *x*-axis. The *y*-coordinate is -6. We must move 6 units in the negative direction (down).



1) On the grid, construct and label the x and y axes with the appropriate scale.

On the axes you made in question 1), graph or plot the following points, and label each with their coordinates.

2) (1, 5)

3) (-2, 4)

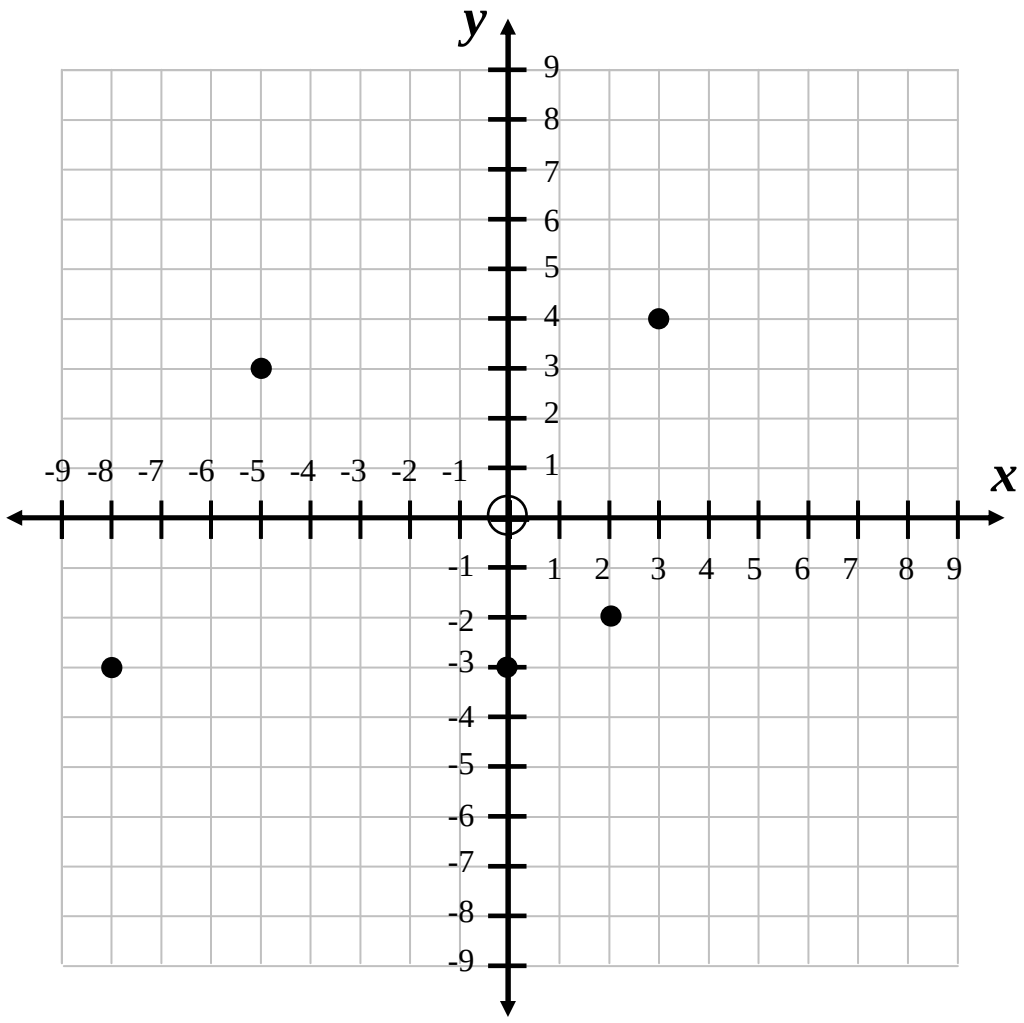
4) (-2, -7)

5) (0, 5)

6) (5, 0)

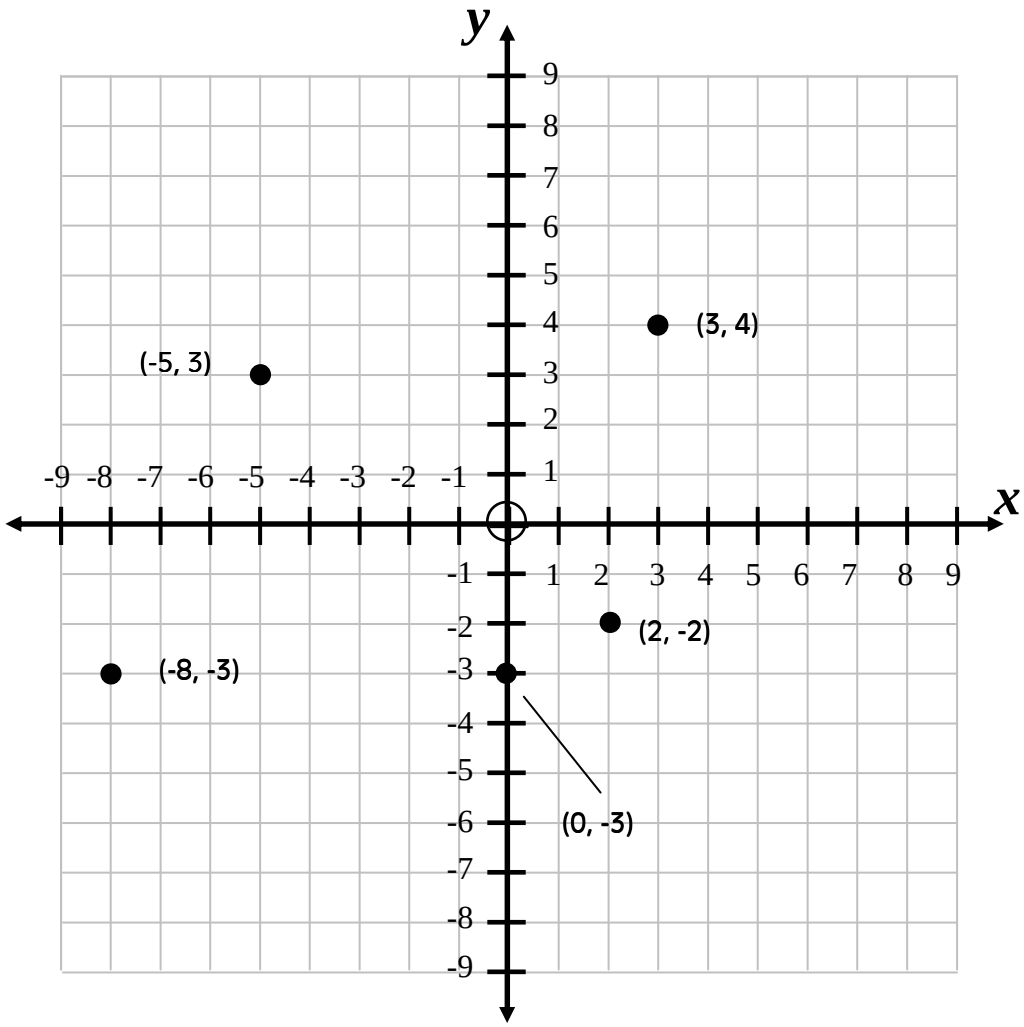
Example

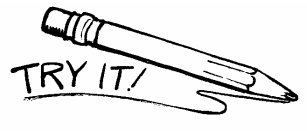
Identify the coordinates of each point on the axes.



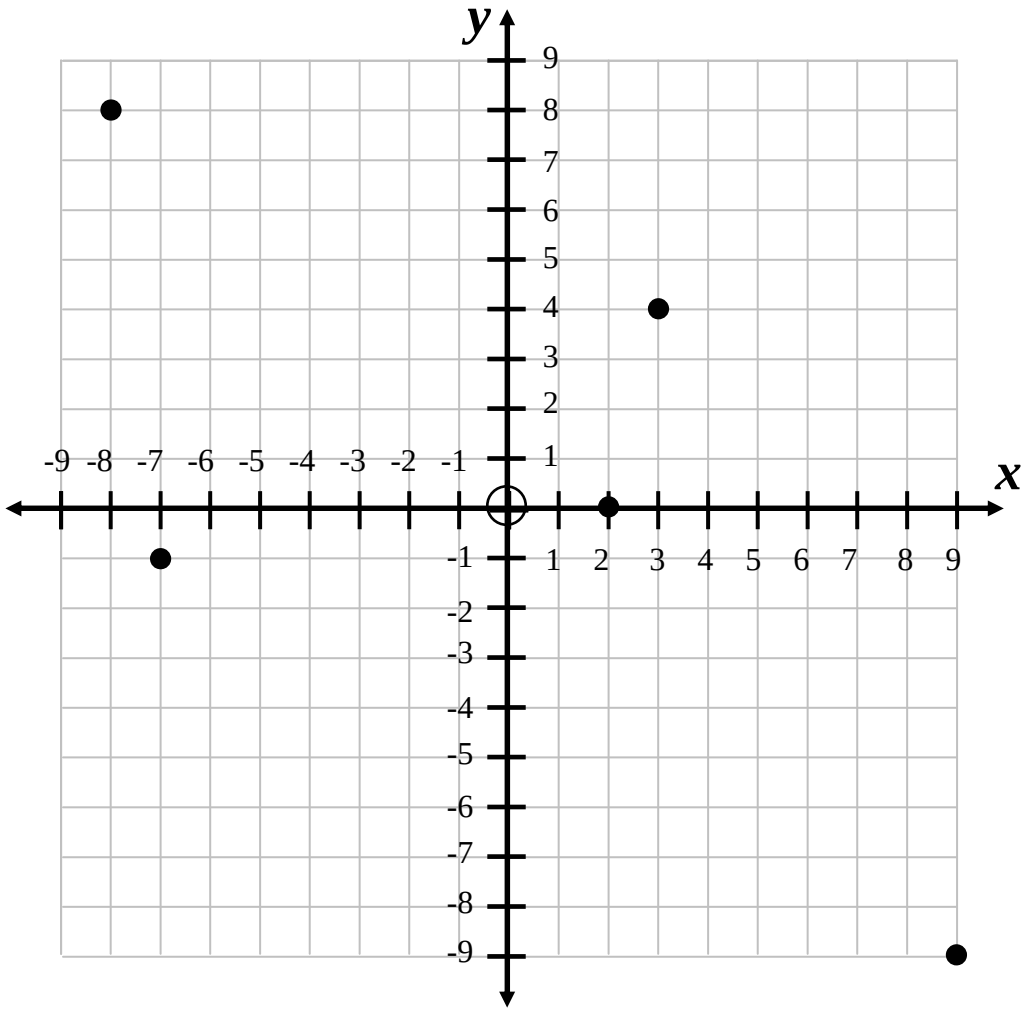
Solution

For each point, we need to ask ourselves two questions. How many units to the left or right of zero is the point on the *x*-axis? How many units up or down from zero is the point on the y-axis? We see on the next page the coordinates are...


y
x
(3, 4)
(-5, 3)
(2, -2)
(-8, -3)
(0, -3)



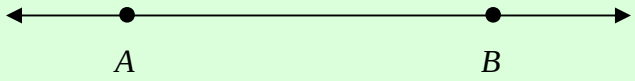
7. Label each point's coordinates.



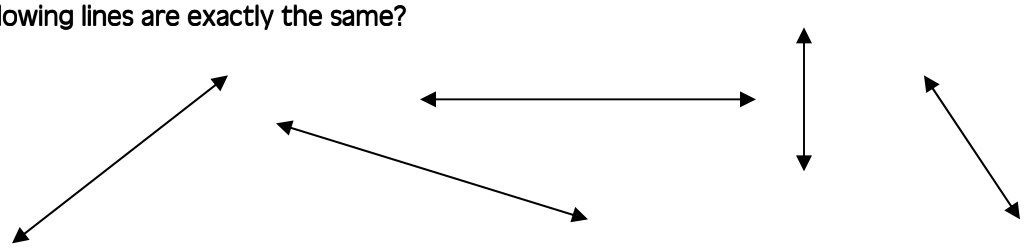
As you can see, we can use a coordinate graph to turn a geometric figure into numbers. For instance, we just turned the graphs of points into ordered pairs of numbers. We can also use the graph of a line to gather information.

To refresh your memory, here is our previous definition of a **line**.

- A **line** goes straight through two points in opposite directions and never ends.

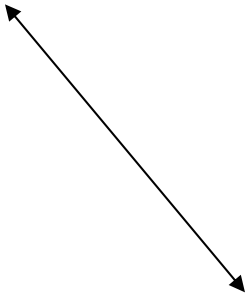


This definition is true, but it does not tell the whole story. For instance, would you say that all of the following lines are exactly the same?



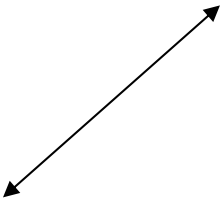
Imagine you are sledding down a hill. Wouldn't your sledding experience be different sledding down a hill such as this:

compared to one like this?

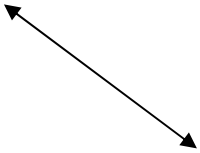


All of these lines have different levels of steepness. Some start high and end low. Some are flat. Some are straight up.

- The steepness of a line, segment, or ray is called its **slope**. *"m"* is the letter most commonly used to represent slope



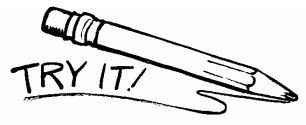
Lines, segments, or rays that rise from left to right, have a positive slope.



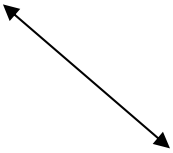
Lines, segments, or rays that go down from left to right, have a negative slope.

A horizontal line, segment, or ray doesn't go up or down from left to right. Because of this, it has a zero slope.

Vertical lines, segments, and rays have an undefined slope.



Classify the slopes of each figure as either positive (+), negative (-), zero (0), or undefined (U).

8. 

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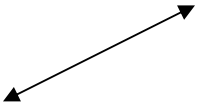
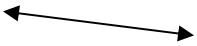
10. 

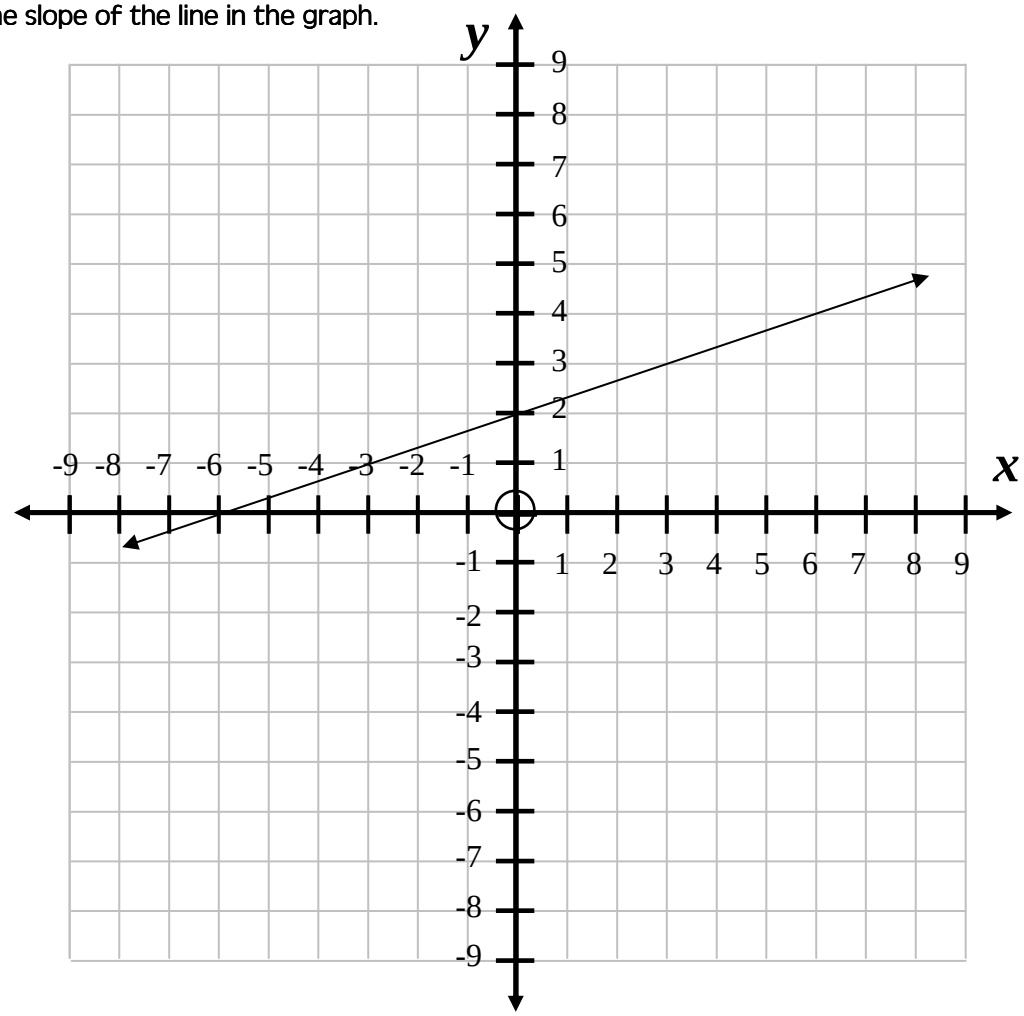
11. 

12. 

13. 

Example

Find the slope of the line in the graph.



Solution

Slope is a measure of steepness. More specifically, it is the ratio of vertical distance traveled to horizontal distance traveled. In other words,

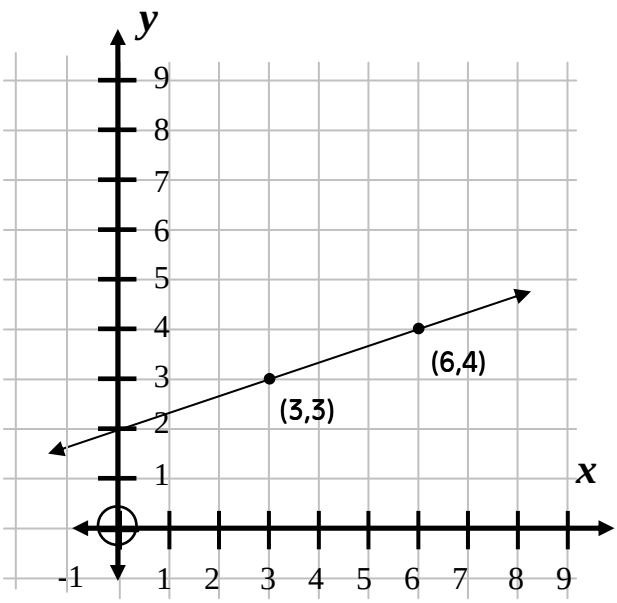
$$m = \frac{\text{vertical distance traveled}}{\text{horizontal distance traveled}} = \frac{rise}{run}$$

Remember: The letter "*m*" is commonly used to denote slope.

We abbreviate the vertical distance with "rise" and the horizontal distance with "run."

Let's apply this on our graph.

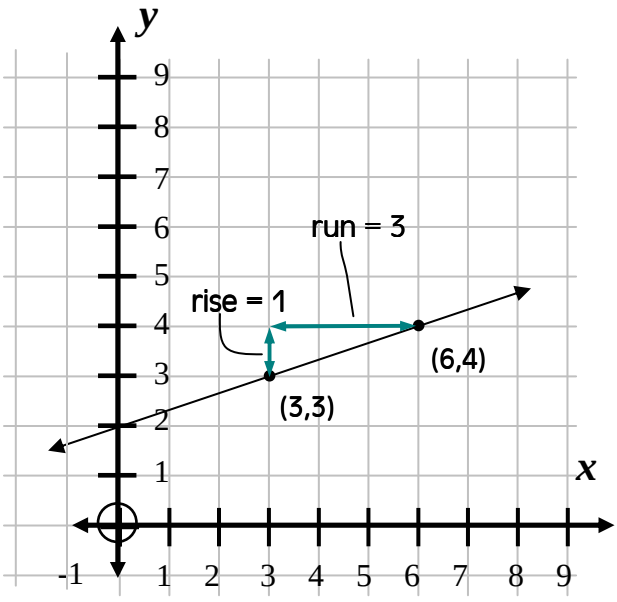
To save space on the paper, a portion of the graph has been chopped off, but it will not change the way we solve these problems.



Step 1: Look at the direction of the slope. Will it be positive, negative, have no slope, or be undefined?

This will be a positive slope because it slants up.

Step 2: Identify the coordinates of any two points the line passes through.
In this case, we select (3,3) and (6,4).



Step 3: Figure out the rise by starting with the leftmost point and counting up to the line on which the second point lies. That is, count up one unit, from 3 to 4. Figure out the run by starting with the same point. Count how many units to the right the second point is. In this case, count to the right three units, from 3 to 6.

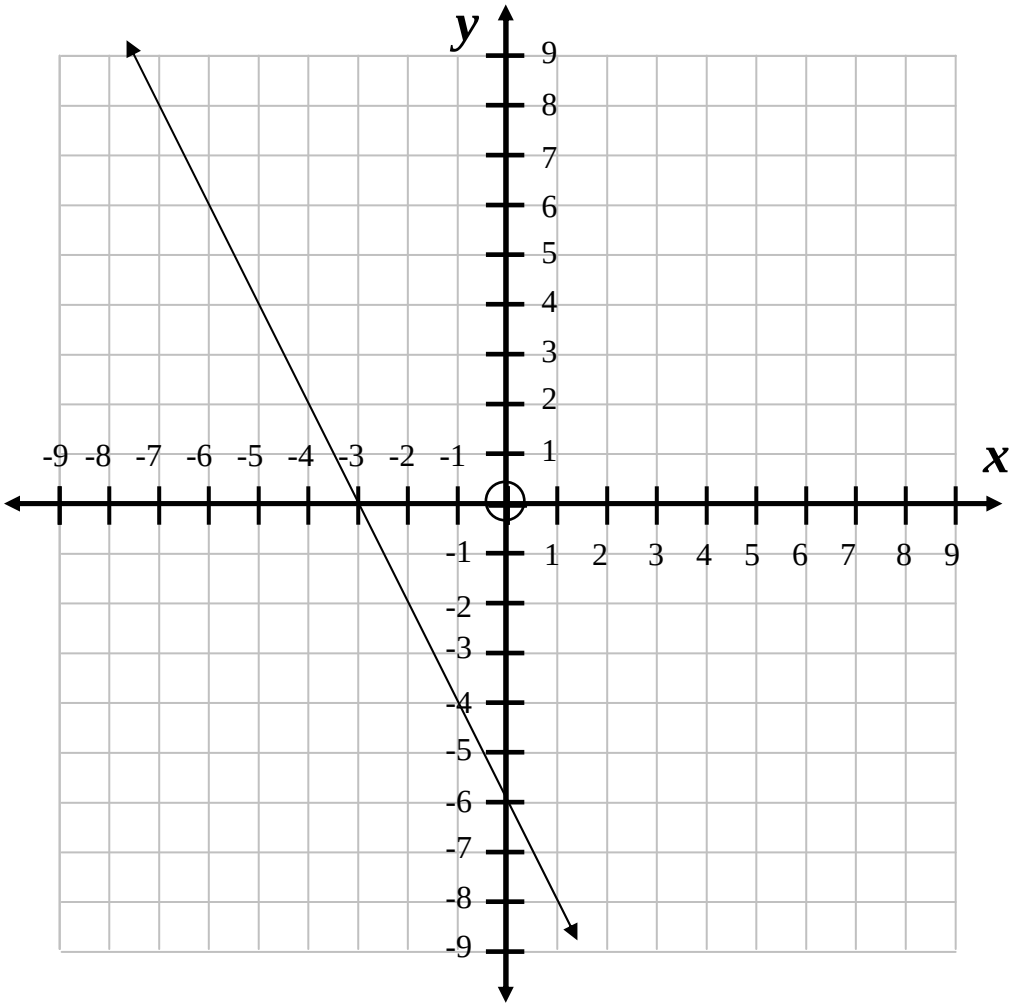
Step 4: Plug these values into the formula for slope.

$$m = \left(\frac{rise}{run}\right)$$

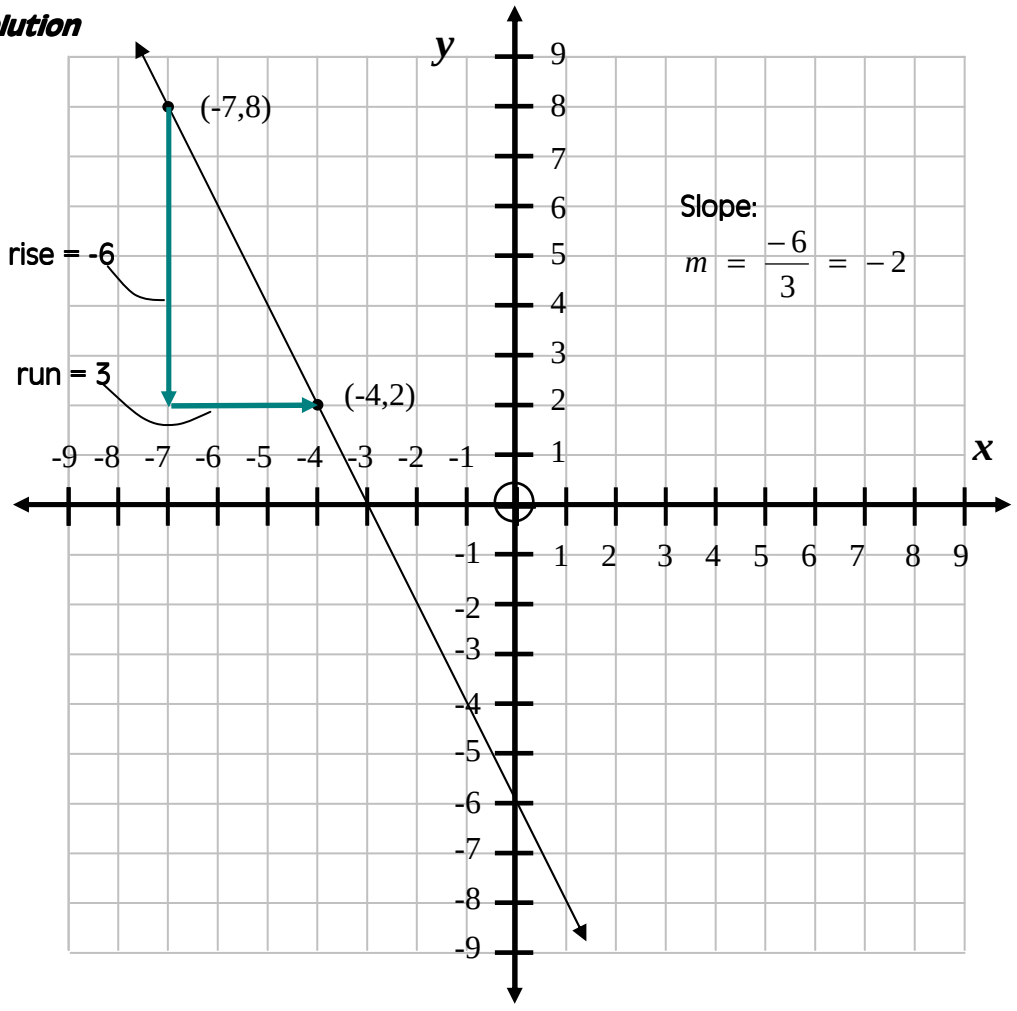
$$m = \frac{1}{3}$$

Example

Find the slope of the following line.



Solution

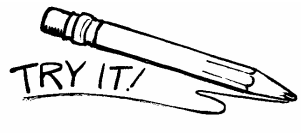


We can see that this will be a negative slope, because the line slants downward, left to right. We picked two points on the line: (-7,8) and (-4,2).

We find the rise by starting with the leftmost point and counting down to the second point. In this case, we count down 6 units, so the rise is -6.

Next, we find the run by counting how many units to the right the second point is from the first. Because we count to the right 3 units, the run is +3.

Now, putting these values into the slope formula, we see $m = \frac{-6}{3}$. Even though we are talking about slope, we have a fraction that can be reduced. So, $m = \frac{-6}{3} = -2$.



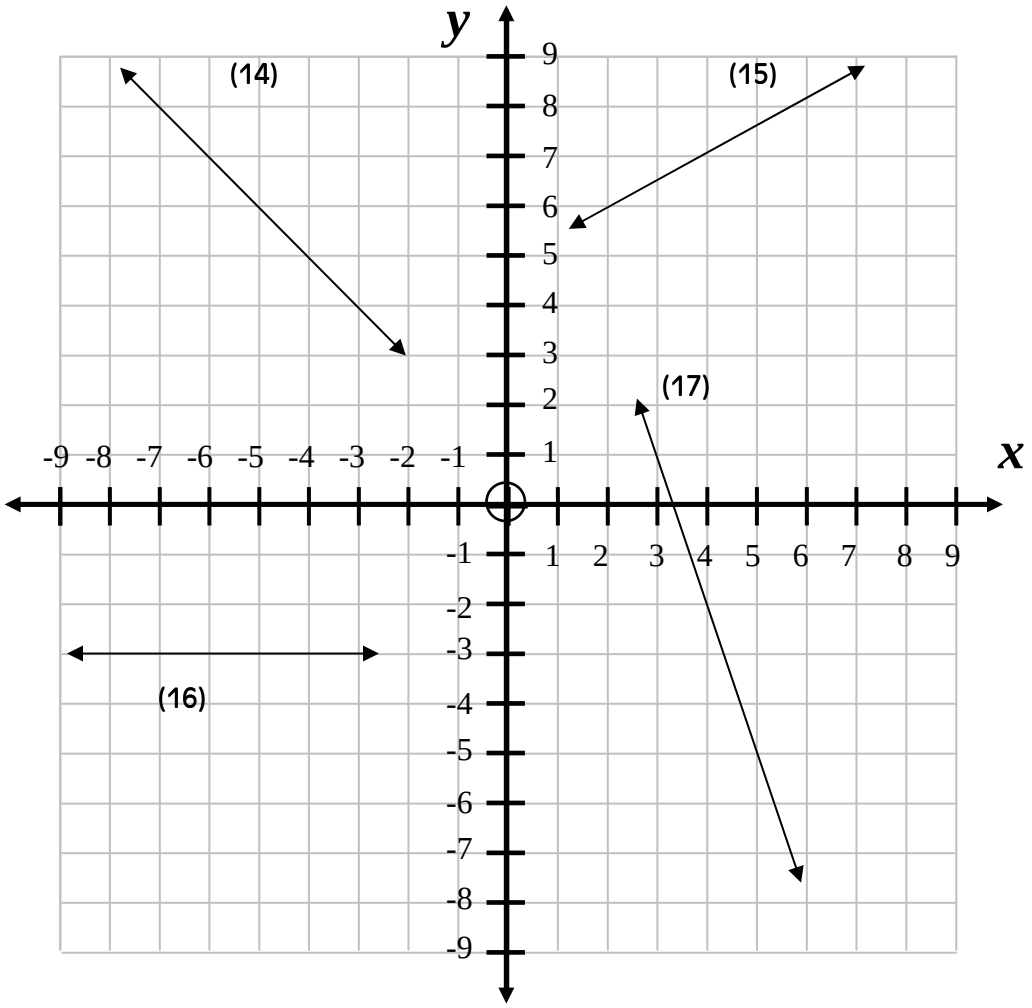
Find the slope of each line.

14.

15.

16.

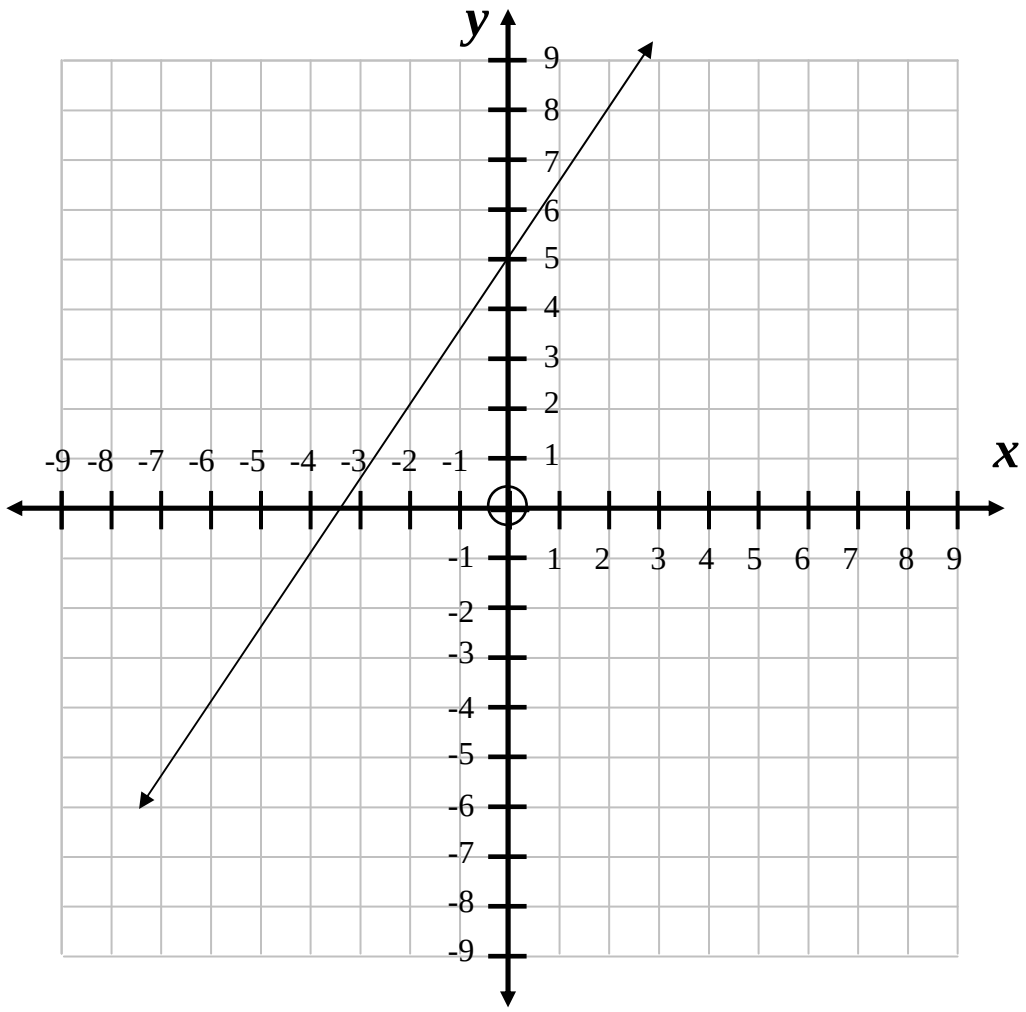
17.



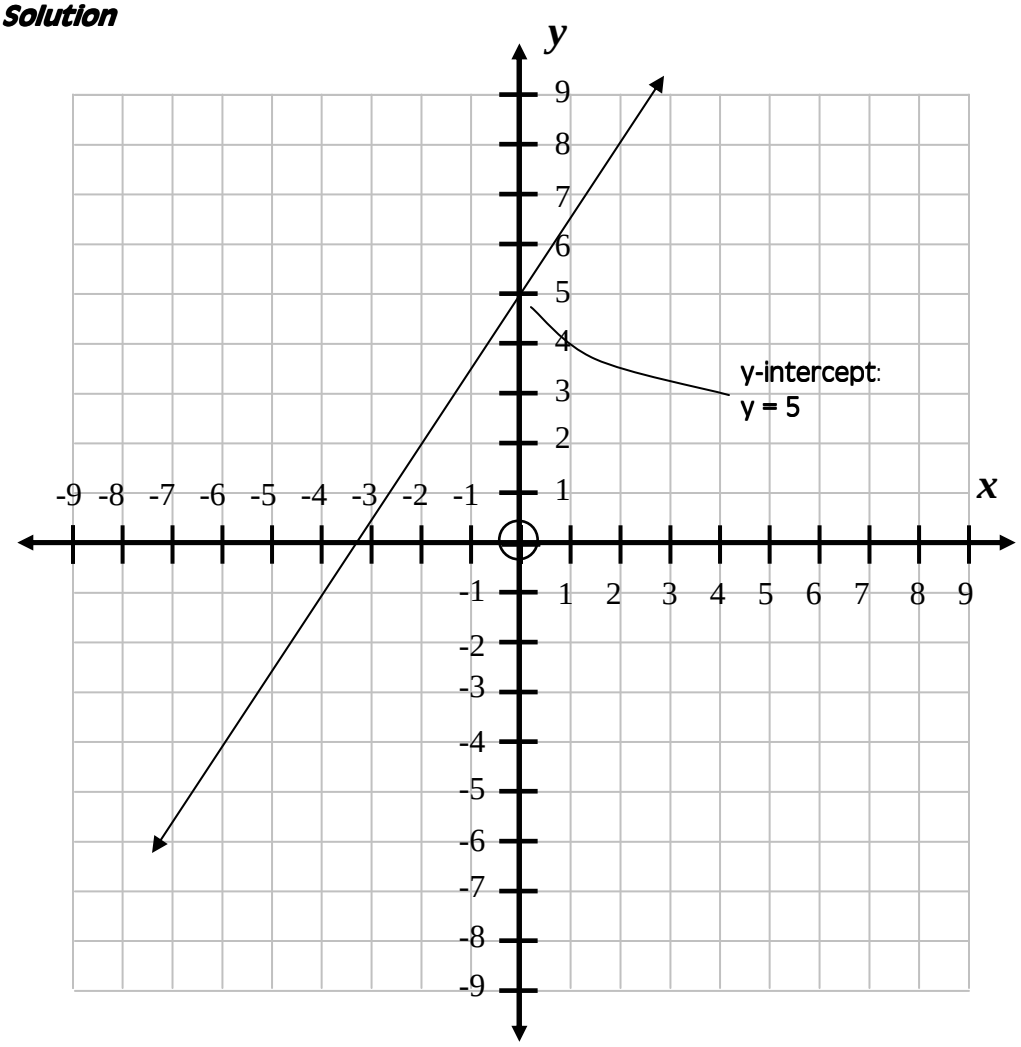
- The **y-intercept** of a line is the point where it passes through the *y*-axis.

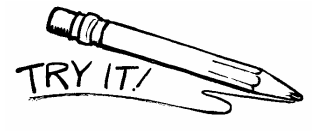
Example

Find the y-intercept of the following line.



Solution





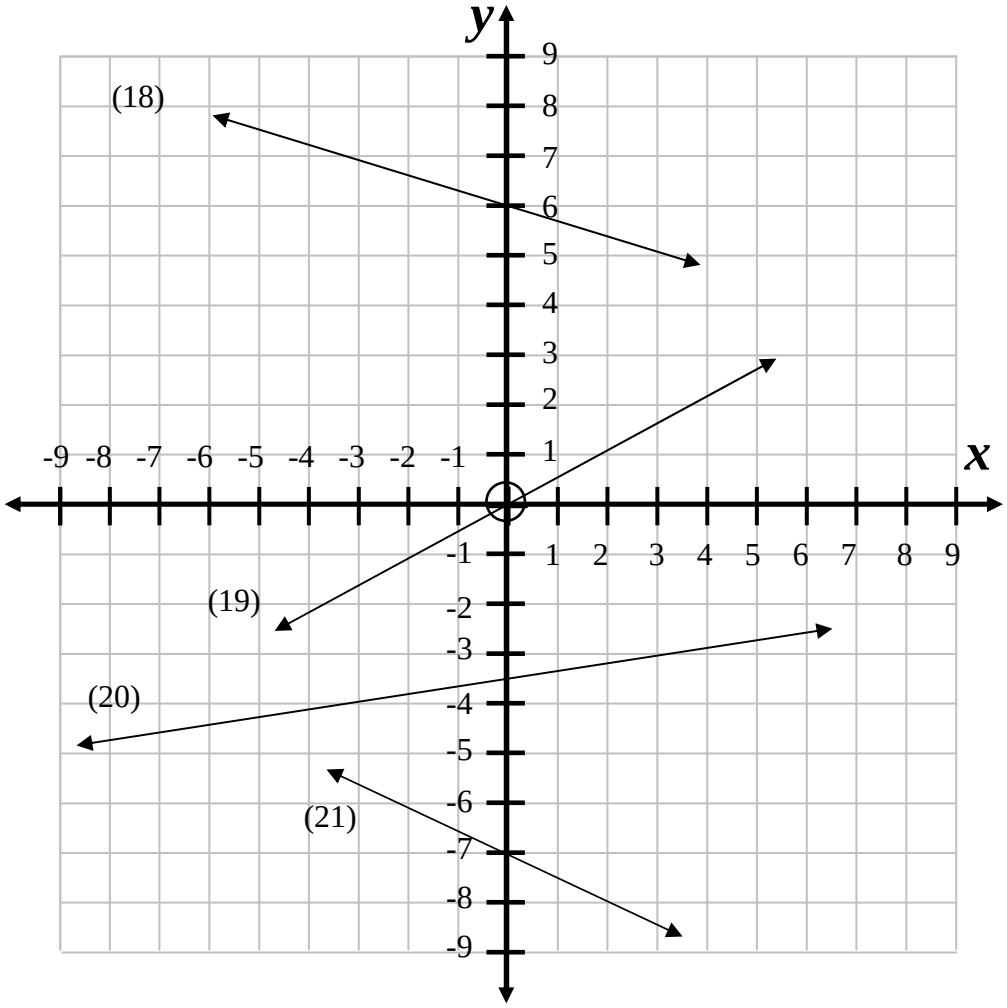
Find the y-intercept of each line.

18.

19.

20.

21.



☑ Review

1. Highlight the following definitions:
    a. coordinate graph
    b. x-axis
    c. y-axis
    d. origin
    e. ordered pair
    f. coordinates
    g. x-coordinate
    h. y-coordinate
    i. line
    j. slope
    k. y-intercept

2. Highlight the Objectives.

3. Write one question you would like to ask your mentor, or one new thing you learned in this lesson.

Practice Problems

Math On the Move Lesson 23

Directions: Write your answers in your math journal. Label this exercise Math On the Move – Lesson 23, Set A and Set B.

Set A

1. Plot and label the following coordinates.

   a. (1, 1) b. (6, 3) c. (-2, 5) d. (9, -8)

   e. (-4, -9) f. (0, -3) g. (5, 0) h. (-7, 0)

   i. (0, 0)

2. Find the slope and y-intercept of each line.

a) slope =

y-intercept =

b) slope =

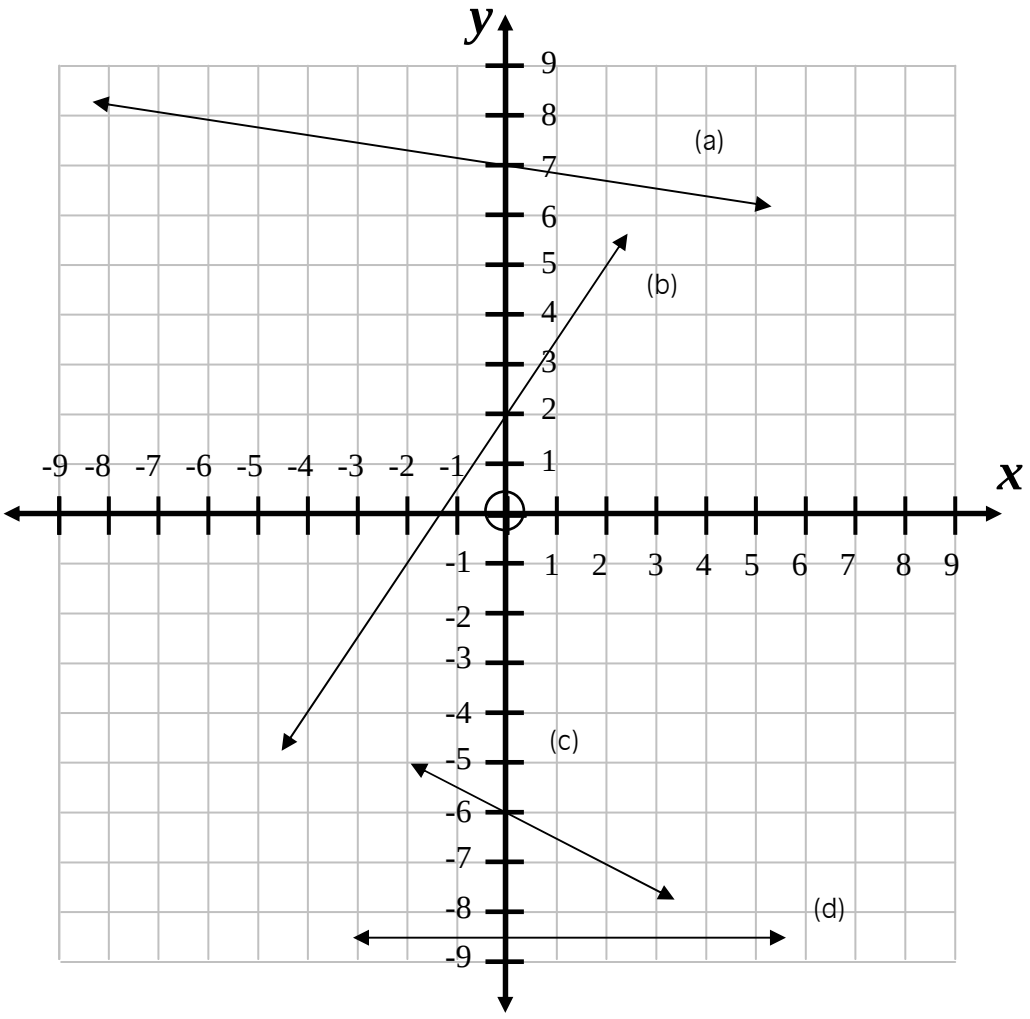
y-intercept =

c) slope =

y-intercept =

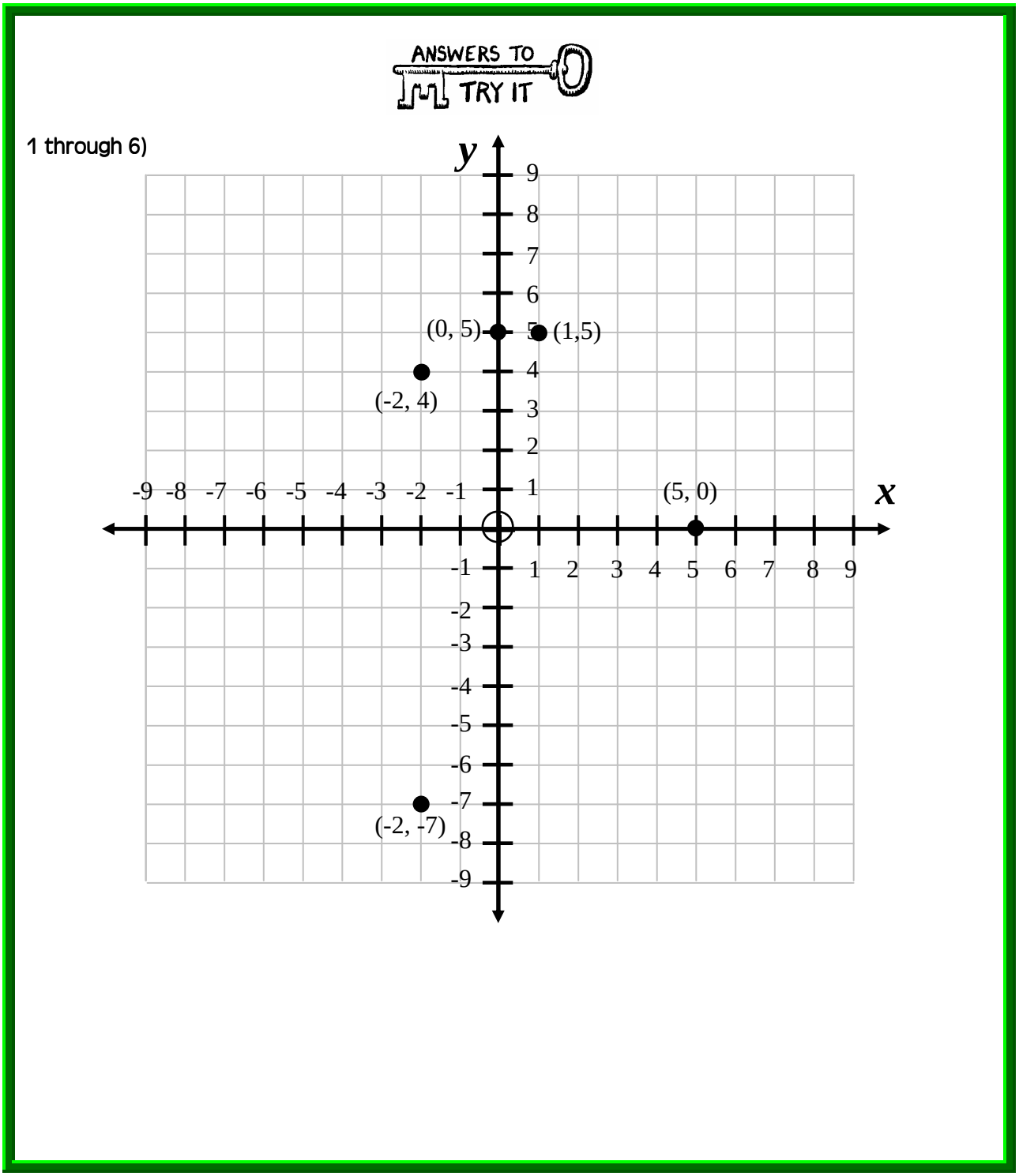
d) slope =

y-intercept =

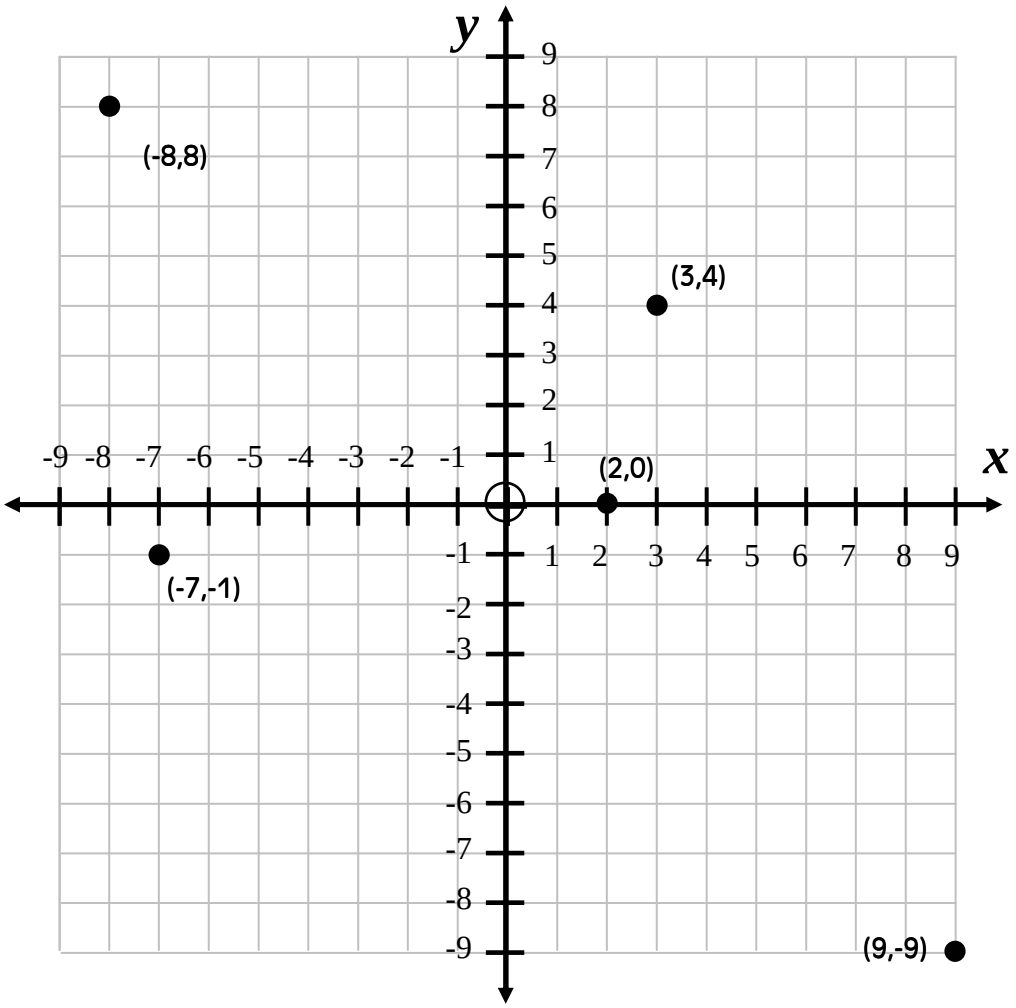


Set B

1. Given the definition of slope as rise over *run*, explain, in your own words, why a vertical line has an undefined slope.



7)



8) - 9) 0 10) + 11) - 12) + 13) U

14) -1 15) $\frac{1}{2}$ 16) 0 17) -3 18) 6 19) 0

20) -3.5 21) -7

End of Lesson 23